History of Modern Mathematics

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ARTICLE TWO: THEORY OF NUMBERS

The Theory of Numbers,[1] a favorite study among the Greeks, had its renaissance in the sixteenth and seventeenth centuries in the labors of Viete, Bachet de Meziriac, and especially Fermat. In the eighteenth century Euler and Lagrange contributed to the theory, and at its close the sub ject began to take scientific form through the great labors of Legendre (1798), and Gauss (1801). With the latter's Disquisitiones Arithmeticæ(1801) may be said to begin the modern theory of numbers. This theory separates into two branches, the one dealing with integers, and concerning itself especially with (1) the study of primes, of congruences, and of residues, and in particular with the law of reciprocity, and (2) the theory of forms, and the other dealing with complex numbers.

The Theory of Primes[2] has attracted many investigators during the nineteenth century, but the results have been detailed rather than general. Tchébichef (1850) was the first to reach any valuable conclusions in the way of ascertaining the number of primes between two given limits. Riemann (1859) also gave a well-known formula for the limit of the number of primes not exceeding a given number.

The Theory of Congruences may be said to start with Gauss's Disquisitiones. He introduced the symbolism a □ b (mod c), and explored most of the field. Tchébichef published in 1847 a work in Russian upon the subject, and in France Serret has done much to make the theory known.

Besides summarizing the labors of his predecessors in the theory of numbers, and adding many original and noteworthy contributions, to Legendre may be assigned the fundamental theorem which bears his name, the Law of Reciprocity of Quadratic Residues. This law, discovered by induction and enunciated by Euler, was first proved by Legendre in his Théorie des Nombres (1798) for special cases. Independently of Euler and Legendre, Gauss discovered the law about 1795, and was the first to give a general proof. To the subject have also contributed Cauchy, perhaps the most versatile of French mathematicians of the century; Dirichlet, whose Vorlesungen über Zahlentheorie, edited by Dedekind, is a classic; Jacobi, who introduced the generalized symbol which bears his name; Liouville, Zeller, Eisenstein, Kummer, and Kronecker.

The theory has been extended to include cubic and biquadratic reciprocity, notably by Gauss, by Jacobi, who first proved the law of cubic reciprocity, and by Kummer.

To Gauss is also due the representation of numbers by binary quadratic forms. Cauchy, Poinsot (1845), Lebesque (1859, 1868), and notably Hermite have added to the subject. In the theory of ternary forms Eisenstein has been a leader, and to him and H. J. S. Smith is also due a noteworthy advance in the theory of forms in general. Smith gave a complete classification of ternary quadratic forms, and extended Gauss's researches concerning real quadratic forms to complex forms. The investigations concerning the representation of numbers by the sum of 4, 5, 6, 7, 8 squares were advanced by Eisenstein and the theory was completed by Smith.

In Germany, Dirichlet was one of the most zealous workers in the theory of numbers, and was the first to lecture upon the subject in a German university. Among his contributions is the extension of Fermat's theorem on xn + yn = zn , which Euler and Legendre had proved for n = 3, 4, Dirichlet showing that x5 + y5 ≠ az5. Among the later French writers are Borel; Poincaré, whose memoirs are numerous and valuable; Tannery, and Stieltjes. Among the leading contributors in Germany are Kronecker, Kummer, Schering, Bachmann, and Dedekind. In Austria Stolz's Vorlesungen über allgemeine Arithmetik (188586), and in England Mathews' Theory of Numbers (Part I, 1892) are among the most scholarly of general works. Genocchi, Sylvester, and J. W. L. Glaisher have also added to the theory.

1 Cantor, M., Geschichte der Mathematik, Vol. III, p. 94; Smith, H. J. S., Report on the theory of numbers; Collected Papers, Vol. I; Stolz, O., Grössen und Zahien, Leipzig. 1891.

2 Brocard, H., Sur la fréquence et la totalité des nombres premiers; Nouvelle Correspondence de Mathématiques, Vols. V and VI; gives recent history to 1879.