ARTICLE 17: ELEMENTARY GEOMETRY

Trigonometry and Elementary Geometry have also been affected by the general mathematical spirit of the century. In trigonometry the general substitution of ratios for lines in the definitions of functions has simplified the treatment, and certain formulas have been improved and others added.1 The convergence of trigonometric series, the introduction of the Fourier series, and the free use of the imaginary have already been mentioned. The definition of the sine and cosine by series, and the systematic development of the theory on this basis, have been set forth by Cauchy (1821), Lobachevsky (1833), and others. The hyperbolic trigonometry,2 already founded by Mayer and Lambert, has been popularized and further developed by Gudermann (1830), Hoüel, and Laisant (1871), and projective formulas and generalized figures have been introduced, notably by Gudermann, Möbius, Poncelet, and Steiner. Recently Study has investigated the formulas of spherical trigonometry from the standpoint of the modern theory of functions and theory of groups, and Macfarlane has generalized the fundamental theorem of trigonometry for three-dimensional space.

Elementary Geometry has been even more affected. Among the many contributions to the theory may be mentioned the following: That of Möbius on the opposite senses of lines, angles, surfaces, and solids; the principle of duality as given by Gergonne and Poncelet; the contributions of De Morgan to the logic of the subject; the theory of transversals as worked out by Monge, Brianchon, Servois, Carnot, Chasles, and others; the theory of the radical axis, a property discovered by the Arabs, but introduced as a definite concept by Gaultier (1813) and used by Steiner under the name of "line of equal power"; the researches of Gauss concerning inscriptible polygons, adding the 17and 257-gon to the list below the 1000-gon; the theory of stellar polyhedra as worked out by Cauchy, Jacobi, Bertrand, Cayley, Möbius, Wiener, Hess, Hersel, and others, so that a whole series of bodies have been added to the four Kepler-Poinsot regular solids; and the researches of Muir on stellar polygons. These and many other improvements now find more or less place in the text-books of the day.

To these must be added the recent Geometry of the Triangle, now a prominent chapter in elementary mathematics. Crelle (1816) made some investigations in this

line, Feuerbach (1822) soon after discovered the properties of the Nine-Point Circle, and Steiner also came across some of the properties of the triangle, but none of these followed up the investigation. Lemoine[3] (1873) was the first to take up the subject in a systematic way, and he has contributed extensively to its development. His theory of "transformation continue" and his "géométrographie" should also be mentioned. Brocard's contributions to the geometry of the triangle began in 1877. Other prominent writers have been Tucker, Neuberg, Vigarié, Emmerich, M'Cay, Longchamps, and H. M. Taylor. The theory is also greatly indebted to Miller's work in The Educational Times, and to Hoffmann's Zeitschrift.

The study of linkages was opened by Peaucellier (1864), who gave the first theoretically exact method for drawing a straight line. Kempe and Sylvester have elaborated the subject.

In recent years the ancient problems of trisecting an angle, doubling the cube, and squaring the circle have all been settled by the proof of their insolubility through the use of compasses and straight edge.[4]

[1] Todhunter, I., History of certain formulas of spherical trigonometry, Philosophical Magazine, 1873.

[2] Gunther, S., Die Lehre von den gewöhnlichen und verallgemeinerten Hyperbelfunktionen, Halle, 1881; Chrystal, G., Algebra, Vol. II, p. 288.

[3] Smith, D. E., Biography of Lemoine, American Mathematical Monthly, Vol. III, p. 29; Mackay, J. S., various articles on modern geometry in Proceedings Edinburgh Mathematical Society, various years; Vigarié, é., Géométrie du triangle. Articles in recent numbers of Journal de Mathématiques spéciales, Mathesis, and Proceedings of the Association française pour l'avancement des sciences.

[4] Klein, F., Vorträge über ausgewählten Fragen; Rudio, F., Das Problem von der Quadratur des Zirkels. Naturforschende Gesellschaft Vierteljahrschrift, 1890; Archimedes, Huygens, Lambert, Legendre (Leipzig, 1892).