Chapter 7

HENRY JOHN STEPHEN SMITH[1]

(1826-1883)

Henry John Stephen Smith was born in Dublin, Ireland, on November 2, 1826. His father, John Smith, was an Irish barrister, who had graduated at Trinity College, Dublin, and had afterwards studied at the Temple, London, as a pupil of Henry John Stephen, the editor of Blackstone's *Commentaries*; hence the given name of the future mathematician. His mother was Mary Murphy, an accomplished and clever Irishwoman, tall and beautiful. Henry was the youngest of four children, and was but two years old when his father died. His mother would have been left in straitened circumstances had she not been successful in claiming a bequest of £10,000 which had been left to her husband but had been disputed. On receiving this money, she migrated to England, and finally settled in the Isle of Wight.

Henry as a child was sickly and very near-sighted. When four years of age he displayed a genius for mastering languages. His first instructor was his mother, who had an accurate knowledge of the classics. When eleven years of age, he, along with his brother and sisters, was placed in the charge of a private tutor, who was strong in the classics; in one year he read a large portion of the Greek and Latin authors commonly studied. His tutor was impressed with his power of memory, quickness of perception, indefatigable diligence, and intuitive grasp of whatever he studied. In their leisure hours the children would improvise plays from Homer, or Robinson Crusoe; and they also became diligent students of animal and insect life. Next year a new tutor was strong in the mathematics, and with his aid Henry became acquainted with advanced arithmetic, and the elements of algebra and geometry. The year following, Mrs. Smith moved to Oxford, and placed Henry under the care of Rev. Mr. Highton, who was not only a sound scholar, but an exceptionally good mathematician. The year following

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Mr. Highton received a mastership at Rugby with a boardinghouse attached to it (which is important from a financial point of view) and he took Henry Smith with him as his first boarder. Thus at the age of fifteen Henry Smith was launched into the life of the English public school, and Rugby was then under the most famous headmaster of the day, Dr. Arnold. Schoolboy life as it was then at Rugby has been depicted by Hughes in "Tom Brown's Schooldays."

Here he showed great and all-around ability. It became his ambition to crown his school career by carrying off an entrance scholarship at Balliol College, Oxford. But as a sister and brother had already died of consumption, his mother did not allow him to complete his third and final year at Rugby, but took him to Italy, where he continued his reading privately. Notwithstanding this manifest disadvantage, he was able to carry off the coveted scholarship; and at the age of nineteen he began residence as a student of Balliol College. The next long vacation was spent in Italy, and there his health broke down. By the following winter he had not recovered enough to warrant his return to Oxford; instead, he went to Paris, and took several of the courses at the Sorbonne and the Collège de France. These studies abroad had much influence on his future career as a mathematician. Thereafter he resumed his undergraduate studies at Oxford, carried off what is considered the highest classical honor, and in 1849, when 23 years old, finished his undergraduate career with a double-first; that is, in the honors examination for bachelor of arts he took first-class rank in the classics, and also first-class rank in the mathematics.

It is not very pleasant to be a double first, for the outwardly envied and distinguished recipient is apt to find himself in the position of the ass between two equally inviting bundles of hay, unless indeed there is some external attraction superior to both. In the case of Smith, the external attraction was the bar, for which he was in many respects well suited; but the feebleness of his constitution led him to abandon that course. So he had a difficulty in deciding between classics and mathematics, and there is a story to the effect that he finally solved the difficulty by tossing up a penny. He certainly used the expression: but the reasons which determined his choice in favor of mathematics were first, his weak sight, which made thinking preferable to reading, and secondly, the opportunity which presented itself.

At that time Oxford was recovering from the excitement which had been produced by the Tractarian movement, and which had ended in Newman going over to the Church of Rome. But a Parliamentary Commission had been appointed to inquire into the working of the University. The old system of close scholarships and fellowships was doomed, and the close preserves of the Colleges were being either extinguished or thrown open to public competition. Resident professors, married tutors or fellows were almost or quite unknown; the heads of the several colleges, then the governing body of the University, formed a little society by themselves. Balliol College (founded by John Balliol, the unfortunate King of Scotland who was willing to sell its independence) was then the most distinguished for intellectual eminence; the master was singular among his compeers for keeping steadily in view the true aim of a college, and he reformed the abuses of privilege and close endowment as far as he legally could. Smith was elected a fellow with the hope that he would consent to reside, and take the further office of tutor in mathematics, which he did. Soon after he became one of the mathematical tutors of Balliol he was asked by his college to deliver a course of lectures on chemistry. For this purpose he took up the study of chemical analysis, and exhibited skill in manipulation and accuracy in work. He had an idea of seeking numerical relations connecting the atomic weights of the elements, and some mathematical basis for their properties which might enable experiments to be predicted by the operation of the mind.

About this time Whewell, the master of Trinity College, Cambridge, wrote *The Plurality of Worlds*, which was at first published anonymously. Whewell pointed out what he called law of waste traceable in the Divine economy; and his argument was that the other planets were waste effects, the Earth the only oasis in the desert of our system, the only world inhabited by intelligent beings; Sir David Brewster, a Scottish physicist, inventor of the kaleidoscope, wrote a fiery answer entitled "More worlds than one, the creed of the philosopher and the hope of the Christian." In 1855 Smith wrote an essay on this subject for a volume of Oxford and Cambridge Essays in which the fallibility both of men of science and of theologians was impartially exposed. It was his first and only effort at popular writing.

His two earliest mathematical papers were on geometrical subjects, but the third concerned that branch of mathematics in which he won fame—the theory of numbers. How he was led to take up this branch of mathematics is not stated on authority, but it was probably as follows: There was then no school of mathematics at Oxford; the symbolical school was flourishing at Cambridge; and Hamilton was lecturing on Quaternions at Dublin. Smith did not estimate either of these very highly; he had studied at Paris under some of the great French analysts; he had lived much on the Continent, and was familiar with the French, German and Italian languages. As a scholar he was drawn to the masterly disquisitions of Gauss, who had made the theory of numbers a principal subject of research. I may quote here his estimate of Gauss and of his work: "If we except the great name of Newton (and the exception is one which Gauss himself would have been delighted to make) it is probable that no mathematician of any age or country has ever surpassed Gauss in the combination of an abundant fertility of invention with an absolute vigorousness in demonstration, which the ancient Greeks themselves might have envied. It may be admitted, without any disparagement to the eminence of such great mathematicians as Euler and Cauchy that they were so overwhelmed with the exuberant wealth of their own creations, and so fascinated by the interest attaching to the results at which they arrived, that they did not greatly care to expend their time in arranging their ideas in a strictly logical order, or even in establishing by irrefragable proof propositions which they instinctively felt, and could almost see to be true. With Gauss the case was otherwise. It may seem paradoxical, but it is probably nevertheless true that it is precisely the effort after a logical perfection of form which has rendered the writings of Gauss open to the charge of obscurity and unnecessary difficulty. The fact is that there is neither obscurity nor difficulty in his writings, as long as we read them in the submissive spirit in which an intelligent schoolboy is made to read his Euclid. Every assertion that is made is fully proved, and the assertions succeed one another in a perfectly just analogical order; there nothing so far of which we can complain. But when we have finished the perusal, we soon begin to feel that our work is but begun, that we are still standing on the threshold of the temple, and that there is a secret which lies behind the veil and is as yet concealed from us. No vestige appears of the process by which the result itself was obtained, perhaps not even a trace of the considerations which suggested the successive steps of the demonstration. Gauss says more than once that for brevity, he gives only the synthesis, and suppresses the analysis of his propositions. *Pauca sed matura*—few but well-matured—were the words with which he delighted to describe the character which he endeavored to impress upon his mathematical writings. If, on the other hand, we turn to a memoir of Euler's, there is a sort of free and luxuriant gracefulness about the whole performance, which tells of the quiet pleasure which Euler must have taken in each step of his work; but we are conscious nevertheless that we are at an immense distance from the severe grandeur of design which is characteristic of all Gauss's greater efforts."

Following the example of Gauss, he wrote his first paper on the theory of numbers in Latin: "De compositione numerorum primorum formæ $4^n + 1$ ex duobus quadratis." In it he proves in an original manner the theorem of Fermat—"That every prime number of the form $4^n + 1$ (n being an integer number) is the sum of two square numbers." In his second paper he gives an introduction to the theory of numbers. "It is probable that the Pythagorean school was acquainted with the definition and nature of prime numbers; nevertheless the arithmetical books of the elements of Euclid contain the oldest extant investigations respecting them; and, in particular the celebrated yet simple demonstration that the number of the primes is infinite. To Eratosthenes of Alexandria, who is for so many other reasons entitled to a place in the history of the sciences, is attributed the invention of the method by which the primes may successively be determined in order of magnitude. It is termed, after him, 'the sieve of Eratosthenes'; and is essentially a method of exclusion, by which all composite numbers are successively erased from the series of natural numbers, and the primes alone are left remaining. It requires only one kind of arithmetical operation; that is to say, the formation of the successive multiples of given numbers, or in other words, addition only. Indeed it may be said to require no arithmetical operation whatever, for if the natural series of numbers be represented by points set off at equal distances along a line, by using a geometrical compass we can determine without calculation the multiples of any given number. And in fact, it was by a mechanical contrivance of this nature that M. Burckhardt calculated his table of the least divisors of the first three millions of numbers."

In 1857 Mrs. Smith died; as the result of her cares and exertions she had seen her son enter Balliol College as a scholar, graduate a double-first, elected a fellow of his college, appointed tutor in mathematics, and enter on his career as an independent mathematician. The brother and sister that were left arranged to keep house in Oxford, the two spending the terms together, and each being allowed complete liberty of movement during the vacations. Thereafter this was the domestic arrangement in which Smith lived and worked; he never married. As the owner of a house, instead of living in rooms in college he was able to satisfy his fondness for pet animals, and also to extend Irish hospitality to visiting friends under his own roof. He had no household cares to destroy the needed serenity for scientific work, excepting that he was careless in money matters, and trusted more to speculation in mining shares than to economic management of his income. Though addicted to the theory of numbers, he was not in any sense a recluse; on the contrary he entered with zest into every form of social enjoyment in Oxford, from croquet parties and picnics to banquets. He had the rare power of utilizing stray hours of leisure, and it was in such odd times that he accomplished most of his scientific work. After attending a picnic in the afternoon, he could mount to those serene heights in the theory of numbers

> "Where never creeps a cloud or moves a wind,
> Nor ever falls the least white star of snow,
> Nor ever lowest roll of thunder moans,
> Nor sound of human sorrow mounts, to mar
> Their sacred everlasting calm."

Then he could of a sudden come down from these heights to attend a dinner, and could conduct himself there, not as a mathematical genius lost in reverie and pointed out as a poor and eccentric mortal, but on the contrary as a thorough man of the world greatly liked by everybody.

In 1860, when Smith was 34 years old, the Savilian professor of geometry at Oxford died. At that time the English universities were so constituted that the teaching was done by the college tutors. The professors were officers of the University; and before reform set in, they not only did not teach, they did not even reside in Oxford. At the present day the lectures of the University professors are in general attended by only a few advanced students. Henry Smith was the only Oxford candidate; there were other candidates from the outside, among them George Boole, then professor of mathematics at Queens College, Cork. Smith's claims and talents were considered so conspicuous by the electors, that they did not consider any other candidates. He did not resign as tutor at Balliol, but continued to discharge the arduous duties, in order that the income of his Fellowship might be continued. With proper financial sense he might have been spared from labors which militated against the discharge of the higher duties of professor.

His freedom during vacation gave him the opportunity of attending the meetings of the British Association, where he was not only a distinguished savant, but an accomplished member of the social organization known as the Red Lions. In 1858 he was selected by that body to prepare a report upon the Theory of Numbers. It was prepared in five parts, extending over the years 1859-1865. It is neither a history nor a treatise, but something intermediate. The author analyzes with remarkable clearness and order the works of mathematicians for the preceding century upon the theory of congruences, and upon that of binary quadratic forms. He returns to the original sources, indicates the principle and sketches the course of the demonstrations, and states the result, often adding something of his own. The work has been pronounced to be the most complete and elegant monument ever erected to the theory of numbers, and the model of what a scientific report ought to be.

During the preparation of the Report, and as a logical consequence of the researches connected therewith, Smith published several original contributions to the higher arithmetic. Some were in complete form and appeared in the *Philosophical Transactions* of the Royal Society of London; others were incomplete, giving only the results without the extended demonstrations, and appeared in the Proceedings of that Society. One of the latter, entitled "On the orders and genera of quadratic forms containing more than three indeterminates," enunciates certain general principles by means of which he solves a problem proposed by Eisenstein, namely, the decomposition of integer numbers into the sum of five squares; and further, the analogous problem for seven squares. It was also indicated that the four, six, and eight-square theorems of Jacobi, Eisenstein and Lionville were deducible from the principles set forth.

In 1868 he returned to the geometrical researches which had first occupied his attention. For a memoir on "Certain cubic and biquadratic problems" the Royal Academy of Sciences of Berlin awarded him the Steiner prize. On account of his ability as a man of affairs, Smith was in great demand for University and scientific work of the day. He was made Keeper of the University Museum; he accepted the office of Mathematical Examiner to the University of London; he was a member of a Royal Commission appointed to report on Scientific Education; a member of the Commission appointed to reform the University of Oxford; chairman of the committee of scientists who were given charge of the Meteorological Office, etc. It was not till 1873, when offered a Fellowship by Corpus Christi College, that he gave up his tutorial duties at Balliol. The demands of these offices and of social functions upon his time and energy necessarily reduced the total output of mathematical work of the highest order; the results of long research lay buried in note-books, and the necessary time was not found for elaborating them into a form suitable for publication. Like his master, Gauss, he had a high ideal of what a scientific memoir ought to be in logical order, vigor of demonstration and literary execution; and it was to his mathematical friends matter of regret that he did not reserve more of his energy for the work for which he was exceptionally fitted.

He was a brilliant talker and wit. Working in the purely speculative region of the theory of numbers, it was perhaps natural that he should take an anti-utilitarian view of mathematical science, and that he should express it in exaggerated terms as a defiance to the grossly utilitarian views then popular. It is reported that once in a lecture after explaining a new solution of an old problem he said, "It is the peculiar beauty of this method, gentlemen, and one which endears it to the really scientific mind, that under no circumstances can it be of the smallest possible utility." I believe that it was at a banquet of the Red Lions that he proposed the toast "Pure mathematics; may it never be of any use to any one."

I may mention some other specimens of his wit. "You take tea in the morning," was the remark with which he once greeted a friend; "if I did that I should be awake all day." Some one mentioned to him the enigmatical motto of Marischal College, Aberdeen: "They say; what say they; let them say." "Ah," said he, "it expresses the three stages of an undergraduate's career. 'They say'—in his first year he accepts everything he is told as if it were inspired. 'What say they'—in his second year he is skeptical and asks that question. 'Let them say' expresses the attitude of contempt characteristic of his third year." Of a brilliant writer but illogical thinker he said "He is never right and never wrong; he is never to the point." Of Lockyer, the astronomer, who has been for many years the editor of the scientific journal *Nature*, he said, "Lockyer sometimes forgets that he is only the editor, not the author, of Nature." Speaking to a newly elected fellow of his college he advised him in a low whisper to write a little and to save a little, adding "I have done neither."

At the jubilee meeting of the British Association held at York in 1881, Prof. Huxley and Sir John Lubbock (now Lord Avebury) strolled down one afternoon to the Minster, which is considered the finest cathedral in England. At the main door they met Prof. Smith coming out, who made a mock movement of surprise. Huxley said, "You seem surprised to see me here." "Yes," said Smith, "going in, you know; I would not have been surprised to see you on one of the pinnacles." Once I was introduced to him at a garden party, given in the grounds of York Minster. He was a tall man, with sandy hair and beard, decidedly good-looking, with a certain intellectual distinction in his features and expression. He was everywhere and known to everyone, the life and soul of the gathering. He retained to the day of his death the simplicity and high spirits of a boy. Socially he was an embodiment of Irish blarney modified by Oxford dignity.

In 1873 the British Association met at Bradford; at which meeting Maxwell delivered his famous "Discourse on Molecules." At the same meeting Smith was the president of the section of mathematics and physics. He did not take up any technical subject in his address; but confined himself to matters of interest in the exact sciences. He spoke of the connection between mathematics and physics, as evidenced by the dual province of the section. "So intimate is the union between mathematics and physics that probably by far the larger part of the accessions to our mathematical knowledge have been obtained by the efforts of mathematicians to solve the problems set to them by experiment, and to create for each successive class of phenomena a new calculus or a new geometry, as the case might be, which might prove not wholly inadequate to the subtlety of nature. Sometimes indeed the mathematician has been before the physicist, and it has happened that when some great and new question has occurred to the experimenter or the observer, he has found in the armory of the mathematician the weapons which he has needed ready made to his hand. But much oftener the questions proposed by the physicist have transcended the utmost powers of the mathematics of the time, and a fresh mathematical creation has been needed to supply the logical instrument required to interpret the new enigma." As an example of the rule he points out that the experiments of Faraday called forth the mathematical theory of Maxwell; as an example of the exception that the work of Apollonius on the conic sections was ready for Kepler in investigating the orbits of the planets.

At the time of the Bradford meeting, education in the public schools and universities of England was practically confined to the classics and pure mathematics. In his address Smith took up the importance of science as an educational discipline in schools; and the following sentences, falling as they did from a profound scholar, produced a powerful effect: "All knowledge of natural science that is imparted to a boy, is, or may be, useful to him in the business of his after-life; but the claim of natural science to a place in education cannot be rested upon its usefulness only. The great object of education is to expand and to train the mental faculties, and it is because we believe that the study of natural science is eminently fitted to further these two objects that we urge its introduction into school studies. Science expands the minds of the young, because it puts before them great and ennobling objects of contemplation; many of its truths are such as a child can understand, and yet such that while in a measure he understands them, he is made to feel something of the greatness, something of the sublime regularity and something of the impenetrable mystery, of the world in which he is placed. But science also trains the growing faculties, for science proposes to itself truth as its only object, and it presents the most varied, and at the same time the most splendid examples of the different mental processes which lead to the attainment of truth, and which make up what we call reasoning. In science error is always possible, often close at hand; and the constant necessity for being on our guard against it is one important part of the education which science supplies. But in science sophistry is impossible; science knows no love of paradox; science has no skill to make the worse appear the better reason; science visits with a not long deferred exposure all our fondness for preconceived opinions, all our partiality for views which we have ourselves maintained; and thus teaches the two best lessons that can well be taught—on the one hand, the love of truth; and on the other, sobriety and watchfulness in the use of the understanding."

The London Mathematical Society was founded in 1865. By going to the meetings Prof. Smith was induced to prepare for publication a number of papers from the materials of his notebooks. He was for two years president, and at the end of his term delivered an address "On the present state and prospects of some branches of pure mathematics." He began by referring to a charge which had been brought against the Society, that its Proceedings showed a partiality in favor of one or two great branches of mathematical science to the comparative neglect and possible disparagement of others. He replies in the language of a miner. "It may be rejoined with great plausibility that ours is not a blamable partiality, but a well-grounded preference. So great (we might contend) have been the triumphs achieved in recent times by that combination of the newer algebra with the direct contemplation of space which constitutes the modern geometry—so large has been the portion of these triumphs, which is due to the genius of a few great English mathematicians; so vast and so inviting has been the field thus thrown open to research, that we do well to press along towards a country which has, we might say, been 'prospected' for us, and in which we know beforehand we cannot fail to find something that will repay our trouble, rather than adventure ourselves into regions where, soon after the first step, we should have no beaten tracks to guide us to the lucky spots, and in which (at the best) the daily earnings of the treasure-seeker are small, and do not always make a great show, even after long years of work. Such regions, however, there are in the realm of pure mathematics, and it cannot be for the interest of science that they should be altogether neglected by the rising generation of English mathematicians. I propose, therefore, in the first instance to direct your attention to some few of these comparatively neglected spots." Since then quite a number of the neglected spots pointed out have been worked.

In 1878 Oxford friends urged him to come forward as a candidate for the representation in Parliament of the University of Oxford, on the principle that a University constituency ought to have for its representative not a mere party politician, but an academic man well acquainted with the special needs of the University. The main question before the electors was the approval or disapproval of the Jingo war policy of the Conservative Government. Henry Smith had always been a Liberal in politics, university administration, and religion. The voting was influenced mainly by party considerations—Beaconsfield or Gladstone—with the result that Smith was defeated by more than 2 to 1; but he had the satisfaction of knowing that his support came mainly from the resident and working members of the University. He did not expect success and he hardly desired it, but he did not shrink when asked to stand forward as the representative of a principle in which he believed. The election over, he devoted himself with renewed energy to the publication of his mathematical researches. His report on the theory of numbers had ended in elliptic functions; and it was this subject which now engaged his attention.

In February, 1882, he was surprised to see in the *Comptes rendus* that the subject proposed by the Paris Academy of Science for the *Grand prix des sciences mathématiques* was the theory of the decomposition of integer numbers into a sum of five squares; and that the attention of competitors was directed to the results announced without demonstration by Eisenstein, whereas nothing was said about his papers dealing with the same subject in the Proceedings of the Royal Society. He wrote to M. Hermite calling his attention to what he had published; in reply he was assured that the members of the commission did not know of the existence of his papers, and he was advised to complete his demonstrations and submit the memoir according to the rules of the competition. According to the rules each manuscript bears a motto, and the corresponding envelope containing the name of the successful author is opened. There were still three months before the closing of the *concours* (1 June, 1882) and Smith set to work, prepared the memoir and despatched it in time.

Meanwhile a political agitation had grown up in favor of extending the franchise in the county constituencies. In the towns the mechanic had received a vote; but in the counties that power remained with the squire and the farmer; poor Hodge, as he is called, was left out. Henry Smith was not merely a Liberal; he felt a genuine sympathy for the poor of his own land. At a meeting in the

Oxford Town Hall he made a speech in favor of the movement, urging justice to all classes. From that platform he went home to die. When he spoke he was suffering from a cold. The exposure and excitement were followed by congestion of the liver, to which he succumbed on February 9, 1883, in the 57th year of his age.

Two months after his death the Paris Academy made their award. Two of the three memoirs sent in were judged worthy of the prize. When the envelopes were opened, the authors were found to be Prof. Smith and M. Minkowski, a young mathematician of Koenigsberg, Prussia. No notice was taken of Smith's previous publication on the subject, and M. Hermite on being written to, said that he forgot to bring the matter to the notice of the commission. It was admitted that there was considerable similarity in the course of the investigation in the two memoirs. The truth seems to be that M. Minkowski availed himself of whatever had been published on the subject, including Smith's paper, but to work up the memoir from that basis cost Smith himself much intellectual labor, and must have cost Minkowski much more. Minkowski is now the chief living authority in that high region of the theory of numbers. Smith's work remains the monument of one of the greatest British mathematicians of the nineteenth century.