Chapter 10

ISAAC TODHUNTER[1]

(1820-1884)

Isaac Todhunter was born at Rye, Sussex, 23 Nov., 1820. He was the second son of George Todhunter, Congregationalist minister of the place, and of Mary his wife, whose maiden name was Hume, a Scottish surname. The minister died of consumption when Isaac was six years old, and left his family, consisting of wife and four boys, in narrow circumstances. The widow, who was a woman of strength, physically and mentally, moved to the larger town of Hastings in the same county, and opened a school for girls. After some years Isaac was sent to a boys' school in the same town kept by Robert Carr, and subsequently to one newly opened by a Mr. Austin from London; for some years he had been unusually backward in his studies, but under this new teacher he made rapid progress, and his career was then largely determined.

After his school days were over, he became an usher or assistant master with Mr. Austin in a school at Peckham; and contrived to attend at the same time the evening classes at University College, London. There he came under the great educating influence of De Morgan, for whom in after years he always expressed an unbounded admiration; to De Morgan "he owed that interest in the history and bibliography of science, in moral philosophy and logic which determined the course of his riper studies." In 1839 he passed the matriculation examination of the University of London, then a merely examining body, winning the exhibition for mathematics (£30 for two years); in 1842 he passed the B.A. examination carrying off a mathematical scholarship (of £50 for three years); and in 1844 obtained the degree of Master of Arts with the gold medal awarded to the candidate who gained the greatest distinction in that examination.

Sylvester was then professor of natural philosophy in University College, and Todhunter studied under him. The writings of Sir John Herschel also had an influence; for Todhunter wrote as follows (*Conflict of Studies*, p. 66): "Let me at the outset record my opinion of mathematics; I cannot do this better than by adopting the words of Sir J. Herschel, to the influence of which I gratefully

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attribute the direction of my own early studies. He says of Astronomy, 'Admission to its sanctuary can only be gained by one means,—sound and sufficient knowledge of mathematics, the great instrument of all exact inquiry, without which no man can ever make such advances in this or any other of the higher departments of science as can entitle him to form an independent opinion on any subject of discussion within their range.' "

When Todhunter graduated as M.A. he was 24 years of age. Sylvester had gone to Virginia, but De Morgan remained. The latter advised him to go through the regular course at Cambridge; his name was now entered at St. John's College. Being somewhat older, and much more brilliant than the honor men of his year, he was able to devote a great part of his attention to studies beyond those prescribed. Among other subjects he took up Mathematical Electricity. In 1848 he took his B.A. degree as senior wrangler, and also won the first Smith's prize.

While an undergraduate Todhunter lived a very secluded life. He contributed along with his brothers to the support of their mother, and he had neither money nor time to spend on entertainments. The following legend was applied to him, if not recorded of him: "Once on a time, a senior wrangler gave a wine party to celebrate his triumph. Six guests took their seats round the table. Turning the key in the door, he placed one bottle of wine on the table asseverating with unction, 'None of you will leave this room while a single drop remains.' "

At the University of Cambridge there is a foundation which provides for what is called the Burney prize. According to the regulations the prize is to be awarded to a graduate of the University who is not of more than three years' standing from admission to his degree and who shall produce the best English essay "On some moral or metaphysical subject, or on the existence, nature and attributes of God, or on the truth and evidence of the Christian religion." Todhunter in the course of his first postgraduate year submitted an essay on the thesis that "The doctrine of a divine providence is inseparable from the belief in the existence of an absolutely perfect Creator." This essay received the prize, and was printed in 1849.

Todhunter now proceeded to the degree of M.A., and unlike his mathematical instructors in University College, De Morgan and Sylvester, he did not parade his non-conformist principles, but submitted to the regulations with as good grace as possible. He was elected a fellow of his college, but not immediately, probably on account of his being a non-conformist, and appointed lecturer on mathematics therein; he also engaged for some time in work as a private tutor, having for one of his pupils P. G. Tait, and I believe E. J. Routh also.

For a space of 15 years he remained a fellow of St. John's College, residing in it, and taking part in the instruction. He was very successful as a lecturer, and it was not long before he began to publish textbooks on the subjects of his lectures. In 1853 he published a textbook on *Analytical Statics*; in 1855 one on *Plane Coordinate Geometry*; and in 1858 *Examples of Analytical Geometry of Three Dimensions*. His success in these subjects induced him to prepare manuals on elementary mathematics; his *Algebra* appeared in 1858, his *Trigonometry* in 1859, his *Theory of Equations* in 1861, and his *Euclid* in 1862. Some of his

textbooks passed through many editions and have been widely used in Great Britain and North America. Latterly he was appointed principal mathematical lecturer in his college, and he chose to drill the freshmen in Euclid and other elementary mathematics.

Within these years he also labored at some works of a more strictly scientific character. Professor Woodhouse (who was the forerunner of the Analytical Society) had written a history of the calculus of variations, ending with the eighteenth century; this work was much admired for its usefulness by Todhunter, and as he felt a decided taste for the history of mathematics, he formed and carried out the project of continuing the history of that calculus during the nineteenth century. It was the first of the great historical works which has given Todhunter his high place among the mathematicians of the nineteenth century. This history was published in 1861; in 1862 he was elected a Fellow of the Royal Society of London. In 1863 he was a candidate for the Sadlerian professorship of Mathematics, to which Cayley was appointed. Todhunter was not a mere mathematical specialist. He was an excellent linguist; besides being a sound Latin and Greek scholar, he was familiar with French, German, Spanish, Italian and also Russian, Hebrew and Sanskrit. He was likewise well versed in philosophy, and for the two years 1863-5 acted as an Examiner for the Moral Science Tripos, of which the chief founders were himself and Whewell.

By 1864 the financial success of his books was such that he was able to marry, a step which involved the resigning of his fellowship. His wife was a daughter of Captain George Davies of the Royal Navy, afterwards Admiral Davies.

As a fellow and tutor of St. John's College he had lived a very secluded life. His relatives and friends thought he was a confirmed bachelor. He had sometimes hinted that the grapes were sour. For art he had little eye; for music no ear. "He used to say he knew two tunes; one was 'God save the Queen,' the other wasn't. The former he recognized by the people standing up." As owls shun the broad daylight he had shunned the glare of parlors. It was therefore a surprise to his friends and relatives when they were invited to his marriage in 1864. Prof. Mayor records that Todhunter wrote to his fiancée, "You will not forget, I am sure, that I have always been a student, and always shall be; but books shall not come into even distant rivalry with you," and Prof. Mayor insinuated that thus forearmed, he calmly introduced to the inner circle of their honeymoon Hamilton on *Quaternions*.

It was now (1865) that the London Mathematical Society was organized under the guidance of De Morgan, and Todhunter became a member in the first year of its existence. The same year he discharged the very onerous duties of examiner for the mathematical tripos—a task requiring so much labor and involving so much interference with his work as an author that he never accepted it again. Now (1865) appeared his *History of the Mathematical Theory of Probability*, and the same year he was able to edit a new edition of Boole's *Treatise on Differential Equations*, the author having succumbed to an untimely death. Todhunter certainly had a high appreciation of Boole, which he shared in common with De Morgan. The work involved in editing the successive editions of his elementary books was great; he did not proceed to stereotype until many inde-

pendent editions gave ample opportunity to correct all errors and misprints. He now added two more textbooks; *Mechanics* in 1867 and *Mensuration* in 1869.

About 1847 the members of St. John's College founded a prize in honor of their distinguished fellow, J. C. Adams. It is awarded every two years, and is in value about £225. In 1869 the subject proposed was "A determination of the circumstances under which Discontinuity of any kind presents itself in the solution of a problem of maximum or minimum in the Calculus of Variations." There had been a controversy a few years previous on this subject in the pages of *Philosophical Magazine* and Todhunter had there advocated his view of the matter. This view is found in the opening sentences of his essay: "We shall find that, generally speaking, discontinuity is introduced, by virtue of some restriction which we impose, either explicitly or implicitly in the statement of the problems which we propose to solve." This thesis he supported by considering in turn the usual applications of the calculus, and pointing out where he considers the discontinuities which occur have been introduced into the conditions of the problem. This he successfully proves in many instances. In some cases, the want of a distinct test of what discontinuity is somewhat obscures the argument. To his essay the prize was awarded; it is published under the title "Researches in the Calculus of Variations"—an entirely different work from his *History of the Calculus of Variations*.

In 1873 he published his *History of the Mathematical Theories of Attraction*. It consists of two volumes of nearly 1000 pages altogether and is probably the most elaborate of his histories. In the same year (1873) he published in book form his views on some of the educational questions of the day, under the title of *The Conflict of Studies, and other essays on subjects connected with education*. The collection contains six essays; they were originally written with the view of successive publication in some magazine, but in fact they were published only in book form. In the first essay, that on the Conflict of Studies—Todhunter gave his opinion of the educative value in high schools and colleges of the different kinds of study then commonly advocated in opposition to or in addition to the old subjects of classics and mathematics. He considered that the Experimental Sciences were little suitable, and that for a very English reason, because they could not be examined on adequately. He says:

"Experimental Science viewed in connection with education, rejoices in a name which is unfairly expressive. A real experiment is a very valuable product of the mind, requiring great knowledge to invent it and great ingenuity to carry it out. When Perrier ascended the Puy de Dôme with a barometer in order to test the influence of change of level on the height of the column of mercury, he performed an experiment, the suggestion of which was worthy of the genius of Pascal and Descartes. But when a modern traveller ascends Mont Blanc, and directs one of his guides to carry a barometer, he cannot be said to perform an experiment in any very exact or very meritorious sense of the word. It is a repetition of an observation made thousands of times before, and we can never recover any of the interest which belonged to the first trial, unless indeed, without having ever heard of it, we succeeded in reconstructing the process of ourselves. In fact, almost always he who first plucks an experimental flower thus

appropriates and destroys its fragrance and its beauty."

At the time when Todhunter was writing the above, the Cavendish Laboratory for Experimental Physics was just being built at Cambridge, and Clerk-Maxwell had just been appointed the professor of the new study; from Todhunter's utterance we can see the state of affairs then prevailing. Consider the corresponding experiment of Torricelli, which can be performed inside a classroom; to every fresh student the experiment retains its fragrance; the sight of it, and more especially the performance of it imparts a kind of knowledge which cannot be got from description or testimony; it imparts accurate conceptions and is a necessary preparative for making a new and original experiment. To Todhunter it may be replied that the flowers of Euclid's *Elements* were plucked at least 2000 years ago, yet, he must admit, they still possess, to the fresh student of mathematics, even although he becomes acquainted with them through a textbook, both fragrance and beauty.

Todhunter went on to write another passage which roused the ire of Professor Tait. "To take another example. We assert that if the resistance of the air be withdrawn a sovereign and a feather will fall through equal spaces in equal times. Very great credit is due to the person who first imagined the well-known experiment to illustrate this; but it is not obvious what is the special benefit now gained by seeing a lecturer repeat the process. It may be said that a boy takes more interest in the matter by seeing for himself, or by performing for himself, that is, by working the handle of the air-pump; this we admit, while we continue to doubt the educational value of the transaction. The boy would also probably take much more interest in football than in Latin grammar; but the measure of his interest is not identical with that of the importance of the subjects. It may be said that the fact makes a stronger impression on the boy through the medium of his sight, that he believes it the more confidently. I say that this ought not to be the case. If he does not believe the statements of his tutor—probably a clergyman of mature knowledge, recognized ability and blameless character—his suspicion is irrational, and manifests a want of the power of appreciating evidence, a want fatal to his success in that branch of science which he is supposed to be cultivating."

Clear physical conceptions cannot be got by tradition, even from a clergyman of blameless character; they are best got directly from Nature, and this is recognized by the modern laboratory instruction in physics. Todhunter would reduce science to a matter of authority; and indeed his mathematical manuals are not free from that fault. He deals with the characteristic difficulties of algebra by authority rather than by scientific explanation. Todhunter goes on to say: "Some considerable drawback must be made from the educational value of experiments, so called, on account of their failure. Many persons must have been present at the exhibitions of skilled performers, and have witnessed an uninterrupted series of ignominious reverses,—they have probably longed to imitate the cautious student who watched an eminent astronomer baffled by Foucault's experiment for proving the rotation of the Earth; as the pendulum would move the wrong way the student retired, saying that he wished to retain his faith in the elements of astronomy."

It is not unlikely that the series of ignominious reverses Todhunter had in his view were what he had seen in the physics classroom of University College when the manipulation was in the hands of a pure mathematician—Prof. Sylvester. At the University of Texas there is a fine clear space about 60 feet high inside the building, very suitable for Foucault's experiment. I fixed up a pendulum, using a very heavy ball, and the turning of the Earth could be seen in two successive oscillations. The experiment, although only a repetition according to Todhunter, was a live and inspiring lesson to all who saw it, whether they came with previous knowledge about it or no. The repetition of any such great experiment has an educative value of which Todhunter had no conception.

Another subject which Todhunter discussed in these essays is the suitability of Euclid's *Elements* for use as the elementary textbook of Geometry. His experience as a college tutor for 25 years; his numerous engagements as an examiner in mathematics; his correspondence with teachers in the large schools gave weight to the opinion which he expressed. The question was raised by the first report of the Association for the Improvement of Geometrical Teaching; and the points which Todhunter made were afterwards taken up and presented in his own unique style by Lewis Carroll in "Euclid and his modern rivals." Up to that time Euclid's manual was, and in a very large measure still is, the authorized introduction to geometry; it is not as in this country where there is perfect liberty as to the books and methods to be employed. The great difficulty in the way of liberty in geometrical teaching is the universal tyranny of competitive examinations. Great Britain is an examination-ridden country. Todhunter referred to one of the most distinguished professors of Mathematics in England; one whose pupils had likewise gained a high reputation as investigators and teachers; his "venerated master and friend," Prof. De Morgan; and pointed out that he recommended the study of Euclid with all the authority of his great attainments and experience.

Another argument used by Todhunter was as follows: In America there are the conditions which the Association desires; there is, for example, a textbook which defines parallel lines as those which *have the same direction.* Could the American mathematicians of that day compare with those of England? He answered no.

While Todhunter could point to one master—De Morgan—as in his favor, he was obliged to quote another master—Sylvester—as opposed. In his presidential address before section A of the British Association at Exeter in 1869, Sylvester had said: "I should rejoice to see . . . Euclid honorably shelved or buried 'deeper than did ever plummet sound' out of the schoolboy's reach; morphology introduced into the elements of algebra; projection, correlation, and motion accepted as aids to geometry; the mind of the student quickened and elevated and his faith awakened by early initiation into the ruling ideas of polarity, continuity, infinity, and familiarization with the doctrine of the imaginary and inconceivable." Todhunter replied: "Whatever may have produced the dislike to Euclid in the illustrious mathematician whose words I have quoted, there is no ground for supposing that he would have been better pleased with the substitutes which are now offered and recommended in its place. But the remark which is natu-

rally suggested by the passage is that nothing prevents an enthusiastic teacher from carrying his pupils to any height he pleases in geometry, even if he starts with the use of Euclid."

Todhunter also replied to the adverse opinion, delivered by some professor (doubtless Tait) in an address at Edinburgh which was as follows: "From the majority of the papers in our few mathematical journals, one would almost be led to fancy that British mathematicians have too much pride to use a simple method, while an unnecessarily complex one can be had. No more telling example of this could be wished for than the insane delusion under which they permit 'Euclid' to be employed in our elementary teaching. They seem voluntarily to weight alike themselves and their pupils for the race." To which Todhunter replied: "The British mathematical journals with the titles of which I am acquainted are the Quarterly Journal of Mathematics, the Mathematical Messenger, and the Philosophical Magazine; to which may be added the Proceedings of the Royal Society and the Monthly Notices of the Astronomical Society. I should have thought it would have been an adequate employment, for a person engaged in teaching, to read and master these periodicals regularly; but that a single mathematician should be able to improve more than half the matter which is thus presented to him fills me with amazement. I take down some of these volumes, and turning over the pages I find article after article by Profs. Cayley, Salmon and Sylvester, not to mention many other highly distinguished names. The idea of amending the elaborate essays of these eminent mathematicians seems to me something like the audacity recorded in poetry with which a superhuman hero climbs to the summit of the Indian Olympus and overturns the thrones of Vishnu, Brahma and Siva. While we may regret that such ability should be exerted on the revolutionary side of the question, here is at least one mournful satisfaction: the weapon with which Euclid is assailed was forged by Euclid himself. The justly celebrated professor, from whose address the quotation is taken, was himself trained by those exercises which he now considers worthless; twenty years ago his solutions of mathematical problems were rich with the fragrance of the Greek geometry. I venture to predict that we shall have to wait some time before a pupil will issue from the reformed school, who singlehanded will be able to challenge more than half the mathematicians of England." Professor Tait, in what he said, had, doubtless, reference to the avoidance of the use of the Quaternion method by his contemporaries in mathematics.

More than half of the Essays is taken up with questions connected with competitive examinations. Todhunter explains the influence of Cambridge in this matter: "Ours is an age of examination; and the University of Cambridge may claim the merit of originating this characteristic of the period. When we hear, as we often do, that the Universities are effete bodies which have lost their influence on the national character, we may point with real or affected triumph to the spread of examinations as a decisive proof that the humiliating assertion is not absolutely true. Although there must have been in schools and elsewhere processes resembling examinations before those of Cambridge had become widely famous, yet there can be little chance of error in regarding our mathematical

tripos as the model for rigor, justice and importance, of a long succession of institutions of a similar kind which have since been constructed." Todhunter makes the damaging admission that "We cannot by our examinations, *create* learning or genius; it is uncertain whether we can infallibly *discover* them; what we detect is simply the examination-passing power."

In England education is for the most part directed to training pupils for examination. One direct consequence is that the memory is cultivated at the expense of the understanding; knowledge instead of being assimilated is crammed for the time being, and lost as soon as the examination is over. Instead of a rational study of the principles of mathematics, attention is directed to problem-making,—to solving ten-minute conundrums. Textbooks are written with the view not of teaching the subject in the most scientific manner, but of passing certain specified examinations. I have seen such a textbook on trigonometry where all the important theorems which required the genius of Gregory and others to discover, are put down as so many definitions. Nominal knowledge, not real, is the kind that suits examinations.

Todhunter possessed a considerable sense of humour. We see this in his Essays; among other stories he tells the following: A youth who was quite unable to satisfy his examiners as to a problem, endeavored to mollify them, as he said, "by writing out book work bordering on the problem." Another youth who was rejected said "if there had been fairer examiners and better papers I should have passed; I knew many things which were not set." Again: "A visitor to Cambridge put himself under the care of one of the self-constituted guides who obtrude their services. Members of the various ranks of the academical state were pointed out to the stranger—heads of colleges, professors and ordinary fellows; and some attempt was made to describe the nature of the functions discharged by the heads and professors. But an inquiry as to the duties of fellows produced and reproduced only the answer, 'Them's fellows I say.' The guide had not been able to attach the notion of even the pretense of duty to a fellowship."

In 1874 Todhunter was elected an honorary fellow of his college, an honor which he prized very highly. Later on he was chosen as an elector to three of the University professorships—Moral Philosophy, Astronomy, Mental Philosophy and Logic. When the University of Cambridge established its new degree of Doctor of Science, restricted to those who have made original contributions to the advancement of science or learning, Todhunter was one of those whose application was granted within the first few months. In 1875 he published his manual *Functions of Laplace, Bessel and Legendre*. Next year he finished an arduous literary task—the preparation of two volumes, the one containing an account of the writings of Whewell, the other containing selections from his literary and scientific correspondence. Todhunter's task was marred to a considerable extent by an unfortunate division of the matter: the scientific and literary details were given to him, while the writing of the life itself was given to another.

In the summer of 1880 Dr. Todhunter first began to suffer from his eyesight, and from that date he gradually and slowly became weaker. But it was not till

September, 1883, when he was at Hunstanton, that the worst symptoms came on. He then partially lost by paralysis the use of the right arm; and, though he afterwards recovered from this, he was left much weaker. In January of the next year he had another attack, and he died on March 1, 1884, in the 64th year of his age.

Todhunter left a *History of Elasticity* nearly finished. The manuscript was submitted, to Cayley for report; it was in 1886 published under the editorship of Karl Pearson. I believe that he had other histories in contemplation; I had the honor of meeting him once, and in the course of conversation on mathematical logic, he said that he had a project of taking up the history of that subject; his interest in it dated from his study under De Morgan. Todhunter had the same ruling passion as Airy—love of order—and was thus able to achieve an immense amount of mathematical work. Prof. Mayor wrote, "Todhunter had no enemies, for he neither coined nor circulated scandal; men of all sects and parties were at home with him, for he was many-sided enough to see good in every thing. His friendship extended even to the lower creatures. The canaries always hung in his room, for he never forgot to see to their wants."