

CHAPTER II

GENERAL ANALYSIS OF PROPOSITIONS

Section 1. Since Logic discusses the proof or disproof, or (briefly) the testing of propositions, we must begin by explaining their nature. A proposition, then, may first be described in the language of grammar as *a sentence indicative*; and it is usually expressed in the present tense.

It is true that other kinds of sentences, optative, imperative, interrogative, exclamatory, if they express or imply an assertion, are not beyond the view of Logic; but before treating such sentences, Logic, for greater precision, reduces them to their equivalent sentences indicative. Thus, *I wish it were summer* may be understood to mean, *The coming of summer is an object of my desire*. *Thou shalt not kill* may be interpreted as *Murderers are in danger of the judgment*. Interrogatories, when used in argument, if their form is affirmative, have negative force, and affirmative force if their form is negative. Thus, *Do hypocrites love virtue?* anticipates the answer, *No*. *Are not traitors the vilest of mankind?* anticipates the answer, *Yes*. So that the logical form of these sentences is, *Hypocrites are not lovers of virtue*; *Traitors are the vilest of mankind*. Impersonal propositions, such as *It rains*, are easily rendered into logical forms of equivalent meaning, thus: *Rain is falling*; or (if that be tautology), *The clouds are raining*. Exclamations may seem capricious, but are often part of the argument. *Shade of Chatham!* usually means *Chatham, being aware of our present foreign policy, is much disgusted*. It is in fact, an appeal to authority, without the inconvenience of stating what exactly it is that the authority declares.

Section 2. But even sentences indicative may not be expressed in the way most convenient to logicians. *Salt dissolves in water* is a plain enough statement; but the logician prefers to have it thus: *Salt is soluble in water*. For he says that a proposition is analysable into three elements: (1) a Subject (as *Salt*) about which something is asserted or denied; (2) a Predicate (as *soluble in water*) which is asserted or denied of the Subject, and (3) the Copula (*is* or *are*, or *is not* or *are not*), the sign of relation between the Subject and

Predicate. The Subject and Predicate are called the Terms of the proposition: and the Copula may be called the sign of predication, using the verb 'to predicate' indefinitely for either 'to affirm' or 'to deny.' Thus *S is P* means that the term *P* is given as related in some way to the term *S*. We may, therefore, further define a Proposition as 'a sentence in which one term is predicated of another.'

In such a proposition as *Salt dissolves*, the copula (*is*) is contained in the predicate, and, besides the subject, only one element is exhibited: it is therefore said to be *secundi adjacentis*. When all three parts are exhibited, as in *Salt is soluble*, the proposition is said to be *tertii adjacentis*.

For the ordinary purposes of Logic, in predicating attributes of a thing or class of things, the copula *is*, or *is not*, sufficiently represents the relation of subject and predicate; but when it is desirable to realise fully the nature of the relation involved, it may be better to use a more explicit form. Instead of saying *Salt—is—soluble*, we may say *Solubility—coinheres with—the nature of salt*, or *The putting of salt in water—is a cause of—its dissolving*: thus expanding the copula into a full expression of the relation we have in view, whether coinherence or causation.

Section 3. The sentences of ordinary discourse are, indeed, for the most part, longer and more complicated than the logical form of propositions; it is in order to prove them, or to use them in the proof of other propositions, that they are in Logic reduced as nearly as possible to such simple but explicit expressions as the above (*tertii adjacentis*). A Compound Proposition, reducible to two or more simple ones, is said to be *exponible*.

The modes of compounding sentences are explained in every grammar-book. One of the commonest forms is the copulative, such as *Salt is both savoury and wholesome*, equivalent to two simple propositions: *Salt is savoury*; *Salt is wholesome*. *Pure water is neither sapid nor odorous*, equivalent to *Water is not sapid*; *Water is not odorous*. Or, again, *Tobacco is injurious, but not when used in moderation*, equivalent to *Much tobacco is injurious*; *a little is not*.

Another form of *Exponible* is the *Exceptive*, as *Kladderadatsch*

is published daily, except on week-days, equivalent to *Kladderadatsch is published on Sunday; it is not published any other day*. Still another Exponible is the Exclusive, as *Only men use fire*, equivalent to *Men are users of fire; No other animals are*. Exceptive and exclusive sentences are, however, equivalent forms; for we may say, *Kladderadatsch is published only on Sunday*; and *No animals use fire, except men*.

There are other compound sentences that are not explicable, since, though they contain two or more verbal clauses, the construction shows that these are inseparable. Thus, *If cats are scarce, mice are plentiful*, contains two verbal clauses; but *if cats are scarce* is conditional, not indicative; and *mice are plentiful* is subject to the condition that *cats are scarce*. Hence the whole sentence is called a Conditional Proposition. For the various forms of Conditional Propositions see chap. v, sec. 4.

But, in fact, to find the logical force of recognised grammatical forms is the least of a logician's difficulties in bringing the discourses of men to a plain issue. Metaphors, epigrams, innuendoes and other figures of speech present far greater obstacles to a lucid reduction whether for approval or refutation. No rules can be given for finding everybody's meaning. The poets have their own way of expressing themselves; sophists, too, have their own way. And the point often lies in what is unexpressed. Thus, "barbarous nations make, the civilised write history," means that civilised nations do not make history, which none is so brazen as openly to assert. Or, again, "Alcibiades is dead, but X is still with us"; the whole meaning of this 'explicable' is that X would be the lesser loss to society. Even an epithet or a suffix may imply a proposition: *This personage* may mean *X is a pretentious nobody*.

How shall we interpret such illusive predications except by cultivating our literary perceptions, by reading the most significant authors until we are at home with them? But, no doubt, to disentangle the compound propositions, and to expand the abbreviations of literature and conversation, is a useful logical exercise. And if it seem a laborious task thus to reduce to its logical elements a long argument in a speech or treatise, it should be observed that, as a rule, in a long discourse only a few sentences are of principal importance to the reasoning, the rest being

explanatory or illustrative digression, and that a close scrutiny of these cardinal sentences will frequently dispense us from giving much attention to the rest.

Section 4. But now, returning to the definition of a Proposition given in Section 2 that it is 'a sentence in which one term is predicated of another,' we must consider what is the import of such predication. For the definition, as it stands, seems to be purely Nominalist. Is a proposition nothing more than a certain synthesis of words; or, is it meant to correspond with something further, a synthesis of ideas, or a relation of facts?

Conceptualist logicians, who speak of judgments instead of propositions, of course define the judgment in their own language. According to Hamilton, it is "a recognition of the relation of congruence or confliction in which two concepts stand to each other." To lighten the sentence, I have omitted one or two qualifications (Hamilton's *Lectures on Logic*, xiii.). "Thus," he goes on "if we compare the thoughts *water*, *iron*, and *rusting*, we find them congruent, and connect them into a single thought, thus: *water rusts iron*—in that case we form a judgment." When a judgment is expressed in words, he says, it is called a proposition.

But has a proposition no meaning beyond the judgment it expresses? Mill, who defines it as "a portion of discourse in which a predicate is affirmed or denied of a subject" (*Logic*, Book 1., chap. iv. Section 1.), proceeds to inquire into the import of propositions (Book 1., chap. v.), and finds three classes of them: (a) those in which one proper name is predicated of another; and of these Hobbes's Nominalist definition is adequate, namely, that a proposition asserts or denies that the predicate is a name for the same thing as the subject, as *Tully is Cicero*.

(b) Propositions in which the predicate means a part (or the whole) of what the subject means, as *Horses are animals*, *Man is a rational animal*. These are Verbal Propositions (see below: chap. v, section 6), and their import consists in affirming or denying a coincidence between the meanings of names, as *The meaning of 'animal' is part of the meaning of 'horse.'* They are partial or complete definitions.

But (c) there are also Real Propositions, whose predicates do not mean the same as their subjects, and whose import consists in affirming or denying one of five different kinds of matter of fact: (1) That the subject exists, or does not; as if we say *The bison exists*, *The great auk is extinct*. (2) Co-existence, as *Man is mortal*; that is, *the being subject to death coinheres with the qualities on account of which we call certain objects men*. (3) Succession, as *Night follows day*. (4) Causation (a particular kind of Succession), as *Water rusts iron*. (5) Resemblance, as *The colour of this geranium is like that of a soldier's coat*, or $A = B$.

On comparing this list of real predications with the list of logical relations given above (chap. i, section 5 (a)), it will be seen that the two differ only in this, that I have there omitted simple Existence. Nothing simply exists, unrelated either in Nature or in knowledge. Such a proposition as *The bison exists* may, no doubt, be used in Logic (subject to interpretation) for the sake of custom or for the sake of brevity; but it means that some specimens are still to be found in N. America, or in Zoological gardens.

Controversy as to the Import of Propositions really turns upon a difference of opinion as to the scope of Logic and the foundations of knowledge. Mill was dissatisfied with the "congruity" of concepts as the basis of a judgment. Clearly, mere congruity does not justify belief. In the proposition *Water rusts iron*, the concepts *water*, *rust* and *iron* may be congruous, but does any one assert their connection on that ground? In the proposition *Murderers are haunted by the ghosts of their victims*, the concepts *victim*, *murderer*, *ghost* have a high degree of congruity; yet, unfortunately, I cannot believe it: there seems to be no such cheap defence of innocence. Now, Mill held that Logic is concerned with the grounds of belief, and that the scope of Logic includes Induction as well as Deduction; whereas, according to Hamilton, Induction is only Modified Logic, a mere appendix to the theory of the "forms of thought as thought." Indeed, Mill endeavoured in his *Logic* to probe the grounds of belief deeper than usual, and introduced a good deal of Metaphysics—either too much or not enough—concerning the ground of axioms. But, at any rate, his great point was that belief, and therefore (for the most part) the Real Proposition, is concerned not merely with the relations of words, or even of ideas, but with matters of fact; that is, both

propositions and judgments point to something further, to the relations of things which we can examine, not merely by thinking about them (comparing them in thought), but by observing them with the united powers of thought and perception. This is what convinces us that *water rusts iron*: and the difficulty of doing this is what prevents our feeling sure that *murderers are haunted by the ghosts of their victims*. Hence, although Mill's definition of a proposition, given above, is adequate for propositions in general; yet that kind of proposition (the Real) with regard to which Logic (in Mill's view) investigates the conditions of proof, may be more explicitly and pertinently defined as 'a predication concerning the relation of matters of fact.'

Section 5. This leads to a very important distinction to which we shall often have to refer in subsequent pages—namely, the distinction between the Form and the Matter of a proposition or of an argument. The distinction between Form and Matter, as it is ordinarily employed, is easily understood. An apple growing in the orchard and a waxen apple on the table may have the same shape or form, but they consist of different materials; two real apples may have the same shape, but contain distinct ounces of apple-stuff, so that after one is eaten the other remains to be eaten. Similarly, tables may have the same shape, though one be made of marble, another of oak, another of iron. The form is common to several things, the matter is peculiar to each. Metaphysicians have carried the distinction further: apples, they say, may have not only the same outward shape, but the same inward constitution, which, therefore, may be called the Form of apple-stuff itself—namely, a certain pulpiness, juiciness, sweetness, *etc.*; qualities common to all dessert apples: yet their Matter is different, one being here, another there—differing in place or time, if in nothing else. The definition of a species is the form of every specimen of it.

To apply this distinction to the things of Logic: it is easy to see how two propositions may have the same Form but different Matter: not using 'Form' in the sense of 'shape,' but for that which is common to many things, in contrast with that which is peculiar to each. Thus, *All male lions are tawny* and *All water is liquid at 50° Fahrenheit*, are two propositions that have the same form, though their matter is entirely different. They both predicate something of the whole of their subjects, though their subjects are

different, and so are the things predicated of them. Again, *All male lions have tufted tails* and *All male lions have manes*, are two propositions having the same form and, in their subjects, the same matter, but different matter in their predicates. If, however, we take two such propositions as these: *All male lions have manes* and *Some male lions have manes*, here the matter is the same in both, but the form is different—in the first, predication is made concerning *every* male lion; in the second of only *some* male lions; the first is *universal*, the second is *particular*. Or, again, if we take *Some tigers are man-eaters* and *Some tigers are not man-eaters*, here too the matter is the same, but the form is different; for the first proposition is *affirmative*, whilst the second is *negative*.

Section 6. Now, according to Hamilton and Whately, pure Logic has to do only with the Form of propositions and arguments. As to their Matter, whether they are really true in fact, that is a question, they said, not for Logic, but for experience, or for the special sciences. But Mill desired so to extend logical method as to test the material truth of propositions: he thought that he could expound a method by which experience itself and the conclusions of the special sciences may be examined.

To this method it may be objected, that the claim to determine Material Truth takes for granted that the order of Nature will remain unchanged, that (for example) water not only at present is a liquid at 50° Fahrenheit, but will always be so; whereas (although we have no reason to expect such a thing) the order of Nature may alter—it is at least supposable—and in that event water may freeze at such a temperature. Any matter of fact, again, must depend on observation, either directly, or by inference—as when something is asserted about atoms or ether. But observation and material inference are subject to the limitations of our faculties; and however we may aid observation by microscopes and micrometers, it is still observation; and however we may correct our observations by repetition, comparison and refined mathematical methods of making allowances, the correction of error is only an approximation to accuracy. Outside of Formal Reasoning, suspense of judgment is your only attitude.

But such objections imply that nothing short of absolute truth has any value; that all our discussions and investigations in science

or social affairs are without logical criteria; that Logic must be confined to symbols, and considered entirely as mental gymnastics. In this book prominence will be given to the character of Logic as a formal science, and it will also be shown that Induction itself may be treated formally; but it will be assumed that logical forms are valuable as representing the actual relations of natural and social phenomena.

Section 7. Symbols are often used in Logic instead of concrete terms, not only in Symbolic Logic where the science is treated algebraically (as by Dr. Venn in his *Symbolic Logic*), but in ordinary manuals; so that it may be well to explain the use of them before going further.

It is a common and convenient practice to illustrate logical doctrines by examples: to show what is meant by a Proposition we may give *salt is soluble*, or *water rusts iron*: the copulative exponible is exemplified by *salt is savoury and wholesome*; and so on. But this procedure has some disadvantages: it is often cumbrous; and it may distract the reader's attention from the point to be explained by exciting his interest in the special fact of the illustration. Clearly, too, so far as Logic is formal, no particular matter of fact can adequately illustrate any of its doctrines. Accordingly, writers on Logic employ letters of the alphabet instead of concrete terms, (say) *X* instead of *salt* or instead of *iron*, and (say) *Y* instead of *soluble* or instead of *rusts by water*; and then a proposition may be represented by *X is Y*. It is still more usual to represent a proposition by *S is (or is not) P*, *S* being the initial of Subject and *P* of Predicate; though this has the drawback that if we argue—*S is P*, therefore *P is S*, the symbols in the latter proposition no longer have the same significance, since the former subject is now the predicate.

Again, negative terms frequently occur in Logic, such as *not-water*, or *not-iron*, and then if *water* or *iron* be expressed by *X*, the corresponding negative may be expressed by *x*; or, generally, if a capital letter stand for a positive term, the corresponding small letter represents the negative. The same device may be adopted to express contradictory terms: either of them being *X*, the other is *x* (see chap. iv., sections 7-8); or the contradictory terms may be expressed by *x* and $\bar{x}$, *y* and $\bar{y}$.

And as terms are often compounded, it may be convenient to express them by a combination of letters: instead of illustrating such a case by *boiling water* or *water that is boiling*, we may write *XY*; or since positive and negative terms may be compounded, instead of illustrating this by *water that is not boiling*, we may write *Xy*.

The convenience of this is obvious; but it is more than convenient; for, if one of the uses of Logic be to discipline the power of abstract thought, this can be done far more effectually by symbolic than by concrete examples; and if such discipline were the only use of Logic it might be best to discard concrete illustrations altogether, at least in advanced text-books, though no doubt the practice would be too severe for elementary manuals. On the other hand, to show the practical applicability of Logic to the arguments and proofs of actual life, or even of the concrete sciences, merely symbolic illustration may be not only useless but even misleading. When we speak of politics, or poetry, or species, or the weather, the terms that must be used can rarely have the distinctness and isolation of *X* and *Y*; so that the perfunctory use of symbolic illustration makes argument and proof appear to be much simpler and easier matters than they really are. Our belief in any proposition never rests on the proposition itself, nor merely upon one or two others, but upon the immense background of our general knowledge and beliefs, full of circumstances and analogies, in relation to which alone any given proposition is intelligible. Indeed, for this reason, it is impossible to illustrate Logic sufficiently: the reader who is in earnest about the cogency of arguments and the limitation of proofs, and is scrupulous as to the degrees of assent that they require, must constantly look for illustrations in his own knowledge and experience and rely at last upon his own sagacity.