CHAPTER XII

Conditional Syllogisms

Section 1. Conditional Syllogisms may be generally described as those that contain conditional propositions. They are usually divided into two classes, Hypothetical and Disjunctive.
A Hypothetical Syllogism is one that consists of a Hypothetical Major Premise, a Categorical Minor Premise, and a Categorical Conclusion. Two Moods are usually recognised the Modus ponens, in which the antecedent of the hypothetical major premise is affirmed; and the Modus tollens, in which its consequent is denied.

(1) Modus ponens, or Constructive.
If A is B, C is D;
A is B:
∴. C is D.

If Aristotle's reasoning is conclusive, Plato's theory of Ideas is erroneous;_Aristotle's reasoning is conclusive:
∴. Plato's theory of Ideas is erroneous.

Rule of the Modus ponens: The antecedent of the major premise being affirmed in the minor premise, the consequent is also affirmed in the conclusion.
(2) Modus tollens, or Destructive.
If A is B, C is D;
C is not D:
∴. A is not B.

If Pythagoras is to be trusted, Justice is a number;
Justice is not a number:
∴. Pythagoras is not to be trusted.

Rule of the Modus tollens: The consequent of the major premise being denied in the minor premise, the antecedent is denied in the conclusion.

By using negative major premises two other forms are obtainable:

then, either by affirming the antecedent or by denying the consequent, we draw a negative conclusion.

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Thus (Modus ponens):    (Modus tollens):
If A is B, C is not D; If A is B, C is not D;
A is B:                  C is D:
.'. C is not D.          .'. A is not B.
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Further, since the antecedent of the major premise, taken by itself, may be negative, it seems possible to obtain four more forms, two in each Mood, from the following major premises:

(1) If A is not B, C is D;
(2) If A is not B, C is not D.

But since the quality of a Hypothetical Proposition is determined by the quality of its consequent, not at all by the quality of its antecedent, we cannot get from these two major premises any really new Moods, that is to say, Moods exhibiting any formal difference from the four previously expounded.
It is obvious that, given the hypothetical major premise–

If A is B, C is D–

we cannot, by denying the antecedent, infer a denial of the consequent. That A is B, is a mark of C being D; but we are not told that it is the sole and indispensable condition of it. If men read good books, they acquire knowledge; but they may acquire knowledge by other means, as by [Pg 149]observation. For the same reason, we cannot by affirming the consequent infer the affirmation of the antecedent: Caius may have acquired knowledge; but we cannot thence conclude that he has read good books.

To see this in another light, let us recall chap. v. Section 4, where it was shown that a hypothetical proposition may be translated into a categorical one; whence it follows that a Hypothetical Syllogism may be translated into a Categorical Syllogism. Treating the above examples thus, we find that the Modus ponens (with affirmative major premise) takes the form of Barbara, and the Modus tollens the form of Camestres:

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     Modus ponens.                              Barbara.
If A is B, C is D;               The case of A being B is a
                                 case of C being D;
     A is B:                     This is a case of A being
                                 B:
.'. C is D                       .'. This is a case of C
                                 being D.
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Now if, instead of this, we affirm the consequent, to form the new minor premise, This is a case of C being D, there will be a Syllogism in the Second Figure with two affirmative premises, and therefore the fallacy of undistributed Middle. Again:

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     Modus tollens.                           Camestres.
  If A is B, C is D;        The case of A being B is a
                            case of C being D:
  C is not D:               This is not a case of C
                            being D:
  .'. A is not B.           .'. This is not a case of A
                                     being B.
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But if, instead of this, we deny the antecedent, to form the new minor premise, This is not a case of A being B, there arises a syllogism in the First Figure with a negative minor premise, and therefore the fallacy of illicit process of the major term.

By thus reducing the Hypothetical Syllogism to the Categorical form, what is lost in elegance is gained in intelligibility. For, first, we may justify ourselves in speaking of the hypothetical premise as the major, and of the categorical premise as the minor; since in the categorical form they contain respectively the major and minor terms. And, secondly, we may justify ourselves in treating the Hypothetical Syllogism as a kind of Mediate Inference, in spite of the fact that it does not exhibit two terms compared by means of a third; since in the Categorical form such terms distinctly appear: a new term ('This') emerges in the position of the minor; the place of the Middle is filled by the antecedent of the major premise in the Modus ponens, and by the consequent in the Modus tollens. The mediate element of the inference in a Hypothetical Syllogism consists in asserting, or denying, the fulfilment of a given condition; just as in a Categorical syllogism to identify the minor term with the Middle is a condition of the major term's being

predicated of it. In the hypothetical proposition– If A is B, C is D–
the Antecedent, A is B, is the conditio sufficiens, or mark, of the
Consequent, C is D; and therefore the Consequent, C is D, is a
conditio sine qua non of the antecedent, A is B; and it is by means
of affirming the former condition, or else denying the latter, that a
conclusion is rendered possible.

Indeed, we need not say that the element of mediation consists in
affirming, or denying, the fulfilment of a given condition: it is
enough to say 'in affirming.' For thus to explain the Modus tollens,
reduce it to the Modus ponens (contrapositing the major premise
and obverting the minor):

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                                        Celarent.
If A is B, C is D:              The case of C being
                               not-D is
.'. If C is not-D, A is not B;  not a case of A being
                               B;
C is not-D:                    This is a case of C
                               being not-D:
.'. A is not B.                .'. This is not a case
                               of A being B.
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The above four forms commonly treated of as Hypothetical
Syllogisms, are called by Ueberweg and Dr. Keynes 'Hypothetico-
Categorical.' Ueberweg restricts the name 'Hypothetical' simply
(and Dr. Keynes the name 'Conditional') to such Syllogisms as the
following, having two Hypothetical Premises:

If C is D, E is F;
If A is B, C is D:
∴. If A is B, E is F.

If we recognise particular hypothetical propositions (see chap. v.
Section 4), it is obvious that such Syllogisms may be constructed
in all the Moods and Figures of the Categorical Syllogism; and of
course they may be translated into Categoricals. We often reason
in this hypothetical way. For example:_If the margin of cultivation
be extended, rents will rise;_If prices of produce rise, the margin of
cultivation will be extended:

∴. If prices of produce rise, rents will rise.

But the function of the Hypothetical Syllogism (commonly so called), as also of the Disjunctive Syllogism (to be discussed in the next section) is to get rid of the conditional element of the premises, to pass from suspense to certainty, and obtain a decisive categorical conclusion; whereas these Syllogisms with two hypothetical premises leave us still with a hypothetical conclusion. This circumstance seems to ally them more closely with Categorical Syllogisms than with those that are discussed in the present chapter. That they are Categoricals in disguise may be seen by considering that the above syllogism is not materially significant, unless in each proposition the word 'If' is equivalent to 'Whenever.' Accordingly, the name 'Hypothetical Syllogism,' is here employed in the older usage.

Section 2. A Disjunctive Syllogism consists of a Disjunctive Major Premise, a Categorical Minor Premise, and a Categorical Conclusion.

How many Moods are to be recognised in this kind of argument depends on whether the alternatives of the Disjunctive Premise are regarded as mutually exclusive or possibly coincident. In saying 'Either A is B, or C is D,' do we mean 'either, but not both,' or 'either, it may be both'? (See chap. v. Section 4.)

When the alternatives of the Disjunctive are not exclusive, we have only the Modus tollendo ponens.

Either A is B, or C is D;
A is not B (or C is not D):
∴. C is D (or A is B).

Either wages fall, or the weaker hands are dismissed;
Wages do not fall:
∴. The weaker hands are dismissed.

But we cannot argue–
Wages fall:
∴. The weaker hands are not dismissed;

since in 'hard times' both events may happen together.
Rule of the Modus tollendo ponens: If one alternative be denied, the other is affirmed.
When, however, the alternatives of the Disjunctive are mutually exclusive, we have also the
Modus ponendo tollens.

Either A is B, or C is D;
A is B (or C is D):
.'. C is not D (or A is not B).

Either the Tories or the Whigs win the election;
The Tories win:
.'. The Whigs do not win.

We may also, of course, argue as above in the Modus tollendo ponens–

The Tories do not win:
.'. The Whigs do.

But in this example, to make the Modus tollendo ponens materially valid, it must be impossible that the election should result in a tie. The danger of the Disjunctive Proposition is that the alternatives may not, between them, exhaust the possible cases. Only contradictory alternatives are sure to cover the whole ground.
Rule of the Modus ponendo tollens: If one alternative be affirmed, the other is denied.

Since a disjunctive proposition may be turned into a hypothetical proposition (chap. v. Section 4,) a Disjunctive Syllogism may be turned into a Hypothetical Syllogism:

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    Modus tollendo ponens.          Modus ponens.
    Either A is B, or C is D;    If A is not B, C is D;
    A is not B:                      A is not B:
    .'. C is D.                      .'. C is D.
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Similarly the Modus ponendo tollens is equivalent to that kind of Modus ponens which may be formed with a negative major premise; for if the alternatives of a disjunctive proposition be

exclusive, the corresponding hypothetical be affirmative or negative:

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Modus ponendo tollens.                Modus ponens.
Either A is B, or C is D;              If A is B, C
                                       is not D;
A is B:                                A is B:
.'. C is not D.                        .'. C is not D.
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Hence, finally, a Disjunctive Syllogism being equivalent to a Hypothetical, and a Hypothetical to a Categorical; a Disjunctive Syllogism is equivalent and reducible to a Categorical. It is a form of Mediate Inference in the same sense as the Hypothetical Syllogism is; that is to say, the conclusion depends upon an affirmation, or denial, of the fulfilment of a condition implied in the disjunctive major premise.

Section 3. The Dilemma is perhaps the most popularly interesting of all forms of proof. It is a favourite weapon of orators and wits; and "impaled upon the horns of a dilemma" is a painful situation in which every one delights to see his adversary. It seems to have been described by Rhetoricians before finding its way into works on Logic; and Logicians, to judge from their diverse ways of defining it, have found some difficulty in making up their minds as to its exact character.

There is a famous Dilemma employed by Demosthenes, from which the general nature of the argument may be gathered:_If Aeschines joined in the public rejoicings, he is inconsistent; if he did not, he is unpatriotic; But either he joined, or he did not join:_Therefore he is either inconsistent or unpatriotic.
That is, reduced to symbols:

If A is B, C is D; and if E is F, G is H:
But either A is B, or E is F;
.'. Either C is D or G is H (Complex Constructive).

This is a compound Conditional Syllogism, which may be analysed as follows:

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                Either A is B or E is F.
   Suppose that E is not F:   Suppose that A is not
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B:
     Then A is B.              Then E is F.
     But if A is B, C is D;     But if E is F, G is H;
     (A is B):                 (E is F):
     .'. C is D.               .'. G is H.
            .'. Either C is D or G is H.
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A Dilemma, then, is a compound Conditional Syllogism, having for its Major Premise two Hypothetical Propositions, and for its Minor Premise a Disjunctive Proposition, whose alternative terms either affirm the Antecedents or deny the Consequents of the two Hypothetical Propositions forming the Major Premise.
The hypothetical propositions in the major premise, may have all four terms distinct (as in the above example); and then the conclusion is a disjunctive proposition, and the Dilemma is said to be Complex. Or the two hypothetical propositions may have a common antecedent or a common consequent; and then the conclusion is a categorical proposition, and the Dilemma is said to be Simple.

Again, the alternatives of the disjunctive minor premise may be affirmative or negative: if affirmative, the Dilemma is called Constructive; and if negative, Destructive.
Using, then, only affirmative hypothetical propositions in the major premise, there are four Moods:

1. The Simple Constructive–
If A is B, C is D; and if E is F, C is D:
But either A is B, or E is F:
.'. C is D.

If the Tories win the election, the Government will avoid innovation; and if the Whigs win, the House of Lords will prevent them innovating:_But either the Tories or the Whigs will win:

.'. There will be no innovation.
If A is B, C is D; and if E is F, G is H:
But either A is B, or E is F:
.'. Either C is D or G is H.

If appearance is all that exists, reality is a delusion; and if there is a

substance beyond consciousness, knowledge of reality is impossible:
But either appearance is all, or there is a substance beyond consciousness:
∴. Either reality is a delusion, or a knowledge of it is impossible.

`The Complex Constructive—`
3. Simple Destructive–
If A is B, C is D; and if A is B, E is F:
But either C is not D, or E is not F:
∴. A is not B.

If table-rappers are to be trusted, the departed are spirits; and they also exert mechanical energy: But either the departed are not spirits, or they do not exert mechanical energy:
∴. Table-rappers are not to be trusted.

4. Complex Destructive–
If A is B, C is D; and if E is F, G is H:
But either C is not D, or G is not H:
∴. Either A is not B, or E is not F.

If poetic justice is observed, virtue is rewarded; and if the mirror is held up to Nature, the villain triumphs:

But either virtue is not rewarded, or the villain does not triumph:
∴. Either poetic justice is not observed, or the mirror is not held up to Nature.

Such are the four Moods of the Dilemma that emerge if we only use affirmative hypotheticals for the major premise; but, certainly, it is often quite as natural to employ two negative hypotheticals (indeed, one might be affirmative and the other negative; but waive that); and then four more moods emerge, all having negative conclusions. It is needless to intimidate the reader by drawing up these four moods in battle array: they always admit of reduction to the foregoing moods by obverting the hypotheticals. Still, by the same process we may greatly decrease the number of moods of the Categorical Syllogism; and just as some Syllogisms are most simply expressed in Celarent or Cesare, so some Dilemmas are

most simply stated with negative major premises–e.g., The example of a Simple Constructive Dilemma above given would run more naturally thus: If the Tories win, the Government will not innovate; and if the Whigs, the Lords will not let them: and similarly Demosthenes' Dilemma–If Aeschines joined, he is not consistent; and if he did not, he is not patriotic. Moreover, the propriety of recognising Dilemmas with negative major premises, follows from the above analysis of the Dilemma into a combination of Conditional Syllogisms, even if (as in Section 1 of this chapter) we take account of only four Moods of the Hypothetical Syllogism.

In the rhetorical use of the Dilemma, it may be observed that the disjunction in the minor premise ought to be obvious, or (at any rate) easily acceptable to the audience. Thus, Either the Tories or the Whigs will win; Either Aeschines joined in the rejoicings, or he did not; such propositions are not likely to be disputed. But if the orator must stop to prove his minor premise, the smacking effect of this figure (if the expression be allowed) will be lost. Hence the minor premises of other examples given above are only fit for a select audience. That Either ghosts are not spirits, or they do not exert mechanical energy, supposes a knowledge of the principle, generally taught by physical philosophers, that only matter is the vehicle of energy; and that Either appearance is all, or there is substance beyond consciousness, is a doctrine which only metaphysical philosophers could be expected to understand, and upon which they could not be expected to agree. However, the chief danger is that a plausible disjunction may not be really such as to exclude any middle ground: Either the Tories or the Whigs win, is bad, if a tie be possible; though in the above argument this is negligible, seeing that a tie cannot directly cause innovations. Either Aeschines joined in the rejoicings, or he did not, does not allow for a decent conformity with the public movement where resistance would be vain; yet such conformity as need not be inconsistent with subsequent condemnation of the proceedings, nor incompatible with patriotic reserve founded on a belief that the rejoicings are premature and ominous.

Another rhetorical consideration is, that the alternatives of the disjunctive conclusion of a Complex Dilemma should both point

the same way, should be equally distasteful or paradoxical. 'Either inconsistent or unpatriotic': horrid words to a politician! 'Either no reality or no possible knowledge of it': very disappointing to an anxious inquirer! Thus the disjunctive conclusion is as bad for an opponent as the categorical one in a Simple Dilemma.

Logicians further speak of the Trilemma, with three Hypotheticals and a corresponding triple Disjunction; and of a Polylemma, with any further number of perplexities. But anyone who has a taste for logical forms may have it amply gratified in numerous text-books.