A Short Account of the History of Mathematics

W. W. Rouse Ball

The Bernoullis

The Bernoullis (or as they are sometimes, and perhaps more correctly, called, the Bernouillis) were a family of Dutch origin, who were driven from Holland by the Spanish persecutions, and finally settled at Bâle in Switzerland. The first member of the family who obtained distinction in mathematics was James.

James Bernoulli

Jacob or James Bernoulli was born at Bâle on December 27, 1654; in 1687 he was appointed to a chair in mathematics in the university there; and occupied it until his death on August 16, 1705.

He was one of the earliest to realize how powerful as an instrument of analysis was the infinitesimal calculus, and he applied it to several problems, but did not himself invent any new processes. His great influence was uniformly and successfully exerted in favour of the use of the differential calculus, and his lessons on it, which were written in the form of two essays in 1691 and are published in the second volume of his works, shew how completely he had even then grasped the principles of the new analysis. These lectures, which contain the earliest use of the term integral, were the first published attempt to construct an integral calculus; for Leibnitz had treated each problem by itself, and had not laid down any general rules on the subject.

The most important discoveries of James Bernoulli were his solution of the problem to find an isochronous curve; his proof that the construction for the catenary which had been given by Leibnitz was correct, and his extension of this to strings of variable density and under a central force; his determination of the form taken by an elastic rod fixed at one end and acted on by a given force at the other, the elastica; also of a flexible rectangular sheet with two sides fixed horizontally and filled with a heavy liquid, the lintearia; and lastly, of a sail filled with wind, the velaria. In 1696 he offered a reward for the general solution of isoperimetrical figures, that is, of figures of a given species and given perimeter which shall include a maximum area: his own solution, published in 1701, is correct as far as it goes. In 1698 he published an essay on the differential calculus and its applications to geometry. He here investigated the

chief properties of the equiangular spiral, and especially noticed the manner in which various curves deduced from it reproduced the original curve: struck by this fact he begged that, in imitation of Archimedes, and equiangular spiral should be engraved on his tombstone with the inscription eadem numero mutata resurgo. He also brought out in 1695 an edition of Descartes's Géometrie. In his Ars Conjectandi, published in 1713, he established the fundamental principles of the calculus of probabilities; in the course of the work he defined the numbers known by his name and explained their use, he also gave some theorems on finite differences. His higher lectures were mostly on the theory of series; these were published by Nicholas Bernoulli in 1713.

John Bernoulli

John Bernoulli, the brother of James Bernoulli, was born at Bâle on August 7, 1667, and died there on January 1, 1748. He occupied the chair of mathematics at Groningen from 1695 to 1705; and at Bâle, where he succeeded his brother, from 1705 to 1748. To all who did not acknowledge his merits in a manner commensurate with his own view of them he behaved most unjustly: as an illustration of his character it may be mentioned that he attempted to substitute for an incorrect solution of his own on the problem of isoperimetrical curves another stolen from his brother James, while he expelled his son Daniel from his house for obtaining a prize from the French Academy which he had expected to receive himself. He was, however, the most successful teacher of his age, and had the faculty of inspiring his pupils with almost as passionate a zeal for mathematics as he felt himself. The general adoption on the continent of the differential rather than the fluxional notation was largely due to his influence.

Leaving out of account his innumerable controversies, the chief discoveries of John Bernoulli were the exponential calculus, the treatment of trigonometry as a branch of analysis, the conditions for a geodesic, the determination of orthogonal trajectories, the solution of the brachistochrone, the statement that a ray of light pursues such a path that □μds is a minimum, and the enunciation of the principle of virtual work. I believe that he was the first to denote the accelerating effect of gravity by an algebraical sign g, and he thus arrived at the formula v2 = 2gh the same result would have been previously expressed by the proportion . The notation □x to indicate a function of x was introduced by him in 1718, and displaced the notation X or □ proposed by him in 1698; but the general adoption of symbols like f, F, □, □, ... to represent functions, seems to be mainly due to Euler and Lagrange.

The Younger Bernoullis

Several members of the same family, but of a younger generation, enriched mathematics by their teaching and writings. The most important of these were the three sons of John; namely Nicholas, Daniel, and John the younger; and the two sons of John the Younger, who bore the names of John and James. To make the account complete I add here their respective dates. Nicholas Bernoulli, the eldest of the three sons of John, was born on Jan. 27, 1695, and was drowned at St. Petersburg, where he was professor, on July 26, 1726. Daniel Bernoulli, the scond son of John, was born on Feb. 9, 1700, and died on March 17, 1782; he was professor first at St. Petersburg and afterwards at Bâle, and shares with Euler the unique distinction of having gained the prize proposed annually by the French Academy no less than ten times. John Bernoulli, the younger, a brother of Nicholas and Daniel, was born on May 18, 1710, and died in 1790; he also was a professor at Bâle. He left two sons, John and James: of these, the former, who was born on Dec. 14, 1744, and died on July 10, 1807, was astronomer-royal, and director of mathematical studies at Berlin; while the latter, who was born on Oct. 17, 1759, and died in July 1789, was successively professor at Bâle, Verona, and St. Petersburg.

Daniel Bernoulli

Daniel Bernoulli, whose name I mentioned above, and who was by far the ablest of the younger Bernoullis, was a contemporary and intimate friend of Euler, whose works are mentioned in the next chapter. Daniel Bernoulli was born on Feb. 9, 1700, and died at Bâle, where he was professor of natural philosophy, on March 17, 1782. He went to St. Petersburg in 1724 as professor of mathematics, but the roughness of the social life was distasteful to him, and he was not sorry when a temporary illness in 1733 allowed him to plead his health as an excuse for leaving. He then returned to Bâle, and held successively chairs of medicine, metaphysics, and natural philosophy there.

His earliest mathematical work was the Exercitationes, published in 1724, which contains a solution of the differential equation proposed by Riccati. Two years later he pointed out for the first time the frequent desirability of resolving a compound motion into motions of translation and motions of rotation. His chief work is his Hydrodynamique, published in 1738; it resembles Lagrange's Méchanique analytique in being arranged so that all the results are consequences of a single principle, namely, in this case, the conservation of energy. This was followed by a memoir on the

theory of the tides, to which, conjointly with the memoirs by Euler and Maclaurin, a prize was awarded by the French Academy: these three memoirs contain all that was done on this subject between the publication of Newton's Principia and the investigations of Laplace. Bernoulli also wrote a large number of papers on various mechanical questions, especially on problems connected with vibrating strings, and the solutions given by Taylor and by D'Alembert. He is the earliest writer who attempted to formulate a kinetic theory of gases, and he applied the idea to explain the law associated with the names of Boyle and Mariotte.