Chapter 5

MISCELLANEOUS NUMBER BASES.

In the development and extension of any series of numbers into a systematic arrangement to which the term *system* may be applied, the first and most indispensable step is the selection of some number which is to serve as a base. When the savage begins the process of counting he invents, one after another, names with which to designate the successive steps of his numerical journey. At first there is no attempt at definiteness in the description he gives of any considerable number. If he cannot show what he means by the use of his fingers, or perhaps by the fingers of a single hand, he unhesitatingly passes it by, calling it many, heap, innumerable, as many as the leaves on the trees, or something else equally expressive and equally indefinite. But the time comes at last when a greater degree of exactness is required. Perhaps the number 11 is to be indicated, and indicated precisely. A fresh mental effort is required of the ignorant child of nature; and the result is “all the fingers and one more,” “both hands and one more,” “one on another count,” or some equivalent circumlocution. If he has an independent word for 10, the result will be simply ten-one. When this step has been taken, the base is established. The savage has, with entire unconsciousness, made all his subsequent progress dependent on the number 10, or, in other words, he has established 10 as the base of his number system. The process just indicated may be gone through with at 5, or at 20, thus giving us a quinary or a vigesimal, or, more probably, a mixed system; and, in rare instances, some other number may serve as the point of departure from simple into compound numeral terms. But the general idea is always the same, and only the details of formation are found to differ.

Without the establishment of some base any *system* of numbers is impossible. The savage has no means of keeping track of his count unless he can at each step refer himself to some well-defined milestone in his course. If, as has been pointed out in the foregoing chapters, confusion results whenever an attempt is made to count any number which carries him above 10, it must at once appear that progress beyond that point would be rendered many times more difficult if it were not for the fact that, at each new step, he has only to indicate the distance he has progressed beyond his base, and not the distance from his original starting-point. Some idea may, perhaps, be gained of the nature of this difficulty by imagining the numbers of our ordinary scale to be represented, each one by a single symbol different from that used to denote any other number. How long would it take the average intellect to master the first 50 even, so that each number could without hesitation be indicated by its appropriate symbol? After the first 50 were once mastered, what of the next 50? and the next? and the next? and so on. The acquisition of a scale for which we had no other means of expression than that just described would be a matter of the extremest difficulty, and could never, save in the most exceptional circumstances, progress beyond the attainment of a limit of a few hundred. If the various numbers in question were designated by words instead of by symbols, the difficulty of the task would be still further increased. Hence, the establishment of some number as a base is not only a matter of the very highest convenience, but of absolute necessity, if any save the first few numbers are ever to be used.

In the selection of a base,—of a number from which he makes a fresh start, and to which he refers the next steps in his count,—the savage simply follows nature when he chooses 10, or perhaps 5 or 20. But it is a matter of the greatest interest to find that other numbers have, in exceptional cases, been used for this purpose. Two centuries ago the distinguished philosopher and mathematician, Leibnitz, proposed a binary system of numeration. The only symbols needed in such a system would be 0 and 1. The number which is now symbolized by the figure 2 would be represented by 10; while 3, 4, 5, 6, 7, 8, etc., would appear in the binary notation as 11, 100, 101, 110, 111, 1000, etc. The difficulty with such a system is that it rapidly grows cumbersome, requiring the use of so many figures for indicating any number. But Leibnitz found in the representation of all numbers by means of the two digits 0 and 1 a fitting symbolization of the creation out of chaos, or nothing, of the entire universe by the power of the Deity. In commemoration of this invention a medal was struck bearing on the obverse the words

Numero Deus impari gaudet,

and on the reverse,

Omnibus ex nihilo ducendis sufficit Unum.

This curious system seems to have been regarded with the greatest affection by its inventor, who used every endeavour in his power to bring it to the notice of scholars and to urge its claims. But it appears to have been received with entire indifference, and to have been regarded merely as a mathematical curiosity.

Unknown to Leibnitz, however, a binary method of counting actually existed during that age; and it is only at the present time that it is becoming extinct. In Australia, the continent that is unique in its flora, its fauna, and its general topography, we find also this anomaly among methods of counting. The natives, who are to be classed among the lowest and the least intelligent of the aboriginal races of the world, have number systems of the most rudimentary nature, and evince a decided tendency to count by twos. This peculiarity, which was to some extent shared by the Tasmanians, the island tribes of the Torres Straits, and other aboriginal races of that region, has by some writers been regarded as peculiar to their part of the world; as though a binary number system were not to be found elsewhere. This attempt to make out of the rude and unusual method of counting which obtained among the Australians a racial characteristic is hardly justified by fuller investigation. Binary number systems, which are given in full on another page, are found in South America. Some of the Dravidian scales are binary; and the marked preference, not infrequently observed among savage races, for counting by pairs, is in itself a sufficient refutation of this theory. Still it is an unquestionable fact that this binary tendency is more pronounced among the Australians than among any other extensive number of kindred races. They seldom count in words above 4, and almost never as high as 7. One of the most careful observers among them expresses his doubt as to a native's ability to discover the loss of two pins, if he were first shown seven pins in a row, and then two were removed without his knowledge.[168] But he believes that if a single pin were removed from the seven, the Blackfellow would become conscious of its loss. This is due to his habit of counting by pairs, which enables him to discover whether any number within reasonable limit is odd or even. Some of the negro tribes of Africa, and of the Indian tribes of America, have the same habit. Progression by pairs may seem to some tribes as natural as progression by single units. It certainly is not at all rare; and in Australia its influence on spoken number systems is most apparent.

Any number system which passes the limit 10 is reasonably sure to have either a quinary, a decimal, or a vigesimal structure. A binary scale could, as it is developed in primitive languages, hardly extend to 20, or even to 10, without becoming exceedingly cumbersome. A binary scale inevitably suggests a wretchedly low degree of mental development, which stands in the way of the formation of any number scale worthy to be dignified by the name of system. Take, for example, one of the dialects found among the western tribes of the Torres Straits, where, in general, but two numerals are found to exist. In this dialect the method of counting is:

1. urapun.

2. okosa.

3. okosa urapun = 2-1.

4. okosa okosa = 2-2.

5. okosa okosa urapun = 2-2-1.

6. okosa okosa okosa = 2-2-2.

Anything above 6 they call *ras*, a lot.

For the sake of uniformity we may speak of this as a “system.” But in so doing, we give to the legitimate meaning of the word a severe strain. The customs and modes of life of these people are not such as to require the use of any save the scanty list of numbers given above; and their mental poverty prompts them to call 3, the first number above a single pair, 2-1. In the same way, 4 and 6 are respectively 2 pairs and 3 pairs, while 5 is 1 more than 2 pairs. Five objects, however, they sometimes denote by *urapuni-getal*, 1 hand. A precisely similar condition is found to prevail respecting the arithmetic of all the Australian tribes. In some cases only two numerals are found, and in others three. But in a very great number of the native languages of that continent the count proceeds by pairs, if indeed it proceeds at all. Hence we at once reject the theory that Australian arithmetic, or Australian counting, is essentially peculiar. It is simply a legitimate result, such as might be looked for in any part of the world, of the barbarism in which the races of that quarter of the world were sunk, and in which they were content to live.

The following examples of Australian and Tasmanian number systems show how scanty was the numerical ability possessed by these tribes, and illustrate fully their tendency to count by twos or pairs.

MURRAY RIVER.

1. enea.

2. petcheval.

3. petchevalenea = 2-1.

4. petcheval peteheval = 2-2.

MAROURA.

1. nukee.

2. barkolo.

3. barkolo nuke = 2-1.

4. barkolo barkolo = 2-2.

LAKE KOPPERAMANA.

1. ngerna.

2. mondroo.

3. barkooloo.

4. mondroo mondroo = 2-2.

MORT NOULAR.

1. gamboden.

2. bengeroo.

3. bengeroganmel = 2-1.

4. bengeroovor bengeroo = 2 + 2.

WIMMERA.

1. keyap.

2. pollit.

3. pollit keyap = 2-1.

4. pollit pollit = 2-2.

POPHAM BAY.

1. motu.

2. lawitbari.

3. lawitbari-motu = 2-1.

KAMILAROI.

1. mal.

2. bularr.

3. guliba.

4. bularrbularr = 2-2.

5. bulaguliba = 2-3.

6. gulibaguliba = 3-3.

PORT ESSINGTON.

1. erad.

2. nargarik.

3. nargarikelerad = 2-1.

4. nargariknargarik = 2-2.

WARREGO.

1. tarlina.

2. barkalo.

3. tarlina barkalo = 1-2.

CROCKER ISLAND.

1. roka.

2. orialk.

3. orialkeraroka = 2-1.

WARRIOR ISLAND.[

1. woorapoo.

2. ocasara.

3. ocasara woorapoo = 2-1.

4. ocasara ocasara = 2-2.

DIPPIL.

1. kalim._2. buller._3. boppa._4. buller gira buller = 2 + 2._5. buller gira buller kalim = 2 + 2 + 1.

FRAZER'S ISLAND.

1. kalim.

2. bulla.

3. goorbunda.

4. bulla-bulla = 2-2.

MORETON'S BAY.

1. kunner.

2. budela.

3. muddan.

4. budela berdelu = 2-2.

ENCOUNTER BAY.

1. yamalaitye.

2. ningenk.

3. nepaldar.

4. kuko kuko = 2-2, or pair pair.

5. kuko kuko ki = 2-2-1.

6. kuko kuko kuko = 2-2-2.

7. kuko kuko kuko ki = 2-2-2-1.

ADELAIDE.

1. kuma.

2. purlaitye, or bula.

3. marnkutye.

4. yera-bula = pair 2.

5. yera-bula kuma = pair 2-1.

6. yera-bula purlaitye = pair 2.2.

WIRADUROI.

1. numbai.

2. bula.

3. bula-numbai = 2-1.

4. bungu = many.

5. bungu-galan = very many.

WIRRI-WIRRI.

1. mooray.

2. boollar.

3. belar mooray = 2-1.

4. boollar boollar = 2-2.

5. mongoonballa.

6. mongun mongun.

COOPER'S CREEK.

1. goona.

2. barkoola.

3. barkoola goona = 2-1.

4. barkoola barkoola = 2-2.

BOURKE, DARLING RIVER.

1. neecha.

2. boolla.

4. boolla neecha = 2-1.

3. boolla boolla = 2-2.

MURRAY RIVER, N.W. BEND.

1. mata.

2. rankool.

3. rankool mata = 2-1.

4. rankool rankool = 2-2.

YIT-THA.

1. mo.

2. thral.

3. thral mo = 2-1.

4. thral thral = 2-2.

PORT DARWIN.

1. kulagook.

2. kalletillick.

3. kalletillick kulagook = 2-1.

4. kalletillick kalletillick = 2-2.

CHAMPION BAY.

1. kootea.

2. woothera.

3. woothera kootea = 2-1.

4. woothera woothera = 2-2.

BELYANDO RIVER.

1. wogin.

2. booleroo.

3. booleroo wogin = 2-1.

4. booleroo booleroo = 2-2.

WARREGO RIVER.

1. onkera.

2. paulludy.

3. paulludy onkera = 2-1.

4. paulludy paulludy = 2-2.

RICHMOND RIVER.

1. yabra.

2. booroora.

3. booroora yabra = 2-1.

4. booroora booroora = 2-2.

PORT MACQUARIE.

1. warcol.

2. blarvo.

3. blarvo warcol = 2-1.

4. blarvo blarvo = 2-2.

HILL END.

1. miko.

2. bullagut.

3. bullagut miko = 2-1.

4. bullagut bullagut = 2-2.

MONEROO

1. boor.

2. wajala, blala.

3. blala boor = 2-1.

4. wajala wajala.

GONN STATION.

1. karp.

2. pellige.

3. pellige karp = 2-1.

4. pellige pellige = 2-2.

UPPER YARRA.

1. kaambo.

2. benjero.

3. benjero kaambo = 2-2.

4. benjero on benjero = 2-2.

OMEO.

1. bore.

2. warkolala.

3. warkolala bore = 2-1.

4. warkolala warkolala = 2-2.

SNOWY RIVER.

1. kootook.

2. boolong.

3. booloom catha kootook = 2 + 1.

4. booloom catha booloom = 2 + 2.

NGARRIMOWRO.

1. warrangen.

2. platir.

3. platir warrangen = 2-1.

4. platir platir = 2-2.

This Australian list might be greatly extended, but the scales selected may be taken as representative examples of Australian binary scales. Nearly all of them show a structure too clearly marked to require comment. In a few cases, however, the systems are to be regarded rather as showing a trace of binary structure, than as perfect examples of counting by twos. Examples of this nature are especially numerous in Curr's extensive list—the most complete collection of Australian vocabularies ever made.

A few binary scales have been found in South America, but they show no important variation on the Australian systems cited above. The only ones I have been able to collect are the following:

BAKAIRI.

1. tokalole.

2. asage.

3. asage tokalo = 2-1.

4. asage asage = 2-2.

ZAPARA.

1. nuquaqui.

2. namisciniqui.

3. haimuckumarachi.

4. namisciniqui ckara maitacka = 2 + 2.

5. namisciniqui ckara maitacka nuquaqui = 2 pairs + 1.

6. haimuckumaracki ckaramsitacka = 3 pairs.

APINAGES.

1. pouchi.

2. at croudou.

3. at croudi-pshi = 2-1.

4. agontad-acroudo = 2-2.

COTOXO.

1. ihueto.

2. ize.

3. ize-te-hueto = 2-1.

4. ize-te-seze = 2-2.

5. ize-te-seze-hue = 2-2-1.

MBAYI.

1. uninitegui.

2. iniguata.

3. iniguata dugani = 2 over.

4. iniguata driniguata = 2-2.

5. oguidi = many.

TAMA.

1. teyo.

2. cayapa.

3. cho-teyo = 2 + 1.

4. cayapa-ria = 2 again.

5. cia-jente = hand.

CURETU.

1. tchudyu.

2. ap-adyu.

3. arayu.

4. apaedyai = 2 + 2.

5. tchumupa.

If the existence of number systems like the above are to be accounted for simply on the ground of low civilization, one might reasonably expect to find ternary and and quaternary scales, as well as binary. Such scales actually exist, though not in such numbers as the binary. An example of the former is the Betoya scale, which runs thus:

1. edoyoyoi.

2. edoi = another.

3. ibutu = beyond.

4. ibutu-edoyoyoi = beyond 1, or 3-1.

5. ru-mocoso = hand.

The Kamilaroi scale, given as an example of binary formation, is partly ternary; and its word for 6, *guliba guliba*, 3-3, is purely ternary. An occasional ternary trace is also found in number systems otherwise decimal or quinary vigesimal; as the *dlkunoutl*, second 3, of the Haida Indians of British Columbia. The Karens of India in a system otherwise strictly decimal, exhibit the following binary-ternary-quaternary vagary:

6. then tho = 3x2.

7. then tho ta = 3x2+1.

8. lwie tho = 4x2.

9. lwie tho ta = 4x2+1.

In the Wokka dialect, found on the Burnett River, Australia, a single ternary numeral is found, thus:

1. karboon.

2. wombura.

3. chrommunda.

4. chrommuda karboon = 3-1.

Instances of quaternary numeration are less rare than are those of ternary, and there is reason to believe that this method of counting has been practised more extensively than any other, except the binary and the three natural methods, the quinary, the decimal, and the vigesimal. The number of fingers on one hand is, excluding the thumb, four. Possibly there have been tribes among which counting by fours arose as a legitimate, though unusual, result of finger counting; just as there are, now and then, individuals who count on their fingers with the forefinger as a starting-point. But no such practice has ever been observed among savages, and such theorizing is the merest guess-work. Still a definite tendency to count by fours is sometimes met with, whatever be its origin. Quaternary traces are repeatedly to be found among the Indian languages of British Columbia. In describing the Columbians, Bancroft says: "Systems of numeration are simple, proceeding by fours, fives, or tens, according to the different languages…." The same preference for four is said to have existed in primitive times in the languages of Central Asia, and that this form of numeration, resulting in scores of 16 and 64, was a development of finger counting.

In the Hawaiian and a few other languages of the islands of the central Pacific, where in general the number systems employed are decimal, we find a most interesting case of the development, within number scales already well established, of both binary and quaternary systems. Their origin seems to have been perfectly natural, but the systems themselves must have been perfected very slowly. In Tahitian, Rarotongan, Mangarevan, and other dialects found in the neighbouring islands of those southern latitudes, certain of the higher units, *tekau*, *rau*, *mano*, which originally signified 10, 100, 1000, have become doubled in value, and now stand for 20, 200, 2000. In Hawaiian and other dialects they have again been doubled, and there they stand for 40, 400, 4000. In the Marquesas group both forms are found, the former in the southern, the latter in the northern, part of the archipelago; and it seems probable that one or both of these methods of numeration are scattered somewhat widely throughout that region. The origin of these methods is probably to be found in the fact that, after the migration from the west toward the east, nearly all the objects the natives would ever count in any great numbers were small,—as yams, cocoanuts, fish, etc.,—and would be most conveniently counted by pairs. Hence the native, as he counted one pair, two pairs, etc., might readily say *one*, *two*, and so on, omitting the word "pair" altogether. Having much more frequent occasion to employ this secondary than the primary meaning of his numerals, the native would easily allow the original significations to fall into disuse, and in the lapse of time to be entirely forgotten. With a subsequent migration to the northward a second duplication might take place, and so produce the singular effect of giving to the same numeral word three different meanings in different parts of Oceania. To illustrate the former or binary method of numeration, the Tahuatan, one of the southern dialects of the Marquesas group, may be employed. Here the ordinary numerals are:

1. tahi,

10. onohuu.

20. takau.

200. au.

2,000. mano.

20,000. tini.

20,000. tufa.

2,000,000. pohi.

In counting fish, and all kinds of fruit, except breadfruit, the scale begins with *tauna*, pair, and then, omitting *onohuu*, they employ the same words again, but in a modified sense. *Takau* becomes 10, *au* 100, etc.; but as the word "pair" is understood in each case, the value is the same as before. The table formed on this basis would be:

2 (units) = 1 tauna = 2.

10 tauna = 1 takau = 20.

10 takau = 1 au = 200.

10 au = 1 mano = 2000.

10 mano = 1 tini = 20,000.

10 tini = 1 tufa = 200,000.

10 tufa = 1 pohi = 2,000,000.

For counting breadfruit they use *pona*, knot, as their unit, breadfruit usually being tied up in knots of four. *Takau* now takes its third signification, 40, and becomes the base of their breadfruit system, so to speak. For some unknown reason the next unit, 400, is expressed by *tauau*, while *au*, which is the term that would regularly stand for that number, has, by a second duplication, come to signify 800. The next unit, *mano*, has in a similar manner been twisted out of its original sense, and in counting breadfruit is made to serve for 8000. In the northern, or Nukuhivan Islands, the decimal-quaternary system is more regular. It is in the counting of breadfruit only,

4 breadfruits = 1 pona = 4.

10 pona = 1 toha = 40.

10 toha = 1 au = 400.

10 au = 1 mano = 4000.

10 mano = 1 tini = 40,000.

10 tini = 1 tufa = 400,000.

10 tufa = 1 pohi = 4,000,000.

In the Hawaiian dialect this scale is, with slight modification, the universal scale, used not only in counting breadfruit, but any other objects as well. The result is a complete decimal-quaternary system, such as is found nowhere else in the world except in this and a few of the neighbouring dialects of the Pacific. This scale, which is almost identical with the Nukuhivan, is

4 units = 1 ha or tauna = 4.

10 tauna = 1 tanaha = 40.

10 tanaha = 1 lau = 400.

10 lau = 1 mano = 4000.

10 mano = 1 tini = 40,000.

10 tini = 1 lehu = 400,000.

The quaternary element thus introduced has modified the entire structure of the Hawaiian number system. Fifty is *tanaha me ta umi*, 40 + 10; 76 is 40 + 20 + 10 + 6; 100 is *ua tanaha ma tekau*, 2_40 + 10; 200 is _lima tanaha_, 5_40; and 864,895 is 2_400,000 + 40,000 + 6_4000 + 2_400 + 2_40 + 10 + 5. Such examples show that this secondary influence, entering and incorporating itself as a part of a well-developed decimal system, has radically changed it by the establishment of 4 as the primary number base. The role which 10 now plays is peculiar. In the natural formation of a quaternary scale new units would be introduced at 16, 64, 256, etc.; that is, at the square, the cube, and each successive power of the base. But, instead of this, the new units are introduced at 10_4, 100 x 4, 1000_4, etc.; that is, at the products of 4 by each successive power of the old base. This leaves the scale a decimal scale still, even while it may justly be called quaternary; and produces one of the most singular and interesting instances of number-system formation that has ever been observed. In this connection it is worth noting that these Pacific island number scales have been developed to very high limits—in some cases into the millions. The numerals for these large numbers do not seem in any way indefinite, but rather to convey to the mind of the native an idea as clear as can well be conveyed by numbers of such magnitude. Beyond the limits given, the islanders have indefinite expressions, but as far as can be ascertained these are only used when the limits given above have actually been passed. To quote one more example, the Hervey Islanders, who have a binary-decimal scale, count as follows:

5 kaviri (bunches of cocoanuts) = 1 takau = 20.

10 takau = 1 rau = 200.

10 rau = 1 mano = 2000.

10 mano = 1 kiu = 20,000.

10 kiu = 1 tini = 200,000.

Anything above this they speak of in an uncertain way, as *mano mano* or *tini tini*, which may, perhaps, be paralleled by our English phrases “myriads upon myriads,” and “millions of millions.” It is most remarkable that the same quarter of the globe should present us with the stunted number sense of the Australians, and, side by side with it, so extended and intelligent an appreciation of numerical values as that possessed by many of the lesser tribes of Polynesia.

The Luli of Paraguay show a decided preference for the base 4. This preference gives way only when they reach the number 10, which is an ordinary digit numeral. All numbers above that point belong rather to decimal than to quaternary numeration. Their numerals are:

1. alapea.

2. tamop.

3. tamlip.

4. lokep.

5. lokep moile alapea = 4 with 1, or is-alapea = hand 1.

6. lokep moile tamop = 4 with 2.

7. lokep moile tamlip = 4 with 3.

8. lokep moile lokep = 4 with 4.

9. lokep moile lokep alapea = 4 with 4-1.

10. is yaoum = all the fingers of hand.

11. is yaoum moile alapea = all the fingers of hand with 1.

20. is elu yaoum = all the fingers of hand and foot.

30. is elu yaoum moile is-yaoum = all the fingers of hand and foot with_all the fingers of hand.

Still another instance of quaternary counting, this time carrying with it a suggestion of binary influence, is furnished by the Mocobi of the Parana region. Their scale is exceedingly rude, and they use the fingers and toes almost exclusively in counting; only using their spoken numerals when, for any reason, they wish to dispense with the aid of their hands and feet. Their first eight numerals are:

1. iniateda.

2. inabaca.

3. inabacao caini = 2 above.

4. inabacao cainiba = 2 above 2; or natolatata.

5. inibacao cainiba iniateda = 2 above 2+1; or natolatata iniateda = 4-1.

6. natolatatata inibaca = 4+2.

7. natolata inibacao-caini = 4+2 above.

8. natolata-natolata = 4+4.

There is probably no recorded instance of a number system formed on 6, 7, 8, or 9 as a base. No natural reason exists for the choice of any of these numbers for such a purpose; and it is hardly conceivable that any race should proceed beyond the unintelligent binary or quaternary stage, and then begin the formation of a scale for counting with any other base than one of the three natural bases to which allusion has already been made. Now and then some anomalous fragment is found imbedded in an otherwise regular system, which carries us back to the time when the savage was groping his way onward in his attempt to give expression to some number greater than any he had ever used before; and now and then one of these fragments is such as to lead us to the border land of the might-have-been, and to cause us to speculate on the possibility of so great a numerical curiosity as a senary or a septenary scale. The Bretons call 18 *triouec'h*, 3-6, but otherwise their language contains no hint of counting by sixes; and we are left at perfect liberty to theorize at will on the existence of so unusual a number word. Pott remarks208 that the Bolans, of western Africa, appear to make some use of 6 as their number base, but their system, taken as a whole, is really a quinary-decimal. The language of the Sundas,[209] or mountaineers of Java, contains traces of senary counting. The Akra words for 7 and 8, *paggu* and *paniu*, appear to mean 6-1 and 7-1, respectively; and the same is true of the corresponding Tambi words *pagu* and *panjo*. The Watji tribe call 6 *andee*, and 7 *anderee*, which probably means 6-1. These words are to be regarded as accidental variations on the ordinary laws of formation, and are no more significant of a desire to count by sixes than is the Wallachian term *deu-maw*, which expresses 18 as 2-9, indicates the existence of a scale of which 9 is the base. One remarkably interesting number system is that exhibited by the Mosquito tribe of Central America, who possess an extensive quinary-vigesimal scale containing one binary and three senary compounds. The first ten words of this singular scale, which has already been quoted, are:

1. kumi.

2. wal.

3. niupa.

4. wal-wal = 2+2.

5. mata-sip = fingers of one hand.

6. matlalkabe.

7. matlalkabe pura kumi = 6 + 1.

8. matlalkabe pura wal = 6 + 2.

9. matlalkabe pura niupa = 6 + 3.

10. mata-wal-sip = fingers of the second hand.

In passing from 6 to 7, this tribe, also, has varied the almost universal law of progression, and has called 7 6-1. Their 8 and 9 are formed in a similar manner; but at 10 the ordinary method is resumed, and is continued from that point onward. Few number systems contain as many as three numerals which are associated with 6 as their base. In nearly all instances we find such numerals singly, or at most in pairs; and in the structure of any system as a whole, they are of no importance whatever. For example, in the Pawnee, a pure decimal scale, we find the following odd sequence:

6. shekshabish.

7. petkoshekshabish = 2-6, *i.e.* 2d 6.

8. touwetshabish = 3-6, *i.e.* 3d 6.

9. loksherewa = 10 – 1.

In the Uainuma scale the expressions for 7 and 8 are obviously referred to 6, though the meaning of 7 is not given, and it is impossible to guess what it really does signify. The numerals in question are:

6. aira-ettagapi.

7. aira-ettagapi-hairiwigani-apecapecapsi.

8. aira-ettagapi-matschahma = 6 + 2.

In the dialect of the Mille tribe a single trace of senary counting appears, as the numerals given below show:

6. dildjidji.

7. dildjidji me djuun = 6 + 1.

Finally, in the numerals used by the natives of the Marshall Islands, the following curiously irregular sequence also contains a single senary numeral:

6. thil thino = 3 + 3.

7. thilthilim-thuon = 6 + 1.

8. rua-li-dok = 10 – 2.

9. ruathim-thuon = 10 – 2 + 1.

Many years ago a statement appeared which at once attracted attention and awakened curiosity. It was to the effect that the Maoris, the aboriginal inhabitants of New Zealand, used as the basis of their numeral system the number 11; and that the system was quite extensively developed, having simple words for 121 and 1331, *i.e*. for the square and cube of 11. No apparent reason existed for this anomaly, and the Maori scale was for a long time looked upon as something quite exceptional and outside all ordinary rules of number-system formation. But a closer and more accurate knowledge of the Maori language and customs served to correct the mistake, and to show that this system was a simple decimal system, and that the error arose from the following habit. Sometimes when counting a number of objects the Maoris would put aside 1 to represent each 10, and then those so set aside would afterward be counted to ascertain the number of tens in the heap. Early observers among this people, seeing them count 10 and then set aside 1, at the same time pronouncing the word *tekau*, imagined that this word meant 11, and that the ignorant savage was making use of this number as his base. This misconception found its way into the early New Zealand dictionary, but was corrected in later editions. It is here mentioned only because of the wide diffusion of the error, and the interest it has always excited.

Aside from our common decimal scale, there exist in the English language other methods of counting, some of them formal enough to be dignified by the term *system*—as the sexagesimal method of measuring time and angular magnitude; and the duodecimal system of reckoning, so extensively used in buying and selling. Of these systems, other than decimal, two are noticed by Tylor, and commented on at some length, as follows:

“One is the well-known dicing set, *ace*, *deuce*, *tray*, *cater*, *cinque*, *size*; thus *size-ace* is 6-1, *cinques* or *sinks*, double 5. These came to us from France, and correspond with the common French numerals, except *ace*, which is Latin *as*, a word of great philological interest, meaning ‘one.’ The other borrowed set is to be found in the *Slang Dictionary*. It appears that the English street-folk have adopted as a means of secret communication a set of Italian numerals from the organ-grinders and image-sellers, or by other ways through which Italian or Lingua Franca is brought into the low neighbourhoods of London. In so doing they have performed a philological operation not only curious but instructive. By copying such expressions as *due soldi*, *tre soldi*, as equivalent to ‘twopence,’ ‘threepence,’ the word *saltee* became a recognized slang term for ‘penny’; and pence are reckoned as follows:

oney saltee 1d. uno soldo.

dooe saltee 2d. due soldi.

tray saltee 3d. tre soldi.

quarterer saltee 4d. quattro soldi.

chinker saltee 5d. cinque soldi.

say saltee 6d. sei soldi.

say oney saltee, or setter saltee 7d. sette soldi.

say dooe saltee, or otter saltee 8d. otto soldi.

say tray saltee, or nobba saltee 9d. nove soldi.

say quarterer saltee, or dacha saltee 10d. dieci soldi.

say chinker saltee or dacha oney saltee 11d. undici soldi.

oney beong 1s.

a beong say saltee 1s. 6d.

dooe beong say saltee, or madza caroon 2s. 6d. (half-crown, mezza corona).

One of these series simply adopts Italian numerals decimally. But the other, when it has reached 6, having had enough of novelty, makes 7 by 6-1, and so forth. It is for no abstract reason that 6 is thus made the turning-point, but simply because the costermonger is adding pence up to the silver sixpence, and then adding pence again up to the shilling. Thus our duodecimal coinage has led to the practice of counting by sixes, and produced a philological curiosity, a real senary notation."

In addition to the two methods of counting here alluded to, another may be mentioned, which is equally instructive as showing how readily any special method of reckoning may be developed out of the needs arising in connection with any special line of work. As is well known, it is the custom in ocean, lake, and river navigation to measure soundings by the fathom. On the Mississippi River, where constant vigilance is needed because of the rapid shifting of sand-bars, a special sounding nomenclature has come into vogue, which the following terms will illustrate:

5 ft. = five feet.

6 ft. = six feet.

9 ft. = nine feet.

10-1/2 ft. = a quarter less twain; *i.e.* a quarter of a fathom less than 2.

12 ft. = mark twain.

13-1/2 ft. = a quarter twain.

16-1/2 ft. = a quarter less three.

18 ft. = mark three.

19-1/2 ft. = a quarter three.

24 ft. = deep four.

As the soundings are taken, the readings are called off in the manner indicated in the table; 10-1/2 feet being “a quarter less twain,” 12 feet “mark twain,” etc. Any sounding above “deep four” is reported as “no bottom.” In the Atlantic and Gulf waters on the coast of this country the same system prevails, only it is extended to meet the requirements of the deeper soundings there found, and instead of “six feet,” “mark twain,” etc., we find the fuller expressions, “by the mark one,” “by the mark two,” and so on, as far as the depth requires. This example also suggests the older and far more widely diffused method of reckoning time at sea by bells; a system in which “one bell,” “two bells,” “three bells,” etc., mark the passage of time for the sailor as distinctly as the hands of the clock could do it. Other examples of a similar nature will readily suggest themselves to the mind.

Two possible number systems that have, for purely theoretical reasons, attracted much attention, are the octonary and the duodecimal systems. In favour of the octonary system it is urged that 8 is an exact power of 2; or in other words, a large number of repeated halves can be taken with 8 as a starting-point, without producing a fractional result. With 8 as a base we should obtain by successive halvings, 4, 2, 1. A similar process in our decimal scale gives 5, 2-1/2, 1-1/4. All this is undeniably true, but, granting the argument up to this point, one is then tempted to ask “What of it?” A certain degree of simplicity would thereby be introduced into the Theory of Numbers; but the only persons sufficiently interested in this branch of mathematics to appreciate the benefit thus obtained are already trained mathematicians, who are concerned rather with the pure science involved, than with reckoning on any special base. A slightly increased simplicity would appear in the work of stockbrokers, and others who reckon extensively by quarters, eighths, and sixteenths. But such men experience no difficulty whatever in performing their mental computations in the decimal system; and they acquire through constant practice such quickness and accuracy of calculation, that it is difficult to see how octonary reckoning would materially assist them. Altogether, the reasons that have in the past been adduced in favour of this form of arithmetic seem trivial. There is no record of any tribe that ever counted by eights, nor is there the slightest likelihood that such a system could ever meet with any general favour. It is said that the ancient Saxons used the octonary system, but how, or for what purposes, is not stated. It is not to be supposed that this was the common system of counting, for it is well known that the decimal scale was in use as far back as the evidence of language will take us. But the field of speculation into which one is led by the octonary scale has proved most attractive to some, and the conclusion has been soberly reached, that in the history of the Aryan race the octonary was to be regarded as the predecessor of the decimal scale. In support of this theory no direct evidence is brought forward, but certain verbal resemblances. Those ignes fatuii of the philologist are made to perform the duty of supporting an hypothesis which would never have existed but for their own treacherous suggestions. Here is one of the most attractive of them:

Between the Latin words *novus*, new, and *novem*, nine, there exists a resemblance so close that it may well be more than accidental. Nine is, then, the *new* number; that is, the first number on a new count, of which 8 must originally have been the base. Pursuing this thought by investigation into different languages, the same resemblance is found there. Hence the theory is strengthened by corroborative evidence. In language after language the same resemblance is found, until it seems impossible to doubt, that in prehistoric times, 9 *was* the new number—the beginning of a second tale. The following table will show how widely spread is this coincidence:

Sanskrit, navan = 9. nava = new.

Persian, nuh = 9. nau = new.

Greek, [Greek: ennea] = 9. [Greek: neos] = new.

Latin, novem = 9. novus = new.

German, neun = 9. neu = new.

Swedish, nio = 9. ny = new.

Dutch, negen = 9. nieuw = new.

Danish, ni = 9. ny = new.

Icelandic, nyr = 9. niu = new.

English, nine = 9. new = new.

French, neuf = 9. nouveau = new.

Spanish, nueve = 9. neuvo = new.

Italian, nove = 9. nuovo = new.

Portuguese, nove = 9. novo = new.

Irish, naoi = 9. nus = new.

Welsh, naw = 9. newydd = new.

Breton, nevez = 9. nuhue = new.

This table might be extended still further, but the above examples show how widely diffused throughout the Aryan languages is this resemblance. The list certainly is an impressive one, and the student is at first thought tempted to ask whether all these resemblances can possibly have been accidental. But a single consideration sweeps away the entire argument as though it were a cobweb. All the languages through which this verbal likeness runs are derived directly or indirectly from one common stock; and the common every-day words, “nine” and “new,” have been transmitted from that primitive tongue into all these linguistic offspring with but little change. Not only are the two words in question akin in each individual language, but *they are akin in all the languages*. Hence all these resemblances reduce to a single resemblance, or perhaps identity, that between the Aryan words for "nine" and "new." This was probably an accidental resemblance, no more significant than any one of the scores of other similar cases occurring in every language. If there were any further evidence of the former existence of an Aryan octonary scale, the coincidence would possess a certain degree of significance; but not a shred has ever been produced which is worthy of consideration. If our remote ancestors ever counted by eights, we are entirely ignorant of the fact, and must remain so until much more is known of their language than scholars now have at their command. The word resemblances noted above are hardly more significant than those occurring in two Polynesian languages, the Fatuhivan and the Nakuhivan, where "new" is associated with the number 7. In the former case 7 is *fitu*, and "new" is *fou*; in the latter 7 is *hitu*, and "new" is *hou*. But no one has, because of this likeness, ever suggested that these tribes ever counted by the senary method. Another equally trivial resemblance occurs in the Tawgy and the Kamassin languages, thus:

TAWGY. KAMASSIN.

8. siti-data = 2x4.

8. sin-the'de = 2x4.

9. nameaitjuma = another.

9. amithun = another.

But it would be childish to argue, from this fact alone, that either 4 or 8 was the number base used.

In a recent antiquarian work of considerable interest, the author examines into the question of a former octonary system of counting among the various races of the world, particularly those of Asia, and brings to light much curious and entertaining material respecting the use of this number. Its use and importance in China, India, and central Asia, as well as among some of the islands of the Pacific, and in Central America, leads him to the conclusion that there was a time, long before the beginning of recorded history, when 8 was the common number base of the world. But his conclusion has no basis in his own material even. The argument cannot be examined here, but any one who cares to investigate it can find there an excellent illustration of the fact that a pet theory may take complete possession of its originator, and reduce him finally to a state of infantile subjugation.

Of all numbers upon which a system could be based, 12 seems to combine in itself the greatest number of advantages. It is capable of division by 2, 3, 4, and 6, and hence admits of the taking of halves, thirds, quarters, and sixths of itself without the introduction of fractions in the result. From a commercial stand-point this advantage is very great; so great that many have seriously advocated the entire abolition of the decimal scale, and the substitution of the duodecimal in its stead. It is said that Charles XII. of Sweden was actually contemplating such a change in his dominions at the time of his death. In pursuance of this idea, some writers have gone so far as to suggest symbols for 10 and 11, and to recast our entire numeral nomenclature to conform to the duodecimal base. Were such a change made, we should express the first nine numbers as at present, 10 and 11 by new, single symbols, and 12 by 10. From this point the progression would be regular, as in the decimal scale—only the same combination of figures in the different scales would mean very different things. Thus, 17 in the decimal scale would become 15 in the duodecimal; 144 in the decimal would become 100 in the duodecimal; and 1728, the cube of the new base, would of course be represented by the figures 1000.

It is impossible that any such change can ever meet with general or even partial favour, so firmly has the decimal scale become intrenched in its position. But it is more than probable that a large part of the world of trade and commerce will continue to buy and sell by the dozen, the gross, or some multiple or fraction of the one or the other, as long as buying and selling shall continue. Such has been its custom for centuries, and such will doubtless be its custom for centuries to come. The duodecimal is not a natural scale in the same sense as are the quinary, the decimal, and the vigesimal; but it is a system which is called into being long after the complete development of one of the natural systems, solely because of the simple and familiar fractions into which its base is divided. It is the scale of civilization, just as the three common scales are the scales of nature. But an example of its use was long sought for in vain among the primitive races of the world. Humboldt, in commenting on the number systems of the various peoples he had visited during his travels, remarked that no race had ever used exclusively that best of bases, 12. But it has recently been announced that the discovery of such a tribe had actually been made, and that the Aphos of Benue, an African tribe, count to 12 by simple words, and then for 13 say 12-1, for 14, 12-2, etc. This report has yet to be verified, but if true it will constitute a most interesting addition to anthropological knowledge.