Chapter 7

MISCELLANEOUS NUMBER BASES.

THE VIGESIMAL SYSTEM.

In its ordinary development the quinary system is almost sure to merge into either the decimal or the vigesimal system, and to form, with one or the other or both of these, a mixed system of counting. In Africa, Oceanica, and parts of North America, the union is almost always with the decimal scale; while in other parts of the world the quinary and the vigesimal systems have shown a decided affinity for each other. It is not to be understood that any geographical law of distribution has ever been observed which governs this, but merely that certain families of races have shown a preference for the one or the other method of counting. These families, disseminating their characteristics through their various branches, have produced certain groups of races which exhibit a well-marked tendency, here toward the decimal, and there toward the vigesimal form of numeration. As far as can be ascertained, the choice of the one or the other scale is determined by no external circumstances, but depends solely on the mental characteristics of the tribes themselves. Environment does not exert any appreciable influence either. Both decimal and vigesimal numeration are found indifferently in warm and in cold countries; in fruitful and in barren lands; in maritime and in inland regions; and among highly civilized or deeply degraded peoples.

Whether or not the principal number base of any tribe is to be 20 seems to depend entirely upon a single consideration; are the fingers alone used as an aid to counting, or are both fingers and toes used? If only the fingers are employed, the resulting scale must become decimal if sufficiently extended. If use is made of the toes in addition to the fingers, the outcome must inevitably be a vigesimal system. Subordinate to either one of these the quinary may and often does appear. It is never the principal base in any extended system.

To the statement just made respecting the origin of vigesimal counting, exception may, of course, be taken. In the case of

numeral scales like the Welsh, the Nahuatl, and many others where the exact meanings of the numerals cannot be ascertained, no proof exists that the ancestors of these peoples ever used either finger or toe counting; and the sweeping statement that any vigesimal scale is the outgrowth of the use of these natural counters is not susceptible of proof. But so many examples are met with in which the origin is clearly of this nature, that no hesitation is felt in putting the above forward as a general explanation for the existence of this kind of counting. Any other origin is difficult to reconcile with observed facts, and still more difficult to reconcile with any rational theory of number system development.
Dismissing from consideration the quinary scale, let us briefly examine once more the natural process of evolution through which the decimal and the vigesimal scales come into being. After the completion of one count of the fingers the savage announces his result in some form which definitely states to his mind the fact that the end of a well-marked series has been reached. Beginning again, he now repeats his count of 10, either on his own fingers or on the fingers of another. With the completion of the second 10 the result is announced, not in a new unit, but by means of a duplication of the term already used. It is scarcely credible that the unit unconsciously adopted at the termination of the first count should now be dropped, and a new one substituted in its place. When the method here described is employed, 20 is not a natural unit to which higher numbers may be referred. It is wholly artificial; and it would be most surprising if it were adopted. But if the count of the second 10 is made on the toes in place of the fingers, the element of repetition which entered into the previous method is now wanting. Instead of referring each new number to the 10 already completed, the savage is still feeling his way along, designating his new terms by such phrases as “1 on the foot,” “2 on the other foot,” etc. And now, when 20 is reached, a single series is finished instead of a double series as before; and the result is expressed in one of the many methods already noticed—”one man,” “hands and feet,” “the feet finished,” “all the fingers of hands and feet,” or some equivalent formula. Ten is no longer the natural base. The number from which the new start is made is 20, and the resulting scale is inevitably vigesimal. If pebbles or sticks are used instead of fingers, the system will probably be decimal. But back of the stick and pebble counting the 10 natural counters always exist, and

to them we must always look for the origin of this scale.

In any collection of the principal vigesimal number systems of the world, one would naturally begin with those possessed by the Celtic races of Europe. These races, the earliest European peoples of whom we have any exact knowledge, show a preference for counting by twenties, which is almost as decided as that manifested by Teutonic races for counting by tens. It has been conjectured by some writers that the explanation for this was to be found in the ancient commercial intercourse which existed between the Britons and the Carthaginians and Phoenicians, whose number systems showed traces of a vigesimal tendency. Considering the fact that the use of vigesimal counting was universal among Celtic races, this explanation is quite gratuitous. The reason why the Celts used this method is entirely unknown, and need not concern investigators in the least. But the fact that they did use it is important, and commands attention. The five Celtic languages, Breton, Irish, Welsh, Manx, and Gaelic, contain the following well-defined vigesimal scales. Only the principal or characteristic numerals are given, those being sufficient to enable the reader to follow intelligently the growth of the systems. Each contains the decimal element also, and is, therefore, to be regarded as a mixed decimal-vigesimal system.

IRISH.

10. deic.

20. fice.

30. triocad = 3-10

40. da ficid = 2-20.

50. caogad = 5-10.

60. tri ficid = 3-20.

70. reactmoga = 7-10.

80. ceitqe ficid = 4-20.

90. nocad = 9-10.

100. cead.

1000. mile.

GAELIC.

10. deich.

20. fichead.

30. deich ar fichead = 10 + 20.

40. da fhichead = 2-20.

50. da fhichead is deich = 40 + 10.

60. tri fichead = 3-20.

70. tri fichead is deich = 60 + 10.

80. ceithir fichead = 4-20.

90. ceithir fichead is deich = 80 + 10.

100. ceud. 1000. mile.

WELSH.

10. deg.

20. ugain.

30. deg ar hugain = 10 + 20.

40. deugain = 2-20.

50. deg a deugain = 10 + 40.

60. trigain = 3-20.

70. deg a thrigain = 10 + 60.

80. pedwar ugain = 4-20.

90. deg a pedwar ugain = 80 + 10.

100. cant.

MANX.

10. jeih.

20. feed.

30. yn jeih as feed = 10 + 20.

40. daeed = 2-20.

50. jeih as daeed = 10 + 40.

60. three-feed = 3-20.

70. three-feed as jeih = 60 + 10.

80. kiare-feed = 4-20.

100. keead.

1000. thousane, or jeih cheead.

BRETON.

10. dec.

20. ueguend.

30. tregond = 3-10.

40. deu ueguend = 2-20.

50. hanter hand = half hundred.

60. tri ueguend = 3-20.

70. dec ha tri ueguend = 10 + 60.

80. piar ueguend = 4-20.

90. dec ha piar ueguend = 10 + 80.

100. cand.

120. hueh ueguend = 6-20.

140. seih ueguend = 7-20.

160. eih ueguend = 8-20.

180. nau ueguend = 9-20.

200. deu gand = 2-100.

240. deuzec ueguend = 12-20.

280. piarzec ueguend = 14-20.

300. tri hand, or pembzec ueguend.

400. piar hand = 4-100.

1000. mil.

These lists show that the native development of the Celtic number systems, originally showing a strong preference for the vigesimal method of progression, has been greatly modified by intercourse with Teutonic and Latin races. The higher numerals in all these languages, and in Irish many of the lower also, are seen at a glance to be decimal. Among the scales here given the Breton, the legitimate descendant of the ancient Gallic, is especially interesting; but here, just as in the other Celtic tongues, when we reach 1000, the familiar Latin term for that number appears in the various corruptions of *mille*, 1000, which was carried into the Celtic countries by missionary and military influences.

In connection with the Celtic language, mention must be made of the persistent vigesimal element which has held its place in French. The ancient Gauls, while adopting the language of their conquerors, so far modified the decimal system of Latin as to replace the natural *septante*, 70, *octante*, 80, *nonante*, 90, by *soixante-dix*, 60-10, *quatre-vingt*, 4-20, and *quatrevingt-dix*, 4-20-10. From 61 to 99 the French method of counting is wholly vigesimal, except for the presence of the one word *soixante*. In old French this element was still more pronounced. *Soixante* had not yet appeared; and 60 and 70 were *treis vinz*, 3-20, and *treis vinz et dis*, 3-20 and 10 respectively. Also, 120 was *six vinz*, 6-20, 140 was *sept-vinz*, etc. How far this method ever extended in the French language proper, it is, perhaps, impossible to say; but from the name of an almshouse, *les quinze-vingts*, which formerly existed in Paris, and was designed as a home for 300 blind persons, and from the *pembzek-ueguent*, 15-20, of the Breton, which still survives, we may infer that it was far enough to make it the current system of common life.

Europe yields one other example of vigesimal counting, in the number system of the Basques. Like most of the Celtic scales, the Basque seems to become decimal above 100. It does not appear to be related to any other European system, but to be quite isolated philologically. The higher units, as *mila*, 1000, are probably borrowed, and not native. The tens in the Basque scale are:

10. hamar.

20. hogei.

30. hogei eta hamar = 20 + 10.

40. berrogei = 2-20.

50. berrogei eta hamar = 2-20 + 10.

60. hirurogei = 3-20.

70. hirurogei eta hamar = 3-20 + 10.

80. laurogei = 4-20.

90. laurogei eta hamar = 4-20 + 10.

100. ehun.

1000. *milla*.

Besides these we find two or three numeral scales in Europe which contain distinct traces of vigesimal counting, though the scales are, as a whole, decidedly decimal. The Danish, one of the essentially Germanic languages, contains the following numerals:

30. tredive = 3-10.

40. fyrretyve = 4-10.

50. halvtredsindstyve = half (of 20) from 3-20.

60. tresindstyve = 3-20.

70. halvfierdsindstyve = half from 4-20.

80. fiirsindstyve = 4-20.

90. halvfemsindstyve = half from 5-20.

100. hundrede.

Germanic number systems are, as a rule, pure decimal systems; and the Danish exception is quite remarkable. We have, to be sure, such expressions in English as *three score*, *four score*, etc., and the

Swedish, Icelandic, and other languages of this group have similar terms. Still, these are not pure numerals, but auxiliary words rather, which belong to the same category as *pair*, *dozen*, *dizaine*, etc., while the Danish words just given are the ordinary numerals which form a part of the every-day vocabulary of that language. The method by which this scale expresses 50, 70, and 90 is especially noticeable. It will be met with again, and further examples of its occurrence given.

In Albania there exists one single fragment of vigesimal numeration, which is probably an accidental compound rather than the remnant of a former vigesimal number system. With this single exception the Albanian scale is of regular decimal formation. A few of the numerals are given for the sake of comparison:

30. tridgiete = 3-10.

40. dizet = 2-20.

50. pesedgiete = 5-10.

60. giastedgiete = 6-10, etc.

Among the almost countless dialects of Africa we find a comparatively small number of vigesimal number systems. The powers of the negro tribes are not strongly developed in counting, and wherever their numeral scales have been taken down by explorers they have almost always been found to be decimal or quinary-decimal. The small number I have been able to collect are here given. They are somewhat fragmentary, but are as complete as it was possible to make them.

AFFADEH.

10. dekang.

20. degumm.

30. piaske.

40. tikkumgassih = 20x2.

50. tikkumgassigokang = 20x2 + 10.

60. tikkumgakro = 20x3.

70. dungokrogokang = 20x3 + 10.

80. dukumgade = 20x4.

90. dukumgadegokang = 20x4 + 10.

100. miah (borrowed from the Arabs).

IBO.

10. iri.

20. ogu.

30. ogu n-iri = 20 + 10, or iri ato = 10x3.

40. ogu abuo = 20x2, or iri anno = 10x4.

100. ogu ise = 20x5.

VEI.

10. tan.

20. mo bande = a person finished.

30. mo bande ako tan = 20 + 10.

40. mo fera bande = 2_20.

100. mo soru bande = 5 persons finished.

YORUBA.

10. duup.

20. ogu.

30. ogbo.

40. ogo-dzi = 20x2.

60. ogo-ta = 20x3.

80. ogo-ri = 20x4.

100. ogo-ru = 20x5.

120. ogo-fa = 20x6.

140. ogo-dze = 20x7.

160. ogo-dzo = 20x8, etc.

EFIK.

10. duup.

20. edip.

30. edip-ye-duup = 20 + 10.

40. aba = 20x2.

60. ata = 20x3.

80. anan = 20x4.

100. ikie.

The Yoruba scale, to which reference has already been made, p. 70, again shows its peculiar structure, by continuing its vigesimal formation past 100 with no interruption in its method of numeral building. It will be remembered that none of the European scales showed this persistency, but passed at that point into decimal numeration. This will often be found to be the case; but now and then a scale will come to our notice whose vigesimal structure is continued, without any break, on into the hundreds and sometimes into the thousands.

BONGO.

10. kih.

20. mbaba kotu = 20x1.

40. mbaba gnorr = 20x2.

100. mbaba mui = 20x5.

MENDE.

10. pu.

20. nu yela gboyongo mai = a man finished.

30. nu yela gboyongo mahu pu = 20 + 10.

40. nu fele gboyongo = 2 men finished.

100. nu lolu gboyongo = 5 men finished.

NUPE.

10. gu-wo.

20. esin.

30. gbonwo.

40. si-ba = 2

20._50. arota.

60. sita = 3x20.

70. adoni.

80. sini = 4x20.

90. sini be-guwo = 80 + 10.

100. sisun = 5x20.

LOGONE.

10. chkan.

20. tkam.

30. tkam ka chkan = 20 + 10.

40. tkam ksde = 20x2.

50. tkam ksde ka chkan = 40 + 10.

60. tkam gachkir = 20x3.

100. mia (from Arabic).

1000. debu.

MUNDO.

10. nujorquoi.

20. tiki bere.

30. tiki bire nujorquoi = 20 + 10.

40. tiki borsa = 20x2.

50. tike borsa nujorquoi = 40 + 10.

MANDINGO.

10. tang.

20. mulu.

30. mulu nintang = 20 + 10.

40. mulu foola = 20x2.

50. mulu foola nintang = 40 + 10.

60. mulu sabba = 20x3.

70. mulu sabba nintang = 60 + 10.

80. mulu nani = 20x4.

90. mulu nani nintang = 80 + 10.

100. kemi.

This completes the scanty list of African vigesimal number systems that a patient and somewhat extended search has yielded. It is remarkable that the number is no greater. Quinary counting is not uncommon in the “Dark Continent,” and there is no apparent reason why vigesimal reckoning should be any less common than quinary. Any one investigating African modes of counting with the material at present accessible, will find himself hampered by the fact that few explorers have collected any except the first ten numerals. This leaves the formation of higher terms entirely unknown, and shows nothing beyond the quinary or non-quinary character of the system. Still, among those which Stanley, Schweinfurth, Salt, and others have collected, by far the greatest number are decimal. As our knowledge of African languages is extended, new examples of the vigesimal method may be brought to light. But our present information leads us to believe that they will be few in number.

In Asia the vigesimal system is to be found with greater frequency than in Europe or Africa, but it is still the exception. As Asiatic languages are much better known than African, it is probable that the future will add but little to our stock of knowledge on this point. New instances of counting by twenties may still be found in northern Siberia, where much ethnological work yet remains to be done, and where a tendency toward this form of numeration has been observed to exist. But the total number of Asiatic vigesimal scales must always remain small—quite insignificant in comparison with those of decimal formation.

In the Caucasus region a group of languages is found, in which all but three or four contain vigesimal systems. These systems are as follows:

ABKHASIA.

10. zpha-ba.

20. gphozpha = 2x10.

30. gphozphei zphaba = 20 + 10.

40. gphin-gphozpha = 2x20.

60. chin-gphozpha = 3x20.

80. phsin-gphozpha = 4x20.

100. sphki.

AVARI

10. antsh-go.

20. qo-go.

30. lebergo.

40. khi-qogo = 2x20.

50. khiqojalda antshgo = 40 + 10.

60. lab-qogo = 3x20.

70. labqojalda antshgo = 60 + 10.

80. un-qogo = 4x20.

100. nusgo.

KURI

10. tshud.

20. chad.

30. channi tshud = 20 + 10.

40. jachtshur.

50. jachtshurni tshud = 40 + 10.

60. put chad = 3x20.

70. putchanni tshud = 60 + 10.

80. kud-chad = 4x20.

90. kudchanni tshud = 80 + 10.

100. wis.

UDI

10. witsh.

20. qa.

30. sa-qo-witsh = 20 + 10.

40. pha-qo = 2x20.

50. pha-qo-witsh = 40 + 10.

60. chib-qo = 3x20.

70. chib-qo-witsh = 60 + 10.

80. bip-qo = 4x20.

90. bip-qo-witsh = 80 + 10.

100. bats._1000. hazar (Persian).

TCHETCHNIA

10. ith.

20. tqa.

30. tqe ith = 20 + 10.

40. sauz-tqa = 2x20.

50. sauz-tqe ith = 40 + 10.

60. chuz-tqa = 3x20.

70. chuz-tqe ith = 60 + 10.

80. w-iez-tqa = 4x20.

90. w-iez-tqe ith = 80 + 10.

100. b’e.

1000. ezir (akin to Persian).

THUSCH

10. itt.

20. tqa.

30. tqa-itt = 20 + 10.

40. sauz-tq = 2x20.

50. sauz-tqa-itt = 40 + 10.

60. chouz-tq = 3x20.

70. chouz-tqa-itt = 60 + 10.

80. dhewuz-tq = 4x20.

90. dhewuz-tqa-itt = 80 + 10.

100. phchauz-tq = 5x20.

200. itsha-tq = 10x20.

300. phehiitsha-tq = 15x20.

1000. satsh tqauz-tqa itshatqa = 2x20 x 20 + 200.

GEORGIA

10. athi.

20. otsi.

30. ots da athi = 20 + 10.

40. or-m-otsi = 2x20.

50. ormots da athi = 40 + 10.

60. sam-otsi = 3x20.

70. samots da athi = 60 + 10.

80. othch-m-otsi = 4x20.

90. othmots da athi = 80 + 10.

100. asi.

1000. ath-asi = 10x100.

LAZI

10. wit.

20. oets.

30. oets do wit = 20+10.

40. dzur en oets = 2x20.

50. dzur en oets do wit = 40 + 10.

60. dzum en oets = 3x20.

70. dzum en oets do wit = 60 + 10.

80. otch-an-oets = 4x20.

100. os.

1000. silia (akin to Greek).

CHUNSAG.

10. ants-go.

20. chogo.

30. chogela antsgo = 20 + 10.

40. kichogo = 2x20.

50. kichelda antsgo = 40 + 10.

60. taw chago = 3x20.

70. taw chogelda antsgo = 60 + 10.

80. uch’ chogo = 4x20.

90. uch’ chogelda antsgo.

100. nusgo.

1000. asargo (akin to Persian).

DIDO.

10. zino.

20. ku.

30. kunozino.

40. kaeno ku = 2x20.

50. kaeno kuno zino = 40 + 10.

60. sonno ku = 3x20.

70. sonno kuno zino = 60 + 10.

80. uino ku = 4x20.

90. uino huno zino = 80 + 10.

100. bischon.

400. kaeno kuno zino = 40x10.

AKARI

10. entzelgu.

20. kobbeggu.

30. lowergu.

40. kokawu = 2x20.

50. kikaldanske = 40 + 10.

60. secikagu.

70. kawalkaldansku = 3x20 + 10.

80. onkuku = 4x20.

90. onkordansku = 4x20 + 10.

100. nosku._1000. askergu (from Persian).

CIRCASSIA

10. psche.

20. to-tsch.

30. totsch-era-pschirre = 20 + 10.

40. ptl'i-sch = 4x10.

50. ptl'isch-era-pschirre = 40 + 10.

60. chi-tsch = 6x10.

70. chitsch-era-pschirre = 60 + 10.

80. toshitl = 20x4?

90. toshitl-era-pschirre = 80 + 10.

100. scheh.

1000. min (Tartar) or schi-psche = 100x10.

The last of these scales is an unusual combination of decimal and vigesimal. In the even tens it is quite regularly decimal, unless 80 is of the structure suggested above. On the other hand, the odd tens are formed in the ordinary vigesimal manner. The reason for this anomaly is not obvious. I know of no other number system that presents the same peculiarity, and cannot give any hypothesis which will satisfactorily account for its presence here. In nearly all the examples given the decimal becomes the leading element in the formation of all units above 100, just as was the case in the Celtic scales already noticed.

Among the northern tribes of Siberia the numeral scales appear to be ruder and less simple than those just examined, and the counting to be more consistently vigesimal than in any scale we have thus far met with. The two following examples are exceedingly interesting, as being among the best illustrations of counting by twenties that are to be found anywhere in the Old World.

TSCHUKSCHI.

10. migitken = both hands.

20. chlik-kin = a whole man.

30. chlikkin mingitkin parol = 20 + 10.

40. nirach chlikkin = 2x20.

100. milin chlikkin = 5x20.

200. mingit chlikkin = 10x20, *i.e.* 10 men.

1000. miligen chlin-chlikkin = 5x200, *i.e.* five (times) 10 men.

AINO.

10. wambi.

20. choz.

30. wambi i-doehoz = 10 from 40.

40. tochoz = 2x20.

50. wambi i-richoz = 10 from 60.

60. rechoz = 3x20.

70. wambi [i?] inichoz = 10 from 80.

80. inichoz = 4x20.

90. wambi aschikinichoz = 10 from 100.

100. aschikinichoz = 5x20.

110. wambi juwanochoz = 10 from 120.

120. juwano choz = 6x20.

130. wambi aruwanochoz = 10 from 140.

140. aruwano choz = 7x20.

150. wambi tubischano choz = 10 from 160.

160. tubischano choz = 8x20.

170. wambi schnebischano choz = 10 from 180.

180. schnebischano choz = 9x20.

190. wambi schnewano choz = 10 from 200.

200. schnewano choz = 10x20.

300. aschikinichoz i gaschima chnewano choz = 5x20+10x20.

400. toschnewano choz = 2 x (10x20).

500. aschikinichoz i gaschima toschnewano choz = 100 + 400.

600. reschiniwano choz = 3x200.

700. aschikinichoz i gaschima reschiniwano choz = 100 + 600.

800. inischiniwano choz = 4x200.

900. aschikinichoz i gaschima inischiniwano choz = 100 + 800.

1000. aschikini schinewano choz = 5x200.

2000. wanu schinewano choz = 10 x (10_20).

This scale is in one sense wholly vigesimal, and in another way it is not to be regarded as pure, but as mixed. Below 20 it is quinary, and, however far it might be extended, this quinary element would remain, making the scale quinary-vigesimal. But in another sense, also, the Aino system is not pure. In any unmixed vigesimal scale the word for 400 must be a simple word, and that number must be taken as the vigesimal unit corresponding to 100 in the decimal scale. But the Ainos have no simple numeral word for any number above 20, forming all higher numbers by combinations through one or more of the processes of addition, subtraction, and multiplication. The only number above 20 which is used as a unit is 200, which is expressed merely as 10 twenties. Any even number of hundreds, or any number of thousands, is then indicated as being so many times 10 twenties; and the odd hundreds are so many times 10 twenties, plus 5 twenties more. This scale is an excellent example of the cumbersome methods used by uncivilized races in extending their number systems beyond the ordinary needs of daily life.

In Central Asia a single vigesimal scale comes to light in the following fragment of the Leptscha scale, of the Himalaya region:

10. kati.

40. kafali = 4x10, or kha nat = 2x20.

50. kafano = 5x10, or kha nat sa kati = 2x20 + 10.

100. gjo, or kat.

Further to the south, among the Dravidian races, the vigesimal element is also found. The following will suffice to illustrate the number systems of these dialects, which, as far as the material at hand shows, are different from each other only in minor particulars:

MUNDARI.

10. gelea.

20. mi hisi.

30. mi hisi gelea = 20 + 10.

40. bar hisi = 2x20.

60. api hisi = 3x20.

80. upun hisi = 4x20.

100. mone hisi = 5x20.

In the Nicobar Islands of the Indian Ocean a well-developed example of vigesimal numeration is found. The inhabitants of these islands are so low in the scale of civilization that a definite numeral system of any kind is a source of some surprise. Their neighbours, the Andaman Islanders, it will be remembered, have but two numerals at their command; their intelligence does not seem in any way inferior to that of the Nicobar tribes, and one is at a loss to account for the superior development of the number sense in the case of the latter. The intercourse of the coast tribes with traders might furnish an explanation of the difficulty were it not for the fact that the numeration of the inland tribes is quite as well developed as that of the coast tribes; and as the former never come in contact with traders and never engage in barter of any kind except in the most limited way, the conclusion seems inevitable that this is merely one of the phenomena of mental development among savage races for which we have at present no adequate explanation. The principal numerals of the inland and of the coast tribes are:[356]

INLAND TRIBES	COAST TRIBES
10. teya.	10. sham.
20. heng-inai.	20. heang-inai.
30. heng-inai-tain	30. heang-inai-tanai_= 20 + 5 (couples). = 20 + 5 (couples).
40. au-inai = 2x20.	40. an-inai = 2x20.
100. tain-inai = 5x20.	100. tanai-inai = 5x20.
200. teya-inai = 10x20.	200. sham-inai = 10x20.
300. teya-tain-inai	300. heang-tanai-inai = (10 + 5) x 20. = (10 + 5) 20.
400. heng-teo.	400. heang-momchiama.

In no other part of the world is vigesimal counting found so perfectly developed, and, among native races, so generally preferred, as in North and South America. In the eastern portions of North America and in the extreme western portions of South America the decimal or the quinary decimal scale is in general use. But in the northern regions of North America, in western Canada and northwestern United States, in Mexico and Central America, and in the northern and western parts of South America, the unit of counting among the great majority of the native races was 20. The ethnological affinities of these races are not yet definitely ascertained; and it is no part of the scope of this work to enter into any discussion of that involved question. But either through contact or affinity, this form of numeration spread in prehistoric times over half or more than half of the western hemisphere. It was the method employed by the rude Eskimos of the north and their equally rude kinsmen of Paraguay and eastern Brazil; by the forest Indians of Oregon and British Columbia, and by their more southern kinsmen, the wild tribes of the Rio Grande and of the Orinoco. And, most striking and interesting of all, it was the method upon which were based the numeral systems of the highly civilized races of Mexico, Yucatan, and New Granada. Some of the systems obtained from the languages of these peoples are perfect, extended examples of vigesimal counting, not to be duplicated in any other quarter of the globe. The ordinary unit was, as would be expected, "one man," and in numerous languages the words for 20 and man are identical. But in other cases the original meaning of that numeral word has been lost; and in others still it has a signification quite remote from that given above. These meanings will be noticed in connection with the scales themselves, which are given, roughly speaking, in their geographical order, beginning with the Eskimo of the far north. The systems of some of the tribes are as follows:

ALASKAN ESKIMOS.

10. koleet.

20. enuenok.

30. enuenok kolinik = 20 + 10.

40. malho kepe ak = 2x20.

50. malho-kepe ak-kolmik che pah ak to = 2x20 + 10.

60. pingi shu-kepe ak = 3x20.

100. tale ma-kepe ak = 5x20.

400. enue nok ke pe ak = 20x20.

TCHIGLIT.

10. krolit.

20. kroleti, or innun = man.

30. innok krolinik-tchikpalik = man + 2 hands.

40. innum mallerok = 2 men.

50. adjigaynarmitoat = as many times 10 as the fingers of the hand.

60. innumipit = 3 men.

70. innunmalloeronik arveneloerit = 7 men?

80. innun pinatcunik arveneloerit = 8 men?

90. innun tcitamanik arveneloerit = 9 men?

100. itchangnerkr.

1000. itchangner-park = great 100.

The meanings for 70, 80, 90, are not given by Father Petitot, but are of such a form that the significations seem to be what are given above. Only a full acquaintance with the Tchiglit language would justify one in giving definite meanings to these words, or in asserting that an error had been made in the numerals. But it is so remarkable and anomalous to find the decimal and vigesimal scales mingled in this manner that one involuntarily suspects either incompleteness of form, or an actual mistake.

TLINGIT.

10. djinkat = both hands?

20. tle ka = 1 man.

30. natsk djinkat = 3x10.

40. dak'on djinkat = 4x10.

50. kedjin djinkat = 5x10.

60. tle durcu djinkat = 6x10.

70. daqa durcu djinkat = 7x10.

80. natska durcu djinkat = 8x10.

90. gocuk durcu djinkat = 9x10.

100. kedjin ka = 5 men, or 5x20.

200. djinkat ka = 10x20.

300. natsk djinkat ka = 30 men.

400. dak'on djinkat ka = 40 men.

This scale contains a strange commingling of decimal and vigesimal counting. The words for 20, 100, and 200 are clear evidence of vigesimal, while 30 to 90, and the remaining hundreds, are equally unmistakable proof of decimal, numeration. The word *ka*, man, seems to mean either 10 or 20; a most unusual occurrence. The fact that a number system is partly decimal and partly vigesimal is found to be of such frequent occurrence that this point in the Tlingit scale need excite no special wonder. But it is remarkable that the same word should enter into numeral composition under such different meanings.

NOOTKA.

10. haiu.

20. tsakeits.

30. tsakeits ic haiu = 20 + 10.

40. atlek = 2x20.

60. katstsek = 3x20.

80. moyek = 4x20.

100. sutc'ek = 5x20.

120. nop'ok = 6x20.

140. atlpok = 7x20.

160. atlakutlek = 8x20.

180. ts'owakutlek = 9x20.

200. haiuk = 10x20.

This scale is quinary-vigesimal, with no apparent decimal element in its composition. But the derivation of some of the terms used is detected with difficulty. In the following scale the vigesimal structure is still more obscure.

TSIMSHIAN.

10. gy'ap.

20. kyedeel = 1 man.

30. gulewulgy'ap.

40. t'epqadalgyitk, or tqalpqwulgyap.

50. kctoncwulgyap.

100. kcenecal.

200. k'pal.

300. k'pal te kcenecal = 200 + 100.

400. kyedal.

500. kyedal te kcenecal = 400 + 100.

600. gulalegyitk.

700. gulalegyitk te kcenecal = 600 + 100.

800. tqalpqtalegyitk.

900. tqalpqtalegyitk te kcenecal = 800 + 100.

1000. k'pal.

To the unobservant eye this scale would certainly appear to contain no more than a trace of the vigesimal in its structure. But Dr. Boas, who is one of the most careful and accurate of investigators, says in his comment on this system: "It will be seen at once that this system is quinary-vigesimal…. In 20 we find the word *gyat*, man. The hundreds are identical with the numerals used in counting men, and then the quinary-vigesimal system is most evident."

RIO NORTE INDIANS.

20. taiguaco.

30. taiguaco co juyopamauj ajte = 20 + 2x5.

40. taiguaco ajte = 20x2.

50. taiguaco ajte co juyopamauj ajte = 20x2 + 5x2.

CARIBS OF ESSIQUIBO, GUIANA

10. oween-abatoro.

20. owee-carena = 1 person.

40. oko-carena = 2 persons.

60. oroowa-carena = 3 persons.

OTOMI

10. ra-tta.

20. na-te.

30. na-te-m'a-ratta = 20 + 10.

40. yo-te = 2x20.

50. yote-m'a-ratta = 2x20 + 10.

60. hiu-te = 3x20.

70. hiute-m'a-ratta = 3x20 + 10.

80. gooho-rate = 4x20.

90. gooho-rate-m'a ratta = 4x20 + 10.

100. cytta-te = 5x20, or nanthebe = 1x100.

MAYA, YUCATAN.

1. hun.

10. lahun = it is finished.

20. hunkal = a measure, or more correctly, a fastening together.

30. lahucakal = 40 – 10?

40. cakal = 2x20.

50. lahuyoxkal = 60 – 10.

60. oxkal = 3x20.

70. lahucankal = 80 – 10.

80. cankal = 4x20.

90. lahuyokal = 100 – 10.

100. hokal = 5x20.

110. lahu uackal = 120 – 10.

120. uackal = 6x20.

130. lahu uuckal = 140 – 10.

140. uuckal = 7x20.

200. lahuncal = 10x20.

300. holhukal = 15x20.

400. hunbak = 1 tying around.

500. hotubak.

600. lahutubak

800. calbak = 2x400.

900. hotu yoxbak.

1000. lahuyoxbak.

1200. oxbak = 3x400.

2000. capic (modern).

8000. hunpic = 1 sack.

16,000. ca pic (ancient).

160,000. calab = a filling full

3,200,000. kinchil.

64,000,000. hunalau.

In the Maya scale we have one of the best and most extended examples of vigesimal numeration ever developed by any race. To show in a more striking and forcible manner the perfect regularity of the system, the following tabulation is made of the various Maya units, which will correspond to the "10 units make one ten, 10 tens make one hundred, 10 hundreds make one thousand," etc., which old-fashioned arithmetic compelled us to learn in childhood. The scale is just as regular by twenties in Maya as by tens in English. It is:

20 hun = 1 kal = 20.

20 kal = 1 bak = 400.

20 bak = 1 pic = 8000.

20 pic = 1 calab = 160,000.

20 calab = 1 { kinchil } = 3,200,000. { tzotzceh }

20 kinchil = 1 alau = 64,000,000.

The original meaning of *pic*, given in the scale as "a sack," was rather "a short petticoat, somtimes used as a sack." The word *tzotzceh* signified "deerskin." No reason can be given for the choice of this word as a numeral, though the appropriateness of the others is sufficiently manifest. No evidence of digital numeration appears in the first 10 units, but, judging from the almost universal practice of the Indian tribes of both North and South America, such may readily have been the origin of Maya counting. Whatever its origin, it certainly expanded and grew into a system whose perfection challenges our admiration. It was worthy of the splendid civilization of this unfortunate race, and, through its simplicity and regularity, bears ample testimony to the intellectual capacity which originated it.

The only example of vigesimal reckoning which is comparable with that of the Mayas is the system employed by their northern neighbours, the Nahuatl, or, as they are more commonly designated, the Aztecs of Mexico. This system is quite as pure and quite as simple as the Maya, but differs from it in some important particulars. In its first 20 numerals it is quinary (see p. 141), and as a system must be regarded as quinary-vigesimal. The Maya scale is decimal through its first 20 numerals, and, if it is to be regarded as a mixed scale, must be characterized as decimal-vigesimal. But in both these instances the vigesimal element preponderates so strongly that these, in common with their kindred number systems of Mexico, Yucatan, and Central America, are always thought of and alluded to as vigesimal scales. On account of its importance, the Nahuatl system is given in fuller detail than most of the other systems I have made use of.

10. matlactli = 2 hands.

20. cempoalli = 1 counting.

21. cempoalli once = 20-1.

22. cempoalli omome = 20-2.

30. cempoalli ommatlactli = 20-10.

31. cempoalli ommatlactli once = 20-10-1.

40. ompoalli = 2x20.

50. ompoalli ommatlactli = 40-10.

60. eipoalli, or epoalli, = 3x20.

70. epoalli ommatlactli = 60-10.

80. nauhpoalli = 4x20.

90. nauhpoalli ommatlactli = 90-10.

100. macuilpoalli = 5x20.

120. chiquacempoalli = 6x20.

140. chicompoalli = 7x20.

160. chicuepoalli = 8x20.

180. chiconauhpoalli = 9x20.

200. matlacpoalli = 10x20.

220. matlactli oncempoalli = 11x20.

240. matlactli omompoalli = 12x20.

260. matlactli omeipoalli = 13x20.

280. matlactli onnauhpoalli = 14x20.

300. caxtolpoalli = 15x20.

320. caxtolli oncempoalli.

399. caxtolli onnauhpoalli ipan caxtolli onnaui = 19x20 + 19.

400. centzontli = 1 bunch of grass, or 1 tuft of hair._800.
ometzontli = 2x400.

1200. eitzontli = 3x400.

7600. caxtolli onnauhtzontli = 19x400.

8000. cenxiquipilli, or cexiquipilli.

160,000. cempoalxiquipilli = 20x8000.

3,200,000. centzonxiquipilli = 400x8000.

64,000,000. cempoaltzonxiquipilli = 20x400 x 8000.

Up to 160,000 the Nahuatl system is as simple and regular in its construction as the English. But at this point it fails in the formation of a new unit, or rather in the expression of its new unit by a simple word; and in the expression of all higher numbers it is forced to resort in some measure to compound terms, just as the English might have done had it not been able to borrow from the Italian. The higher numeral terms, under such conditions, rapidly become complex and cumbersome, as the following analysis of the number 1,279,999,999 shows.[366] The analysis will be readily understood when it is remembered that *ipan* signifies plus. _Caxtolli onnauhpoaltzonxiquipilli ipan caxtolli onnauhtzonxiquipilli ipan caxtolli onnauhpoalxiquipilli ipan caxtolli onnauhxiquipilli ipan caxtolli onnauhtzontli ipan caxtolli onnauhpoalli ipan caxtolli onnaui;_ *i.e*. 1,216,000,000 + 60,800,000 + 3,040,000 + 152,000 + 7600 + 380 + 19. To show the compounding which takes place in the higher numerals, the analysis may be made more literally, thus: + (15 + 4) x 400_800 + (15 + 4) x 20 x 8000 + (15 + 4) x 8000 + (15 + 4) x 400 + (15 + 4) x 20 + 15 + 4. Of course this resolution suffers from the fact that it is given in digits arranged in accordance with decimal notation, while the Nahuatl numerals express values by a base twice as great. This gives the effect of a complexity and awkwardness greater than really existed in the actual use of the scale. Except for the presence of the quinary element the number just given is really expressed with just as great simplicity as it could be in English words if our words “million” and “billion” were replaced by “thousand thousand” and “thousand thousand thousand.” If Mexico had remained undisturbed by Europeans, and science and commerce had been left to their natural growth and development, uncompounded words would undoubtedly have been found for the higher units, 160,000, 3,200,000, etc., and the system thus rendered as simple as it is possible for a quinary-vigesimal system to be.

Other number scales of this region are given as follows:

HUASTECA.

10. laluh._20. hum-inic = 1 man._30. hum-inic-lahu = 1 man 10._40. tzab-inic = 2 men._50. tzab-inic-lahu = 2 men 10._60. ox-inic = 3 men._70. ox-inic-lahu = 3 men 10._80. tze-tnic = 4 men._90. tze-ynic-kal-laluh = 4 men and 10._100. bo-inic = 5 men._200. tzab-bo-inic = 2_5 men._300. ox-bo-inic = 3_5 men._400. tsa-bo-inic = 4_5 men._600. acac-bo-inic = 6_5 men._800. huaxic-bo-inic = 8_5 men._1000. xi._8000. huaxic-xi = 8-1000.

The essentially vigesimal character of this system changes in the formation of some of the higher numerals, and a suspicion of the decimal enters. One hundred is *boinic*, 5 men; but 200, instead of being simply *lahuh-inic*, 10 men, is *tsa-bo-inic*, 2_100, or more strictly, 2 times 5 men. Similarly, 300 is 3_100, 400 is 4_100, etc. The word for 1000 is simple instead of compound, and the thousands appear to be formed wholly on the decimal base. A comparison of this scale with that of the Nahuatl shows how much inferior it is to the latter, both in simplicity and consistency.

TOTONACO.

10. cauh.

20. puxam.

30. puxamacauh = 20 + 10.

40. tipuxam = 2x20.

50. tipuxamacauh = 40 + 10.

60. totonpuxam = 3x20.

100. quitziz puxum = 5x20.

200. copuxam = 10x20.

400. tontaman.

1000. titamanacopuxam = 2x400 + 200.

The essential character of the vigesimal element is shown by the last two numerals. *Tontamen*, the square of 20, is a simple word, and 1000 is, as it should be, 2 times 400, plus 200. It is most unfortunate that the numeral for 8000, the cube of 20, is not given.

CORA.

10. tamoamata.

20. cei-tevi.

30. ceitevi apoan tamoamata = 20 + 10.

40. huapoa-tevi = 2x20.

60. huaeica-tevi = 3x20.

100. anxu-tevi = 5x20.

400. ceitevi-tevi = 20x20.

Closely allied with the Maya numerals and method of counting are those of the Quiches of Guatemala. The resemblance is so obvious that no detail in the Quiche scale calls for special mention.

QUICHE.

10. lahuh.

20. hu-uinac = 1 man.

30. hu-uinac-lahuh = 20 + 10.

40. ca-uinac = 2 men.

50. lahu-r-ox-kal = -10 + 3x20.

60. ox-kal = 3x20.

70. lahu-u-humuch = -10 + 80.

80. humuch.

90. lahu-r-ho-kal = -10 + 100.

100. hokal._1000. o-tuc-rox-o-kal.

Among South American vigesimal systems, the best known is that of the Chibchas or Muyscas of the Bogota region, which was obtained at an early date by the missionaries who laboured among them. This system is much less extensive than that of some of the more northern races; but it is as extensive as almost any other South American system with the exception of the Peruvian, which was, however, a pure decimal system. As has already been stated, the native races of South America were, as a rule, exceedingly deficient in regard to the number sense. Their scales are rude, and show great poverty, both in formation of numeral words and in the actual extent to which counting was carried. If extended as far as 20, these scales are likely to become vigesimal, but many stop far short of that limit, and no inconsiderable number of them fail to reach even 5. In this respect we are reminded of the Australian scales, which were so rudimentary as really to preclude any proper use of the word “system” in connection with them. Counting among the South American tribes was often equally limited, and even less regular. Following are the significant numerals of the scale in question:

CHIBCHA, OR MUYSCA.

10. hubchibica.

20. quihica ubchihica = thus says the foot, 10 = 10-10, or gueta = house.

30. guetas asaqui ubchihica = 20 + 10.

40. gue-bosa = 20x2.

60. gue-mica = 20x3.

80. gue-muyhica = 20x4.

100. gue-hisca = 20x5.

NAGRANDA.

10. guha.

20. dino.

30. ‘badinoguhanu = 20 + 10.

40. apudino = 2x20.

50. apudinoguhanu = 2x20 + 10.

60. asudino = 3x20.

70. asudinoguhanu = 3x20 + 10.

80. acudino = 4x20.

90. acudinoguhanu = 4x20 + 10.

100. huisudino = 5x20, or guhamba = great 10.

200. guahadino = 10x20.

400. dinoamba = great 20.

1000. guhaisudino = 10x5 x 20.

2000. hisudinoamba = 5 great 20’s.

4000. guhadinoamba = 10 great 20’s.

In considering the influence on the manners and customs of any people which could properly be ascribed to the use among them of any other base than 10, it must not be forgotten that no races, save those using that base, have ever attained any great degree of civilization, with the exception of the ancient Aztecs and their immediate neighbours, north and south. For reasons already pointed out, no highly civilized race has ever used an exclusively quinary system; and all that can be said of the influence of this mode of counting is that it gives rise to the habit of collecting objects in groups of five, rather than of ten, when any attempt is being made to ascertain their sum. In the case of the subsidiary base 12, for which the Teutonic races have always shown such a fondness, the dozen and gross of commerce, the divisions of English money, and of our common weights and measures are probably an outgrowth of this preference; and the Babylonian base, 60, has fastened upon the world forever a sexagesimal method of dividing time, and of measuring the circumference of the circle.

The advanced civilization attained by the races of Mexico and Central America render it possible to see some of the effects of vigesimal counting, just as a single thought will show how our entire lives are influenced by our habit of counting by tens. Among the Aztecs the universal unit was 20. A load of cloaks, of dresses, or other articles of convenient size, was 20. Time was divided into periods of 20 days each. The armies were numbered by divisions of 8000; and in countless other ways the vigesimal element of numbers entered into their lives, just as the decimal enters into ours; and it is to be supposed that they found it as useful and as convenient for all measuring purposes as we find our own system; as the tradesman of to-day finds the duodecimal system of commerce; or as the Babylonians of old found that singularly curious system, the sexagesimal. Habituation, the laws which the habits and customs of every-day life impose upon us, are so powerful, that our instinctive readiness to make use of any concept depends, not on the intrinsic perfection or imperfection which pertains to it, but on the familiarity with which previous use has invested it. Hence, while one race may use a decimal, another a quinary-vigesimal, and another a sexagesimal scale, and while one system may actually be inherently superior to another, no user of one method of reckoning need ever think of any other method as possessing practical inconveniences, of which those employing it are ever conscious. And, to cite a single instance which illustrates the unconscious daily use of two modes of reckoning in one scale, we have only to think of the singular vigesimal fragment which remains to this day imbedded in the numeral scale of the French. In counting from 70 to 100, or in using any number which lies between those limits, no Frenchman is conscious of employing a method of numeration less simple or less convenient in any particular, than when he is at work with the strictly decimal portions of his scale. He passes from the one style of counting to the other, and from the second back to the first again, entirely unconscious of any break or change; entirely unconscious, in fact, that he is using any particular system, except that which the daily habit of years has made a part himself.

Deep regret must be felt by every student of philology, that the primitive meanings of simple numerals have been so generally lost. But, just as the pebble on the beach has been worn and rounded by the beating of the waves and by other pebbles, until no trace of its original form is left, and until we can say of it now only that it is quartz, or that it is diorite, so too the numerals of many languages have suffered from the attrition of the ages, until all semblance of their origin has been lost, and we can say of them only that they are numerals. Beyond a certain point we can carry the study neither of number nor of number words. At that point both the mathematician and the philologist must pause, and leave everything beyond to the speculations of those who delight in nothing else so much as in pure theory.

THE END.