Philosophy and Fun of Algebra

CHAPTER 3: SIMULTANEOUS PROBLEMS

It often happens that two or three problems are so entangled up together that it seems impossible to solve any one of them until the others have been solved.

For instance, we might get out three answers of this kind:

x equals half of y;
y equals twice x;
z equals x multiplied by y.
The value of each depends on the value of the others.

When we get into a predicament of this kind, three courses are open to us. We can begin to make slap-dash guesses, and each argue to prove that his guess is the right one; and go on quarrelling; and so on; as I described people doing about arithmetic before Algebra was invented.

Or we might write down something of this kind:—

The values cannot be known. There is no answer to our problem.

We might write:—

x is the unknowable;
y is non-existent;
z is imaginary,

and accept those as answers and give them forth to the world with all the authority which is given by big print, wide margins, a handsome binding, and a publisher in a large way of business; and so make a great many foolish people believe we are very wise.

Some people call this way of settling things Philosophy; others call it arrogant conceit. Whatever it is, it is not Algebra. The Algebra way of managing is this:—

We say: Suppose that x were Unity (1); what would become of y and z? Then we write out our problem as before; only that, wherever there was x, we now write 1.

If the result of doing so is to bring out some such ridiculous answer as "2 and 3 make 7," we then know that x cannot be 1. We now add to our column of data, "x cannot be 1."

But if we come to a truism, such as "2 and 3 make 5," we add to our column of data, "x may be 1." Some people add to their column of data, "x is 1," but that again is not Algebra. Next we try the experiment of supposing x to be equal to zero (0), and go over the ground again.

Then we go over the same ground, trying y as 1 and as 0.

And then we try the same with z. Some people think that it is waste of time to go over all this ground so carefully, when all you get by it is either nonsense, such as "2 and 3 are 7"; or truisms, such as "2 and 3 are 5." But it is not waste of time. For, even if we never arrive at finding out the value of x, or y, or z, every conscientious attempt such as I have described adds to our knowledge of the structure of Algebra, and assists us in solving other problems.

Such suggestions as "suppose x were Unity" are called "working hypotheses," or "hypothetical data." In Algebra we are very careful to distinguish clearly between actual data and hypothetical data.

This is only part of the essence of Algebra, which, as I told you, consists in preserving a constant, reverent, and conscientious awareness of our own ignorance.

When we have exhausted all the possible hypotheses connected with Unity and Zero, we next begin to experiment with other values of x; e.g.—suppose x were 2, suppose x were 3, suppose it were 4. Then, suppose it were one half, or one and a half, and so on, registering among our data, each time, either "x may be so and so," or "x cannot be so and so."

The method of finding out what x cannot be, by showing that certain suppositions or hypotheses lead to a ridiculous statement, is called the method of reductio ad absurdum. It is largely used by Euclid.