PHILOSOPHY AND FUN OF ALGEBRA

CHAPTER 16: INFINITY

You remember the angel who looks like this, $\sqrt{-1}$. Now I am going to introduce you to another angel. It is called "Infinity." When you come to it, remember what I told you before—Angels are messengers from the great world of the "As-Yet-Unknown." They never gossip about their private affairs, or those of other angels. They come to tell you either about what you are to do next, or about something you had better not do next; and if you ask them impertinent questions about things that do not concern you for the time being, they will give you headaches and make your head spin: just to teach you to mind your own business. This particular angel always comes with a message about a broken link or a loosened chain. It comes, when an hypothesis has been fully worked out, to tell you that you are now free from the bonds of that hypothesis and at liberty to start experimenting on a fresh one. But its message is never: "You have got out of that particular house of bondage and therefore you may, for all the rest of your life, run riot, and eat, and drink, and do, whatever you please." Its message always is: "You have outgrown that master; now you may take a holiday and have a fling before you go into a higher class; but, just because you are set free, look out for danger traps; and mind your Ten Commandments."

You will understand Infinity's messages better if you will read carefully what is written about it in Chapter XV of The Logic of Arithmetic. It brought the answer to the question: "How many children could pass through a school-room without the apples all being eaten up, supposing that none of the children ate any?"

Let us go over that ground again. Suppose there is a cake on the table. How many children can go through the room without the cake being all eaten up? Well, that depends on two things: the size of the cake, and the share which each child eats. If the cake weighs two pounds, and each child eats two ounces, it will be all eaten up when sixteen children have gone through the room. If the cake weighs only one pound, it will be eaten up when eight children have gone through the room. But if each child eats only one ounce, then again sixteen children will have to go through the room before the cake is eaten up, and so on. Many questions could be asked, all depending on the size of the cake and the size of each child's share.

All this time you are tied to an hypothesis that the children eat cake (more or less).

But now suppose we are freed from that hypothesis. Suppose no cake is given to the children. How many can pass through the room before it is all eaten up?

The answer to that is: “An infinite number.” Infinity does not mean any particular number, or a very large number. It means a loosened chain, a dis- carded hypothesis, escape from the rule we were working under. Something else, not the size of the cake, determines the number of children. Infinity does not mean that there are enough children in the world now to go on passing through the room for ever, but that the number of children who pass through the room, now that the share of each child is 0 (zero), will have to be determined by the number of children that there are in the school, or the parish, or wherever it is that the children are supposed to come from; and not by the size of the cake. The size of the cake has no longer anything to do with answering the question: “How many children can pass through the room before the cake is all eaten?”