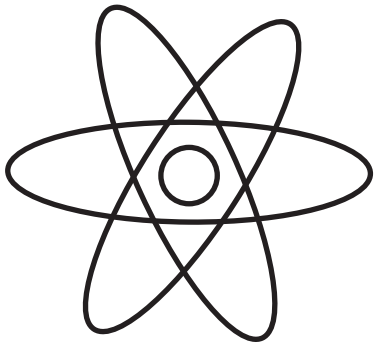

PHYSICS BY ARISTOTLE

Book 6

1

Now if the terms 'continuous', 'in contact', and 'in succession' are understood as defined above things being 'continuous' if their extremities are one, 'in contact' if their extremities are together, and 'in succession' if there is nothing of their own kind intermediate between them-nothing that is continuous can be composed 'of indivisibles': e.g. a line cannot be composed of points, the line being continuous and the point indivisible. For the extremities of two points can neither be one (since of an indivisible there can be no extremity as distinct from some other part) nor together (since that which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct).

Moreover, if that which is continuous is composed of points, these points must be either continuous or in contact with one another: and the same reasoning applies in the case of all indivisibles. Now for the reason given above they cannot be continuous: and one thing can be in contact with another only if whole is in contact with whole or part with part or part with whole. But since indivisibles have no parts, they must be in contact with one another as whole with whole. And if they are in contact with one another as whole with whole, they will not be continuous: for that which is continuous has distinct parts: and these parts into which it is divisible are different in this way, i.e. spatially separate.

Nor, again, can a point be in succession to a point or a moment to a moment in such a way that length can be composed of points or time of moments: for things are in succession if there is nothing of their own kind intermediate between them, whereas that which is intermediate between points is always a line and that which is intermediate between moments is always a period of time.

Again, if length and time could thus be composed of indivisibles, they could be divided into indivisibles, since each is divisible into the parts of which it is composed. But, as we

saw, no continuous thing is divisible into things without parts. Nor can there be anything of any other kind intermediate between the parts or between the moments: for if there could be any such thing it is clear that it must be either indivisible or divisible, and if it is divisible, it must be divisible either into indivisibles or into divisibles that are infinitely divisible, in which case it is continuous.

Moreover, it is plain that everything continuous is divisible into divisibles that are infinitely divisible: for if it were divisible into indivisibles, we should have an indivisible in contact with an indivisible, since the extremities of things that are continuous with one another are one and are in contact.

The same reasoning applies equally to magnitude, to time, and to motion: either all of these are composed of indivisibles and are divisible into indivisibles, or none. This may be made clear as follows. If a magnitude is composed of indivisibles, the motion over that magnitude must be composed of corresponding indivisible motions: e.g. if the magnitude ABG is composed of the indivisibles A, B, G, each corresponding part of the motion DEZ of O over ABG is indivisible. Therefore, since where there is motion there must be something that is in motion, and where there is something in motion there must be motion, therefore the being-moved will also be composed of indivisibles. So O traversed A when its motion was D, B when its motion was E, and G similarly when its motion was Z. Now a thing that is in motion from one place to another cannot at the moment when it was in motion both be in motion and at the same time have completed its motion at the place to which it was in motion: e.g. if a man is walking to Thebes, he cannot be walking to Thebes and at the same time have completed his walk to Thebes: and, as we saw, O traverses a the partless section A in virtue of the presence of the motion D. Consequently, if O actually passed through A after being in process of passing through, the motion must be divisible: for at the time when O was passing through, it neither was at rest nor had completed its passage but was in an intermediate state: while if it is passing through and has completed its passage at the same moment, then that which is walking will at the moment when it is walking have completed its walk and will be in the place to which it is walking; that is to say, it will have completed its motion at the place to which it is in motion. And if a thing is in motion over the whole KBG and its motion is the three D, E, and Z, and if it is not in motion at all over the partless section A but has completed its motion over it, then the motion will consist not of motions but of starts, and will take place by a thing's having completed a motion without being in motion: for on this assumption it has completed its passage through A without passing through it. So it will be possible for a thing to have completed a walk without ever walking: for on this assumption it has

completed a walk over a particular distance without walking over that distance. Since, then, everything must be either at rest or in motion, and O is therefore at rest in each of the sections A, B, and G, it follows that a thing can be continuously at rest and at the same time in motion: for, as we saw, O is in motion over the whole ABG and at rest in any part (and consequently in the whole) of it. Moreover, if the indivisibles composing DEZ are motions, it would be possible for a thing in spite of the presence in it of motion to be not in motion but at rest, while if they are not motions, it would be possible for motion to be composed of something other than motions.

And if length and motion are thus indivisible, it is neither more nor less necessary that time also be similarly indivisible, that is to say be composed of indivisible moments: for if the whole distance is divisible and an equal velocity will cause a thing to pass through less of it in less time, the time must also be divisible, and conversely, if the time in which a thing is carried over the section A is divisible, this section A must also be divisible.

2

And since every magnitude is divisible into magnitudes-for we have shown that it is impossible for anything continuous to be composed of indivisible parts, and every magnitude is continuous-it necessarily follows that the quicker of two things traverses a greater magnitude in an equal time, an equal magnitude in less time, and a greater magnitude in less time, in conformity with the definition sometimes given of 'the quicker'. Suppose that A is quicker than B. Now since of two things that which changes sooner is quicker, in the time ZH, in which A has changed from G to D, B will not yet have arrived at D but will be short of it: so that in an equal time the quicker will pass over a greater magnitude. More than this, it will pass over a greater magnitude in less time: for in the time in which A has arrived at D, B being the slower has arrived, let us say, at E. Then since A has occupied the whole time ZH in arriving at D, will have arrived at O in less time than this, say ZK. Now the magnitude GO that A has passed over is greater than the magnitude GE, and the time ZK is less than the whole time ZH: so that the quicker will pass over a greater magnitude in less time. And from this it is also clear that the quicker will pass over an equal magnitude in less time than the slower. For since it passes over the greater magnitude in less time than the slower, and (regarded by itself) passes over LM the greater in more time than LX the lesser, the time PRh in which it passes over LM will be more than the time PS, which it passes over LX: so that, the time PRh being less than the time PCh in which the slower passes over LX, the time PS will also be less than the time PX: for it is less than the time PRh, and that which is less than something else

that is less than a thing is also itself less than that thing. Hence it follows that the quicker will traverse an equal magnitude in less time than the slower. Again, since the motion of anything must always occupy either an equal time or less or more time in comparison with that of another thing, and since, whereas a thing is slower if its motion occupies more time and of equal velocity if its motion occupies an equal time, the quicker is neither of equal velocity nor slower, it follows that the motion of the quicker can occupy neither an equal time nor more time. It can only be, then, that it occupies less time, and thus we get the necessary consequence that the quicker will pass over an equal magnitude (as well as a greater) in less time than the slower.

And since every motion is in time and a motion may occupy any time, and the motion of everything that is in motion may be either quicker or slower, both quicker motion and slower motion may occupy any time: and this being so, it necessarily follows that time also is continuous. By continuous I mean that which is divisible into divisibles that are infinitely divisible: and if we take this as the definition of continuous, it follows necessarily that time is continuous. For since it has been shown that the quicker will pass over an equal magnitude in less time than the slower, suppose that A is quicker and B slower, and that the slower has traversed the magnitude GD in the time ZH. Now it is clear that the quicker will traverse the same magnitude in less time than this: let us say in the time ZO. Again, since the quicker has passed over the whole D in the time ZO, the slower will in the same time pass over GK, say, which is less than GD. And since B, the slower, has passed over GK in the time ZO, the quicker will pass over it in less time: so that the time ZO will again be divided. And if this is divided the magnitude GK will also be divided just as GD was: and again, if the magnitude is divided, the time will also be divided. And we can carry on this process for ever, taking the slower after the quicker and the quicker after the slower alternately, and using what has been demonstrated at each stage as a new point of departure: for the quicker will divide the time and the slower will divide the length. If, then, this alternation always holds good, and at every turn involves a division, it is evident that all time must be continuous. And at the same time it is clear that all magnitude is also continuous; for the divisions of which time and magnitude respectively are susceptible are the same and equal.

Moreover, the current popular arguments make it plain that, if time is continuous, magnitude is continuous also, inasmuch as a thing asses over half a given magnitude in half the time taken to cover the whole: in fact without qualification it passes over a less magnitude in less time; for the divisions of time and of magnitude will be the same. And if

either is infinite, so is the other, and the one is so in the same way as the other; i.e. if time is infinite in respect of its extremities, length is also infinite in respect of its extremities: if time is infinite in respect of divisibility, length is also infinite in respect of divisibility: and if time is infinite in both respects, magnitude is also infinite in both respects.

Hence Zeno's argument makes a false assumption in asserting that it is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite: and so we find that the time occupied by the passage over the infinite is not a finite but an infinite time, and the contact with the infinites is made by means of moments not finite but infinite in number.

The passage over the infinite, then, cannot occupy a finite time, and the passage over the finite cannot occupy an infinite time: if the time is infinite the magnitude must be infinite also, and if the magnitude is infinite, so also is the time. This may be shown as follows. Let AB be a finite magnitude, and let us suppose that it is traversed in infinite time G, and let a finite period GD of the time be taken. Now in this period the thing in motion will pass over a certain segment of the magnitude: let BE be the segment that it has thus passed over. (This will be either an exact measure of AB or less or greater than an exact measure: it makes no difference which it is.) Then, since a magnitude equal to BE will always be passed over in an equal time, and BE measures the whole magnitude, the whole time occupied in passing over AB will be finite: for it will be divisible into periods equal in number to the segments into which the magnitude is divisible. Moreover, if it is the case that infinite time is not occupied in passing over every magnitude, but it is possible to ass over some magnitude, say BE, in a finite time, and if this BE measures the whole of which it is a part, and if an equal magnitude is passed over in an equal time, then it follows that the time like the magnitude is finite. That infinite time will not be occupied in passing over BE is evident if the time be taken as limited in one direction: for as the part will be passed over in less time than the whole, the time occupied in traversing this part must be finite, the limit in one direction being given. The same reasoning will also show the falsity of the assumption that infinite length can be traversed in a finite time. It is evident, then, from what has been said that neither a line nor a surface nor in fact anything continuous can be indivisible.

This conclusion follows not only from the present argument but from the consideration that the opposite assumption implies the divisibility of the indivisible. For since the distinction of quicker and slower may apply to motions occupying any period of time and in an equal time the quicker passes over a greater length, it may happen that it will pass over a length twice, or one and a half times, as great as that passed over by the slower: for their respective velocities may stand to one another in this proportion. Suppose, then, that the quicker has in the same time been carried over a length one and a half times as great as that traversed by the slower, and that the respective magnitudes are divided, that of the quicker, the magnitude ABGD, into three indivisibles, and that of the slower into the two indivisibles EZ, ZH. Then the time may also be divided into three indivisibles, for an equal magnitude will be passed over in an equal time. Suppose then that it is thus divided into KL, LM, MN. Again, since in the same time the slower has been carried over EZ, ZH, the time may also be similarly divided into two. Thus the indivisible will be divisible, and that which has no parts will be passed over not in an indivisible but in a greater time. It is evident, therefore, that nothing continuous is without parts.

3

The present also is necessarily indivisible-the present, that is, not in the sense in which the word is applied to one thing in virtue of another, but in its proper and primary sense; in which sense it is inherent in all time. For the present is something that is an extremity of the past (no part of the future being on this side of it) and also of the future (no part of the past being on the other side of it): it is, as we have said, a limit of both. And if it is once shown that it is essentially of this character and one and the same, it will at once be evident also that it is indivisible.

Now the present that is the extremity of both times must be one and the same: for if each extremity were different, the one could not be in succession to the other, because nothing continuous can be composed of things having no parts: and if the one is apart from the other, there will be time intermediate between them, because everything continuous is such that there is something intermediate between its limits and described by the same name as itself. But if the intermediate thing is time, it will be divisible: for all time has been shown to be divisible. Thus on this assumption the present is divisible. But if the present is divisible, there will be part of the past in the future and part of the future in the past: for past time will be marked off from future time at the actual point of division. Also the present will be a present not in the proper sense but in virtue of something else: for

the division which yields it will not be a division proper. Furthermore, there will be a part of the present that is past and a part that is future, and it will not always be the same part that is past or future: in fact one and the same present will not be simultaneous: for the time may be divided at many points. If, therefore, the present cannot possibly have these characteristics, it follows that it must be the same present that belongs to each of the two times. But if this is so it is evident that the present is also indivisible: for if it is divisible it will be involved in the same implications as before. It is clear, then, from what has been said that time contains something indivisible, and this is what we call a present.

We will now show that nothing can be in motion in a present. For if this is possible, there can be both quicker and slower motion in the present. Suppose then that in the present N the quicker has traversed the distance AB. That being so, the slower will in the same present traverse a distance less than AB, say AG. But since the slower will have occupied the whole present in traversing AG, the quicker will occupy less than this in traversing it. Thus we shall have a division of the present, whereas we found it to be indivisible. It is impossible, therefore, for anything to be in motion in a present.

Nor can anything be at rest in a present: for, as we were saying, only can be at rest which is naturally designed to be in motion but is not in motion when, where, or as it would naturally be so: since, therefore, nothing is naturally designed to be in motion in a present, it is clear that nothing can be at rest in a present either.

Moreover, inasmuch as it is the same present that belongs to both the times, and it is possible for a thing to be in motion throughout one time and to be at rest throughout the other, and that which is in motion or at rest for the whole of a time will be in motion or at rest as the case may be in any part of it in which it is naturally designed to be in motion or at rest: this being so, the assumption that there can be motion or rest in a present will carry with it the implication that the same thing can at the same time be at rest and in motion: for both the times have the same extremity, viz. the present.

Again, when we say that a thing is at rest, we imply that its condition in whole and in part is at the time of speaking uniform with what it was previously: but the present contains no 'previously': consequently, there can be no rest in it.

It follows then that the motion of that which is in motion and the rest of that which is at rest must occupy time.

4

Further, everything that changes must be divisible. For since every change is from something to something, and when a thing is at the goal of its change it is no longer changing, and when both it itself and all its parts are at the starting-point of its change it is not changing (for that which is in whole and in part in an unvarying condition is not in a state of change); it follows, therefore, that part of that which is changing must be at the starting-point and part at the goal: for as a whole it cannot be in both or in neither. (Here by 'goal of change' I mean that which comes first in the process of change: e.g. in a process of change from white the goal in question will be grey, not black: for it is not necessary that that that which is changing should be at either of the extremes.) It is evident, therefore, that everything that changes must be divisible.

Now motion is divisible in two senses. In the first place it is divisible in virtue of the time that it occupies. In the second place it is divisible according to the motions of the several parts of that which is in motion: e.g. if the whole AG is in motion, there will be a motion of AB and a motion of BG. That being so, let DE be the motion of the part AB and EZ the motion of the part BG. Then the whole DZ must be the motion of AG: for DZ must constitute the motion of AG inasmuch as DE and EZ severally constitute the motions of each of its parts. But the motion of a thing can never be constituted by the motion of something else: consequently the whole motion is the motion of the whole magnitude.

Again, since every motion is a motion of something, and the whole motion DZ is not the motion of either of the parts (for each of the parts DE, EZ is the motion of one of the parts AB, BG) or of anything else (for, the whole motion being the motion of a whole, the parts of the motion are the motions of the parts of that whole: and the parts of DZ are the motions of AB, BG and of nothing else: for, as we saw, a motion that is one cannot be the motion of more things than one): since this is so, the whole motion will be the motion of the magnitude ABG.

Again, if there is a motion of the whole other than DZ, say the the of each of the arts may be subtracted from it: and these motions will be equal to DE, EZ respectively: for the motion of that which is one must be one. So if the whole motion OI may be divided into the motions of the parts, OI will be equal to DZ: if on the other hand there is any remainder, say KI, this will be a motion of nothing: for it can be the motion neither of the whole nor of the parts (as the motion of that which is one must be one) nor of anything

else: for a motion that is continuous must be the motion of things that are continuous. And the same result follows if the division of OI reveals a surplus on the side of the motions of the parts. Consequently, if this is impossible, the whole motion must be the same as and equal to DZ.

This then is what is meant by the division of motion according to the motions of the parts: and it must be applicable to everything that is divisible into parts.

Motion is also susceptible of another kind of division, that according to time. For since all motion is in time and all time is divisible, and in less time the motion is less, it follows that every motion must be divisible according to time. And since everything that is in motion is in motion in a certain sphere and for a certain time and has a motion belonging to it, it follows that the time, the motion, the being-in-motion, the thing that is in motion, and the sphere of the motion must all be susceptible of the same divisions (though spheres of motion are not all divisible in a like manner: thus quantity is essentially, quality accidentally divisible). For suppose that A is the time occupied by the motion B. Then if all the time has been occupied by the whole motion, it will take less of the motion to occupy half the time, less again to occupy a further subdivision of the time, and so on to infinity. Again, the time will be divisible similarly to the motion: for if the whole motion occupies all the time half the motion will occupy half the time, and less of the motion again will occupy less of the time.

In the same way the being-in-motion will also be divisible. For let G be the whole being-in-motion. Then the being-in-motion that corresponds to half the motion will be less than the whole being-in-motion, that which corresponds to a quarter of the motion will be less again, and so on to infinity. Moreover by setting out successively the being-in-motion corresponding to each of the two motions DG (say) and GE, we may argue that the whole being-in-motion will correspond to the whole motion (for if it were some other being-in-motion that corresponded to the whole motion, there would be more than one being-in motion corresponding to the same motion), the argument being the same as that whereby we showed that the motion of a thing is divisible into the motions of the parts of the thing: for if we take separately the being-in motion corresponding to each of the two motions, we shall see that the whole being-in motion is continuous.

The same reasoning will show the divisibility of the length, and in fact of everything that forms a sphere of change (though some of these are only accidentally divisible because

that which changes is so): for the division of one term will involve the division of all. So, too, in the matter of their being finite or infinite, they will all alike be either the one or the other. And we now see that in most cases the fact that all the terms are divisible or infinite is a direct consequence of the fact that the thing that changes is divisible or infinite: for the attributes 'divisible' and 'infinite' belong in the first instance to the thing that changes. That divisibility does so we have already shown: that infinity does so will be made clear in what follows?

5

Since everything that changes changes from something to something, that which has changed must at the moment when it has first changed be in that to which it has changed. For that which changes retires from or leaves that from which it changes: and leaving, if not identical with changing, is at any rate a consequence of it. And if leaving is a consequence of changing, having left is a consequence of having changed: for there is a like relation between the two in each case.

One kind of change, then, being change in a relation of contradiction, where a thing has changed from not-being to being it has left not-being. Therefore it will be in being: for everything must either be or not be. It is evident, then, that in contradictory change that which has changed must be in that to which it has changed. And if this is true in this kind of change, it will be true in all other kinds as well: for in this matter what holds good in the case of one will hold good likewise in the case of the rest.

Moreover, if we take each kind of change separately, the truth of our conclusion will be equally evident, on the ground that that that which has changed must be somewhere or in something. For, since it has left that from which it has changed and must be somewhere, it must be either in that to which it has changed or in something else. If, then, that which has changed to B is in something other than B, say G, it must again be changing from G to B: for it cannot be assumed that there is no interval between G and B, since change is continuous. Thus we have the result that the thing that has changed, at the moment when it has changed, is changing to that to which it has changed, which is impossible: that which has changed, therefore, must be in that to which it has changed. So it is evident likewise that that that which has come to be, at the moment when it has come to be, will be, and that which has ceased to be will not-be: for what we have said applies universally to every kind of change, and its truth is most obvious in the case of contradictory change.

It is clear, then, that that which has changed, at the moment when it has first changed, is in that to which it has changed.

We will now show that the 'primary when' in which that which has changed effected the completion of its change must be indivisible, where by 'primary' I mean possessing the characteristics in question of itself and not in virtue of the possession of them by something else belonging to it. For let AG be divisible, and let it be divided at B. If then the completion of change has been effected in AB or again in BG, AG cannot be the primary thing in which the completion of change has been effected. If, on the other hand, it has been changing in both AB and BG (for it must either have changed or be changing in each of them), it must have been changing in the whole AG: but our assumption was that AG contains only the completion of the change. It is equally impossible to suppose that one part of AG contains the process and the other the completion of the change: for then we shall have something prior to what is primary. So that in which the completion of change has been effected must be indivisible. It is also evident, therefore, that that that in which that which has ceased to be has ceased to be and that in which that which has come to be has come to be are indivisible.

But there are two senses of the expression 'the primary when in which something has changed'. On the one hand it may mean the primary when containing the completion of the process of change—the moment when it is correct to say 'it has changed': on the other hand it may mean the primary when containing the beginning of the process of change. Now the primary when that has reference to the end of the change is something really existent: for a change may really be completed, and there is such a thing as an end of change, which we have in fact shown to be indivisible because it is a limit. But that which has reference to the beginning is not existent at all: for there is no such thing as a beginning of a process of change, and the time occupied by the change does not contain any primary when in which the change began. For suppose that AD is such a primary when. Then it cannot be indivisible: for, if it were, the moment immediately preceding the change and the moment in which the change begins would be consecutive (and moments cannot be consecutive). Again, if the changing thing is at rest in the whole preceding time GA (for we may suppose that it is at rest), it is at rest in A also: so if AD is without parts, it will simultaneously be at rest and have changed: for it is at rest in A and has changed in D. Since then AD is not without parts, it must be divisible, and the changing thing must have changed in every part of it (for if it has changed in neither of the two parts into which AD is divided, it has not changed in the whole either: if, on the other hand, it is in

process of change in both parts, it is likewise in process of change in the whole: and if, again, it has changed in one of the two parts, the whole is not the primary when in which it has changed: it must therefore have changed in every part). It is evident, then, that with reference to the beginning of change there is no primary when in which change has been effected: for the divisions are infinite.

So, too, of that which has changed there is no primary part that has changed. For suppose that of AE the primary part that has changed is AZ (everything that changes having been shown to be divisible): and let OI be the time in which DZ has changed. If, then, in the whole time DZ has changed, in half the time there will be a part that has changed, less than and therefore prior to DZ: and again there will be another part prior to this, and yet another, and so on to infinity. Thus of that which changes there cannot be any primary part that has changed. It is evident, then, from what has been said, that neither of that which changes nor of the time in which it changes is there any primary part.

With regard, however, to the actual subject of change-that is to say that in respect of which a thing changes-there is a difference to be observed. For in a process of change we may distinguish three terms-that which changes, that in which it changes, and the actual subject of change, e.g. the man, the time, and the fair complexion. Of these the man and the time are divisible: but with the fair complexion it is otherwise (though they are all divisible accidentally, for that in which the fair complexion or any other quality is an accident is divisible). For of actual subjects of change it will be seen that those which are classed as essentially, not accidentally, divisible have no primary part. Take the case of magnitudes: let AB be a magnitude, and suppose that it has moved from B to a primary 'where' G. Then if BG is taken to be indivisible, two things without parts will have to be contiguous (which is impossible): if on the other hand it is taken to be divisible, there will be something prior to G to which the magnitude has changed, and something else again prior to that, and so on to infinity, because the process of division may be continued without end. Thus there can be no primary 'where' to which a thing has changed. And if we take the case of quantitative change, we shall get a like result, for here too the change is in something continuous. It is evident, then, that only in qualitative motion can there be anything essentially indivisible.

6

Now everything that changes changes time, and that in two senses: for the time in which a thing is said to change may be the primary time, or on the other hand it may have an

extended reference, as e.g. when we say that a thing changes in a particular year because it changes in a particular day. That being so, that which changes must be changing in any part of the primary time in which it changes. This is clear from our definition of 'primary', in which the word is said to express just this: it may also, however, be made evident by the following argument. Let ChRh be the primary time in which that which is in motion is in motion: and (as all time is divisible) let it be divided at K. Now in the time ChK it either is in motion or is not in motion, and the same is likewise true of the time KRh. Then if it is in motion in neither of the two parts, it will be at rest in the whole: for it is impossible that it should be in motion in a time in no part of which it is in motion. If on the other hand it is in motion in only one of the two parts of the time, ChRh cannot be the primary time in which it is in motion: for its motion will have reference to a time other than ChRh. It must, then, have been in motion in any part of ChRh.

And now that this has been proved, it is evident that everything that is in motion must have been in motion before. For if that which is in motion has traversed the distance KL in the primary time ChRh, in half the time a thing that is in motion with equal velocity and began its motion at the same time will have traversed half the distance. But if this second thing whose velocity is equal has traversed a certain distance in a certain time, the original thing that is in motion must have traversed the same distance in the same time. Hence that which is in motion must have been in motion before.

Again, if by taking the extreme moment of the time-for it is the moment that defines the time, and time is that which is intermediate between moments-we are enabled to say that motion has taken place in the whole time ChRh or in fact in any period of it, motion may likewise be said to have taken place in every other such period. But half the time finds an extreme in the point of division. Therefore motion will have taken place in half the time and in fact in any part of it: for as soon as any division is made there is always a time defined by moments. If, then, all time is divisible, and that which is intermediate between moments is time, everything that is changing must have completed an infinite number of changes.

Again, since a thing that changes continuously and has not perished or ceased from its change must either be changing or have changed in any part of the time of its change, and since it cannot be changing in a moment, it follows that it must have changed at every moment in the time: consequently, since the moments are infinite in number, everything that is changing must have completed an infinite number of changes.

And not only must that which is changing have changed, but that which has changed must also previously have been changing, since everything that has changed from something to something has changed in a period of time. For suppose that a thing has changed from A to B in a moment. Now the moment in which it has changed cannot be the same as that in which it is at A (since in that case it would be in A and B at once): for we have shown above that that that which has changed, when it has changed, is not in that from which it has changed. If, on the other hand, it is a different moment, there will be a period of time intermediate between the two: for, as we saw, moments are not consecutive. Since, then, it has changed in a period of time, and all time is divisible, in half the time it will have completed another change, in a quarter another, and so on to infinity: consequently when it has changed, it must have previously been changing.

Moreover, the truth of what has been said is more evident in the case of magnitude, because the magnitude over which what is changing changes is continuous. For suppose that a thing has changed from G to D. Then if GD is indivisible, two things without parts will be consecutive. But since this is impossible, that which is intermediate between them must be a magnitude and divisible into an infinite number of segments: consequently, before the change is completed, the thing changes to those segments. Everything that has changed, therefore, must previously have been changing: for the same proof also holds good of change with respect to what is not continuous, changes, that is to say, between contraries and between contradictories. In such cases we have only to take the time in which a thing has changed and again apply the same reasoning. So that which has changed must have been changing and that which is changing must have changed, and a process of change is preceded by a completion of change and a completion by a process: and we can never take any stage and say that it is absolutely the first. The reason of this is that no two things without parts can be contiguous, and therefore in change the process of division is infinite, just as lines may be infinitely divided so that one part is continually increasing and the other continually decreasing.

So it is evident also that that that which has become must previously have been in process of becoming, and that which is in process of becoming must previously have become, everything (that is) that is divisible and continuous: though it is not always the actual thing that is in process of becoming of which this is true: sometimes it is something else, that is to say, some part of the thing in question, e.g. the foundation-stone of a house. So, too, in the case of that which is perishing and that which has perished: for that which becomes and that which perishes must contain an element of infiniteness as

an immediate consequence of the fact that they are continuous things: and so a thing cannot be in process of becoming without having become or have become without having been in process of becoming. So, too, in the case of perishing and having perished: perishing must be preceded by having perished, and having perished must be preceded by perishing. It is evident, then, that that which has become must previously have been in process of becoming, and that which is in process of becoming must previously have become: for all magnitudes and all periods of time are infinitely divisible.

Consequently no absolutely first stage of change can be represented by any particular part of space or time which the changing thing may occupy.

7

Now since the motion of everything that is in motion occupies a period of time, and a greater magnitude is traversed in a longer time, it is impossible that a thing should undergo a finite motion in an infinite time, if this is understood to mean not that the same motion or a part of it is continually repeated, but that the whole infinite time is occupied by the whole finite motion. In all cases where a thing is in motion with uniform velocity it is clear that the finite magnitude is traversed in a finite time. For if we take a part of the motion which shall be a measure of the whole, the whole motion is completed in as many equal periods of the time as there are parts of the motion. Consequently, since these parts are finite, both in size individually and in number collectively, the whole time must also be finite: for it will be a multiple of the portion, equal to the time occupied in completing the aforesaid part multiplied by the number of the parts.

But it makes no difference even if the velocity is not uniform. For let us suppose that the line AB represents a finite stretch over which a thing has been moved in the given time, and let GD be the infinite time. Now if one part of the stretch must have been traversed before another part (this is clear, that in the earlier and in the later part of the time a different part of the stretch has been traversed: for as the time lengthens a different part of the motion will always be completed in it, whether the thing in motion changes with uniform velocity or not: and whether the rate of motion increases or diminishes or remains stationary this is none the less so), let us then take AE a part of the whole stretch of motion AB which shall be a measure of AB. Now this part of the motion occupies a certain period of the infinite time: it cannot itself occupy an infinite time, for we are assuming that that is occupied by the whole AB. And if again I take another part equal to

AE, that also must occupy a finite time in consequence of the same assumption. And if I go on taking parts in this way, on the one hand there is no part which will be a measure of the infinite time (for the infinite cannot be composed of finite parts whether equal or unequal, because there must be some unity which will be a measure of things finite in multitude or in magnitude, which, whether they are equal or unequal, are none the less limited in magnitude); while on the other hand the finite stretch of motion AB is a certain multiple of AE: consequently the motion AB must be accomplished in a finite time. Moreover it is the same with coming to rest as with motion. And so it is impossible for one and the same thing to be infinitely in process of becoming or of perishing. The reasoning he will prove that in a finite time there cannot be an infinite extent of motion or of coming to rest, whether the motion is regular or irregular. For if we take a part which shall be a measure of the whole time, in this part a certain fraction, not the whole, of the magnitude will be traversed, because we assume that the traversing of the whole occupies all the time. Again, in another equal part of the time another part of the magnitude will be traversed: and similarly in each part of the time that we take, whether equal or unequal to the part originally taken. It makes no difference whether the parts are equal or not, if only each is finite: for it is clear that while the time is exhausted by the subtraction of its parts, the infinite magnitude will not be thus exhausted, since the process of subtraction is finite both in respect of the quantity subtracted and of the number of times a subtraction is made. Consequently the infinite magnitude will not be traversed in finite time: and it makes no difference whether the magnitude is infinite in only one direction or in both: for the same reasoning will hold good.

This having been proved, it is evident that neither can a finite magnitude traverse an infinite magnitude in a finite time, the reason being the same as that given above: in part of the time it will traverse a finite magnitude and in each several part likewise, so that in the whole time it will traverse a finite magnitude.

And since a finite magnitude will not traverse an infinite in a finite time, it is clear that neither will an infinite traverse a finite in a finite time. For if the infinite could traverse the finite, the finite could traverse the infinite; for it makes no difference which of the two is the thing in motion; either case involves the traversing of the infinite by the finite. For when the infinite magnitude A is in motion a part of it, say GD, will occupy the finite and then another, and then another, and so on to infinity. Thus the two results will coincide: the infinite will have completed a motion over the finite and the finite will have traversed the infinite: for it would seem to be impossible for the motion of the infinite over the finite

to occur in any way other than by the finite traversing the infinite either by locomotion over it or by measuring it. Therefore, since this is impossible, the infinite cannot traverse the finite.

Nor again will the infinite traverse the infinite in a finite time. Otherwise it would also traverse the finite, for the infinite includes the finite. We can further prove this in the same way by taking the time as our starting-point.

Since, then, it is established that in a finite time neither will the finite traverse the infinite, nor the infinite the finite, nor the infinite the infinite, it is evident also that in a finite time there cannot be infinite motion: for what difference does it make whether we take the motion or the magnitude to be infinite? If either of the two is infinite, the other must be so likewise: for all locomotion is in space.

8

Since everything to which motion or rest is natural is in motion or at rest in the natural time, place, and manner, that which is coming to a stand, when it is coming to a stand, must be in motion: for if it is not in motion it must be at rest: but that which is at rest cannot be coming to rest. From this it evidently follows that coming to a stand must occupy a period of time: for the motion of that which is in motion occupies a period of time, and that which is coming to a stand has been shown to be in motion: consequently coming to a stand must occupy a period of time.

Again, since the terms 'quicker' and 'slower' are used only of that which occupies a period of time, and the process of coming to a stand may be quicker or slower, the same conclusion follows.

And that which is coming to a stand must be coming to a stand in any part of the primary time in which it is coming to a stand. For if it is coming to a stand in neither of two parts into which the time may be divided, it cannot be coming to a stand in the whole time, with the result that that that which is coming to a stand will not be coming to a stand. If on the other hand it is coming to a stand in only one of the two parts of the time, the whole cannot be the primary time in which it is coming to a stand: for it is coming to a stand in the whole time not primarily but in virtue of something distinct from itself, the argument being the same as that which we used above about things in motion.

And just as there is no primary time in which that which is in motion is in motion, so too there is no primary time in which that which is coming to a stand is coming to a stand, there being no primary stage either of being in motion or of coming to a stand. For let AB be the primary time in which a thing is coming to a stand. Now AB cannot be without parts: for there cannot be motion in that which is without parts, because the moving thing would necessarily have been already moved for part of the time of its movement: and that which is coming to a stand has been shown to be in motion. But since AB is therefore divisible, the thing is coming to a stand in every one of the parts of AB: for we have shown above that it is coming to a stand in every one of the parts in which it is primarily coming to a stand. Since then, that in which primarily a thing is coming to a stand must be a period of time and not something indivisible, and since all time is infinitely divisible, there cannot be anything in which primarily it is coming to a stand.

Nor again can there be a primary time at which the being at rest of that which is at rest occurred: for it cannot have occurred in that which has no parts, because there cannot be motion in that which is indivisible, and that in which rest takes place is the same as that in which motion takes place: for we defined a state of rest to be the state of a thing to which motion is natural but which is not in motion when (that is to say in that in which) motion would be natural to it. Again, our use of the phrase 'being at rest' also implies that the previous state of a thing is still unaltered, not one point only but two at least being thus needed to determine its presence: consequently that in which a thing is at rest cannot be without parts. Since, then it is divisible, it must be a period of time, and the thing must be at rest in every one of its parts, as may be shown by the same method as that used above in similar demonstrations.

So there can be no primary part of the time: and the reason is that rest and motion are always in a period of time, and a period of time has no primary part any more than a magnitude or in fact anything continuous: for everything continuous is divisible into an infinite number of parts.

And since everything that is in motion is in motion in a period of time and changes from something to something, when its motion is comprised within a particular period of time essentially-that is to say when it fills the whole and not merely a part of the time in question-it is impossible that in that time that which is in motion should be over against some particular thing primarily. For if a thing-itself and each of its parts-occupies the same space for a definite period of time, it is at rest: for it is in just these circumstances

that we use the term 'being at rest'-when at one moment after another it can be said with truth that a thing, itself and its parts, occupies the same space. So if this is being at rest it is impossible for that which is changing to be as a whole, at the time when it is primarily changing, over against any particular thing (for the whole period of time is divisible), so that in one part of it after another it will be true to say that the thing, itself and its parts, occupies the same space. If this is not so and the aforesaid proposition is true only at a single moment, then the thing will be over against a particular thing not for any period of time but only at a moment that limits the time. It is true that at any moment it is always over against something stationary: but it is not at rest: for at a moment it is not possible for anything to be either in motion or at rest. So while it is true to say that that which is in motion is at a moment not in motion and is opposite some particular thing, it cannot in a period of time be over against that which is at rest: for that would involve the conclusion that that which is in locomotion is at rest.

9

Zeno's reasoning, however, is fallacious, when he says that if everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless. This is false, for time is not composed of indivisible moments any more than any other magnitude is composed of indivisibles.

Zeno's arguments about motion, which cause so much disquietude to those who try to solve the problems that they present, are four in number. The first asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal. This we have discussed above.

The second is the so-called 'Achilles', and it amounts to this, that in a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead. This argument is the same in principle as that which depends on bisection, though it differs from it in that the spaces with which we successively have to deal are not divided into halves. The result of the argument is that the slower is not overtaken: but it proceeds along the same lines as the bisection-argument (for in both a division of the space in a certain way leads to the result that the goal is not reached, though the 'Achilles' goes further in that it affirms that even the quickest runner in legendary tradition must fail in his pursuit of the slowest), so

that the solution must be the same. And the axiom that that which holds a lead is never overtaken is false: it is not overtaken, it is true, while it holds a lead: but it is overtaken nevertheless if it is granted that it traverses the finite distance prescribed. These then are two of his arguments.

The third is that already given above, to the effect that the flying arrow is at rest, which result follows from the assumption that time is composed of moments: if this assumption is not granted, the conclusion will not follow.

The fourth argument is that concerning the two rows of bodies, each row being composed of an equal number of bodies of equal size, passing each other on a race-course as they proceed with equal velocity in opposite directions, the one row originally occupying the space between the goal and the middle point of the course and the other that between the middle point and the starting-post. This, he thinks, involves the conclusion that half a given time is equal to double that time. The fallacy of the reasoning lies in the assumption that a body occupies an equal time in passing with equal velocity a body that is in motion and a body of equal size that is at rest; which is false. For instance (so runs the argument), let A, A...be the stationary bodies of equal size, B, B...the bodies, equal in number and in size to A, A...,originally occupying the half of the course from the starting-post to the middle of the A's, and G, G...those originally occupying the other half from the goal to the middle of the A's, equal in number, size, and velocity to B, B....Then three consequences follow:

First, as the B's and the G's pass one another, the first B reaches the last G at the same moment as the first G reaches the last B. Secondly at this moment the first G has passed all the A's, whereas the first B has passed only half the A's, and has consequently occupied only half the time occupied by the first G, since each of the two occupies an equal time in passing each A. Thirdly, at the same moment all the B's have passed all the G's: for the first G and the first B will simultaneously reach the opposite ends of the course, since (so says Zeno) the time occupied by the first G in passing each of the B's is equal to that occupied by it in passing each of the A's, because an equal time is occupied by both the first B and the first G in passing all the A's. This is the argument, but it presupposed the aforesaid fallacious assumption.

Nor in reference to contradictory change shall we find anything unanswerable in the argument that if a thing is changing from not-white, say, to white, and is in neither

condition, then it will be neither white nor not-white: for the fact that it is not wholly in either condition will not preclude us from calling it white or not-white. We call a thing white or not-white not necessarily because it is be one or the other, but cause most of its parts or the most essential parts of it are so: not being in a certain condition is different from not being wholly in that condition. So, too, in the case of being and not-being and all other conditions which stand in a contradictory relation: while the changing thing must of necessity be in one of the two opposites, it is never wholly in either.

Again, in the case of circles and spheres and everything whose motion is confined within the space that it occupies, it is not true to say the motion can be nothing but rest, on the ground that such things in motion, themselves and their parts, will occupy the same position for a period of time, and that therefore they will be at once at rest and in motion. For in the first place the parts do not occupy the same position for any period of time: and in the second place the whole also is always changing to a different position: for if we take the orbit as described from a point A on a circumference, it will not be the same as the orbit as described from B or G or any other point on the same circumference except in an accidental sense, the sense that is to say in which a musical man is the same as a man. Thus one orbit is always changing into another, and the thing will never be at rest. And it is the same with the sphere and everything else whose motion is confined within the space that it occupies.

10

Our next point is that that which is without parts cannot be in motion except accidentally: i.e. it can be in motion only in so far as the body or the magnitude is in motion and the partless is in motion by inclusion therein, just as that which is in a boat may be in motion in consequence of the locomotion of the boat, or a part may be in motion in virtue of the motion of the whole. (It must be remembered, however, that by 'that which is without parts' I mean that which is quantitatively indivisible (and that the case of the motion of a part is not exactly parallel): for parts have motions belonging essentially and severally to themselves distinct from the motion of the whole. The distinction may be seen most clearly in the case of a revolving sphere, in which the velocities of the parts near the centre and of those on the surface are different from one another and from that of the whole; this implies that there is not one motion but many). As we have said, then, that which is without parts can be in motion in the sense in which a man sitting in a boat is in motion when the boat is travelling, but it cannot be in motion of itself. For suppose that

it is changing from AB to BG-either from one magnitude to another, or from one form to another, or from some state to its contradictory-and let D be the primary time in which it undergoes the change. Then in the time in which it is changing it must be either in AB or in BG or partly in one and partly in the other: for this, as we saw, is true of everything that is changing. Now it cannot be partly in each of the two: for then it would be divisible into parts. Nor again can it be in BG: for then it will have completed the change, whereas the assumption is that the change is in process. It remains, then, that in the time in which it is changing, it is in AB. That being so, it will be at rest: for, as we saw, to be in the same condition for a period of time is to be at rest. So it is not possible for that which has no parts to be in motion or to change in any way: for only one condition could have made it possible for it to have motion, viz. that time should be composed of moments, in which case at any moment it would have completed a motion or a change, so that it would never be in motion, but would always have been in motion. But this we have already shown above to be impossible: time is not composed of moments, just as a line is not composed of points, and motion is not composed of starts: for this theory simply makes motion consist of indivisibles in exactly the same way as time is made to consist of moments or a length of points.

Again, it may be shown in the following way that there can be no motion of a point or of any other indivisible. That which is in motion can never traverse a space greater than itself without first traversing a space equal to or less than itself. That being so, it is evident that the point also must first traverse a space equal to or less than itself. But since it is indivisible, there can be no space less than itself for it to traverse first: so it will have to traverse a distance equal to itself. Thus the line will be composed of points, for the point, as it continually traverses a distance equal to itself, will be a measure of the whole line. But since this is impossible, it is likewise impossible for the indivisible to be in motion.

Again, since motion is always in a period of time and never in a moment, and all time is divisible, for everything that is in motion there must be a time less than that in which it traverses a distance as great as itself. For that in which it is in motion will be a time, because all motion is in a period of time; and all time has been shown above to be divisible. Therefore, if a point is in motion, there must be a time less than that in which it has itself traversed any distance. But this is impossible, for in less time it must traverse less distance, and thus the indivisible will be divisible into something less than itself, just as the time is so divisible: the fact being that the only condition under which that which

is without parts and indivisible could be in motion would have been the possibility of the infinitely small being in motion in a moment: for in the two questions-that of motion in a moment and that of motion of something indivisible-the same principle is involved.

Our next point is that no process of change is infinite: for every change, whether between contradictories or between contraries, is a change from something to something. Thus in contradictory changes the positive or the negative, as the case may be, is the limit, e.g. being is the limit of coming to be and not-being is the limit of ceasing to be: and in contrary changes the particular contraries are the limits, since these are the extreme points of any such process of change, and consequently of every process of alteration: for alteration is always dependent upon some contraries. Similarly contraries are the extreme points of processes of increase and decrease: the limit of increase is to be found in the complete magnitude proper to the peculiar nature of the thing that is increasing, while the limit of decrease is the complete loss of such magnitude. Locomotion, it is true, we cannot show to be finite in this way, since it is not always between contraries. But since that which cannot be cut (in the sense that it is inconceivable that it should be cut, the term 'cannot' being used in several senses)-since it is inconceivable that that which in this sense cannot be cut should be in process of being cut, and generally that that which cannot come to be should be in process of coming to be, it follows that it is inconceivable that that which cannot complete a change should be in process of changing to that to which it cannot complete a change. If, then, it is to be assumed that that which is in locomotion is in process of changing, it must be capable of completing the change. Consequently its motion is not infinite, and it will not be in locomotion over an infinite distance, for it cannot traverse such a distance.

It is evident, then, that a process of change cannot be infinite in the sense that it is not defined by limits. But it remains to be considered whether it is possible in the sense that one and the same process of change may be infinite in respect of the time which it occupies. If it is not one process, it would seem that there is nothing to prevent its being infinite in this sense; e.g. if a process of locomotion be succeeded by a process of alteration and that by a process of increase and that again by a process of coming to be: in this way there may be motion for ever so far as the time is concerned, but it will not be one motion, because all these motions do not compose one. If it is to be one process, no motion can be infinite in respect of the time that it occupies, with the single exception of rotatory locomotion.