History of Modern Mathematics

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ARTICLE ONE: INTRODUCTION

In considering the history of modern mathematics two questions at once arise: (1) what limitations shall be placed upon the term Mathematics; (2) what force shall be assigned to the word Modern? In other words, how shall Modern Mathematics be defined?

In these pages the term Mathematics will be limited to the domain of pure science. Questions of the applications of the various branches will be considered only incidentally. Such great contributions as those of Newton in the realm of mathematical physics, of Laplace in celestial mechanics, of Lagrange and Cauchy in the wave theory, and of Poisson, Fourier, and Bessel in the theory of heat, belong rather to the field of applications.

In particular, in the domain of numbers reference will be made to certain of the contributions to the general theory, to the men who have placed the study of irrational and transcendent numbers upon a scientific foundation, and to those who have developed the modern theory of complex numbers and its elaboration in the field of quaternions and Ausdehnungslehre. In the theory of equations the names of some of the leading investigators will be mentioned, together with a brief statement of the results which they secured. The impossibility of solving the quintic will lead to a consideration of the names of the founders of the group theory and of the doctrine of determinants. This phase of higher algebra will be followed by the theory of forms, or quantics. The later development of the calculus, leading to differential equations and the theory of functions, will complete the algebraic side, save for a brief reference to the theory of probabilities. In the domain of geometry some of the contributors to the later development of the analytic and synthetic fields will be mentioned, together with the most noteworthy results of their labors. Had the author's space not been so strictly limited he would have given lists of those who have worked in other important lines, but the topics considered have been thought to have the best right to prominent place under any reasonable definition of Mathematics.

Modern Mathematics is a term by no means well defined. Algebra cannot be called modern, and yet the theory of equations has received some of its most important

additions during the nineteenth century, while the theory of forms is a recent creation. Similarly with elementary geometry; the labors of Lobachevsky and Bolyai during the second quarter of the century threw a new light upon the whole sub ject, and more recently the study of the triangle has added another chapter to the theory. Thus the history of modern mathematics must also be the modern history of ancient branches, while sub jects which seem the product of late generations have root in other centuries than the present.

How unsatisfactory must be so brief a sketch may be inferred from a glance at the Index du Repértoire Bibliographique des Sciences Mathématiques (Paris, 1893), whose seventy-one pages contain the mere enumeration of sub jects in large part modern, or from a consideration of the twenty-six volumes of the Jahrbuch über die Fortschritte der Mathematik, which now devotes over a thousand pages a year to a record of the progress of the science. 1

The seventeenth and eighteenth centuries laid the foundations of much of the sub ject as known to-day. The discovery of the analytic geometry by Descartes, the contributions to the theory of numbers by Fermat, to algebra by Harriot, to geometry and to mathematical physics by Pascal, and the discovery of the differential calculus by Newton and Leibniz, all contributed to make the seventeenth century memorable. The eighteenth century was naturally one of great activity. Euler and the Bernoulli family in Switzerland, d'Alembert, Lagrange, and Laplace in Paris, and Lambert in Germany, popularized Newton's great discovery, and extended both its theory and its applications. Accompanying this activity, however, was a too implicit faith in the calculus and in the inherited principles of mathematics, which left the foundations insecure and necessitated their strengthening by the succeeding generation.

The nineteenth century has been a period of intense study of first principles, of the recognition of necessary limitations of various branches, of a great spread of mathematical knowledge, and of the opening of extensive fields for applied mathematics. Especially influential has been the establishment of scientific schools and journals and university chairs. The great renaissance of geometry is not a little due to the foundation of the École Polytechnique in Paris (1794-5), and the similar schools in Prague (1806), Vienna (1815), Berlin (1820), Karlsruhe (1825), and numerous other cities. About the middle of the century these schools began to exert a still a greater influence through the custom of calling to them mathematicians of high repute, thus making Zürich, Karlsruhe, Munich, Dresden, and other cities well known as mathematical centers.

In 1796 appeared the first number of the Journal de l' École Polytechnique. Crelle's Journal für die reine und angewandte Mathematik appeared in 1826, and ten years later Liouville began the publication of the Journal de Mathématiques pures et appliquées, which has been continued by Resal and Jordan. The Cambridge Mathematical Journal was established in 1839, and merged into the Cambridge and Dublin Mathematical Journal in 1846. Of the other periodicals which have contributed to the spread of mathematical knowledge, only a few can be mentioned: the Nouvelles Annales de Mathématiques (1842), Grunert's Archiv der Mathematik (1843), Tortolini's Annali di Scienze Matematiche e Fisiche (1850), Schlömilch's Zeitschrift für Mathematik und Physik (1856), the Quarterly Journal of Mathematics (1857), Battaglini's Giornale di Matematiche (1863), the Mathematische Annalen (1869), the Bulletin des Sciences Mathématiques (1870), the American Journal of Mathematics (1878), the Acta Mathematica (1882), and the Annals of Mathematics (1884). 2 To this list should be added a recent venture, unique in its aims, namely, L'Intermédiaire des Mathématiciens (1894), and two annual publications of great value, the Jahrbuch already mentioned (1868), and the Jahresbericht der deutschen Mathematiker-Vereinigung (1892).

To the influence of the schools and the journals must be added that of the various learned societies3 whose published proceedings are widely known, together with the increasing liberality of such societies in the preparation of complete works of a monumental character.

The study of first principles, already mentioned, was a natural consequence of the reckless application of the new calculus and the Cartesian geometry during the eighteenth century. This development is seen in theorems relating to infinite series, in the fundamental principles of number, rational, irrational, and complex, and in the concepts of limit, contiunity, function, the infinite, and the infinitesimal. But the nineteenth century has done more than this. It has created new and extensive branches of an importance which promises much for pure and applied mathematics. Foremost among these branches stands the theory of functions founded by Cauchy, Riemann, and Weierstrass, followed by the descriptive and pro jective geometries, and the theories of groups, of forms, and of determinants.

The nineteenth century has naturally been one of specialization. At its opening one might have hoped to fairly compass the mathematical, physical, and astronomical sciences, as did Lagrange, Laplace, and Gauss. But the advent of the new generation, with Monge and Carnot, Poncelet and Steiner, Galois, Abel, and Jacobi, tended to split mathematics into branches between which the relations were long to remain obscure.

In this respect recent years have seen a reaction, the unifying tendency again becoming prominent through the theories of functions and groups. 4

1 The foot-notes give only a few of the authorities which might easily be cited. They are thought to include those from which considerable extracts have been made, the necessary condensation of these extracts making any other form of acknowledgment impossible.

2 For a list of current mathematical journals see the Jahrbuch über die Fortschritte der Mathematik. A small but convenient list of standard periodicals is given in Carr's Synopsis of Pure Mathematics, p. 843; Mackay, J. S., Notice sur le journalisme mathématique en Angleterre, Association française pour l'Avancement des Sciences, 1893, II, 303; Cajori, F., Teaching and History of Mathematics in the United States, pp. 94, 277; Hart, D. S., History of American Mathematical Periodicals, The Analyst, Vol. II, p. 131.

3 For a list of such societies consult any recent number of the Philosophical Transactions of Royal Society of London. Dyck, W., Einleitung zu dem für den mathematischen Teil der deutschen Universitätsausstellung ausgegebenen Specialkatalog, Mathematical Papers Chicago Congress (New York, 1896), p. 41.

4 Klein, F., The Present State of Mathematics, Mathematical Papers of Chicago Congress (New York, 1896), p. 133.