Article 8: Determinants

The Theory of Determinants1 may be said to take its origin with Leibniz (1693), following whom Cramer (1750) added slightly to the theory, treating the subject, as did his predecessor, wholly in relation to sets of equations. The recurrent law was first announced by Bezout (1764). But it was Vandermonde (1771) who first recognized determinants as independent functions. To him is due the first connected exposition of the theory, and he may be called its formal founder. Laplace (1772) gave the general method of expanding a determinant in terms of its complementary minors, although Vandermonde had already given a special case. Immediately following, Lagrange (1773) treated determinants of the second and third order, possibly stopping here because the idea of hyperspace was not then in vogue. Although contributing nothing to the general theory, Lagrange was the first to apply determinants to questions foreign to eliminations, and to him are due many special identities which have since been brought under well-known theorems. During the next quarter of a century little of importance was done. Hindenburg (1784) and Rothe (1800) kept the subject open, but Gauss (1801) made the next advance. Like Lagrange, he made much use of determinants in the theory of numbers. He introduced the word "determinants" (Laplace had used "resultant"), though not in the present signification,2 but rather as applied to the discriminant of a quantic. Gauss also arrived at the notion of reciprocal determinants, and came very near the multiplication theorem. The next contributor of importance is Binet (1811, 1812), who formally stated the theorem relating to the product of two matrices of m columns and n rows, which for the special case of m = n reduces to the multiplication theorem. On the same day (Nov. 30, 1812) that Binet presented his paper to the Academy, Cauchy also presented one on the subject. In this he used the word "determinant" in its present sense, summarized and simplified what was then known on the subject, improved the notation, and gave the multiplication theorem with a proof more satisfactory than Binet's. He was the first to grasp the subject as a whole; before him there were determinants, with him begins their theory in its generality.

The next great contributor, and the greatest save Cauchy, was Jacobi (from 1827). With him the word "determinant" received its final acceptance. He early used the functional determinant which Sylvester has called the “Jacobian,” and in his famous memoirs in Crelle for 1841 he specially treats this subject, as well as that class of alternating functions which Sylvester has called “Alternants.” But about the time of Jacobi’s closing memoirs, Sylvester (1839) and Cayley began their great work, a work which it is impossible to briefly summarize, but which represents the development of the theory to the present time.

The study of special forms of determinants has been the natural result of the completion of the general theory. Axi-symmetric determinants have been studied by Lebesgue, Hesse, and Sylvester; per-symmetric determinants by Sylvester and Hankel; circulants by Catalan, Spottiswoode, Glaisher, and Scott; skew determinants and Pfaffians, in connection with the theory of orthogonal transformation, by Cayley; continuants by Sylvester; Wronskians (so called by Muir) by Christoffel and Frobenius; compound determinants by Sylvester, Reiss, and Picquet; Jacobians and Hessians by Sylvester; and symmetric gauche determinants by Trudi. Of the text-books on the subject Spottiswoode’s was the first. In America, Hanus (1886) and Weld (1893) have published treatises.

1 Muir, T., Theory of Determinants in the Historical Order of its Development, Part I, 1890; Baltzer, R., Theorie und Anwendung der Determinanten. 1881. The writer is under obligations to Professor Weld, who contributes Chap. II, for valuable assistance in compiling this article.

2 “Numerum bb – ac, cuius indole proprietates formæ(a, b, c) imprimis pendere in sequen tibus docebimus, determinantem huius uocabimus.”