History of Modern Mathematics

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ARTICLE 10: CALCULUS

The Differential and Integral Calculus,1 dating from Newton and Leibniz, was quite complete in its general range at the close of the eighteenth century. Aside from the study of first principles, to which Gauss, Cauchy, Jordan, Picard, Méray, and those whose names are mentioned in connection with the theory of functions, have contributed, there must be mentioned the development of symbolic methods, the theory of definite integrals, the calculus of variations, the theory of differential equations, and the numerous applications of the Newtonian calculus to physical problems. Among those who have prepared noteworthy general treatises are Cauchy (1821), Raabe (1839-47), Duhamel (1856), Sturm (1857-59), Bertrand (1864), Serret (1868), Jordan (2d ed., 1893), and Picard (1891-93). A recent contribution to analysis which promises to be valuable is Oltramare's Calcul de Généralization (1893).

Abel seems to have been the first to consider in a general way the question as to what differential expressions can be integrated in a finite form by the aid of ordinary functions, an investigation extended by Liouville. Cauchy early undertook the general theory of determining definite integrals, and the subject has been prominent during the century. Frullani's theorem (1821), Bierens de Haan's work on the theory (1862) and his elaborate tables (1867), Dirichlet's lectures (1858) embodied in Meyer's treatise (1871), and numerous memoirs of Legendre, Poisson, Plana, Raabe, Sohncke, Schlömilch, Elliott, Leudesdorf, and Kronecker are among the noteworthy contributions.

Eulerian Integrals were first studied by Euler and afterwards investigated by Legendre, by whom they were classed as Eulerian integrals of the first and second species, as follows: , although these were not the exact forms of Euler's study. If n is integral, it follows that , but if n is fractional it is a transcendent function. To it Legendre assigned the symbol □, and it is now called the gamma function. To the subject Dirichlet has contributed an important theorem (Liouville, 1839), which has been elaborated by Liouville, Catalan, Leslie Ellis, and others. On the evaluation of □x and log □x Raabe (1843-44), Bauer (1859), and Gudermann (1845) have written. Legendre's great table appeared in 1816.

Symbolic Methods may be traced back to Taylor, and the analogy between successive differentiation and ordinary exponentials had been observed by numerous writers before the nineteenth century. Arbogast (1800) was the first, however, to separate the symbol of operation from that of quantity in a differential equation. François (1812) and Servois (1814) seem to have been the first to give correct rules on the subject. Hargreave (1848) applied these methods in his memoir on differential equations, and Boole freely employed them. Grassmann and Hankel made great use of the theory, the former in studying equations, the latter in his theory of complex numbers.

The Calculus of Variations2 may be said to begin with a problem of Johann Bernoulli's (1696). It immediately occupied the attention of Jakob Bernoulli and the Marquis de l'Hˆopital, but Euler first elaborated the subject. His contributions began in 1733, and his Elementa Calculi Variationum gave to the science its name. Lagrange contributed extensively to the theory, and Legendre (1786) laid down a method, not entirely satisfactory, for the discrimination of maxima and minima. To this discrimination Brunacci (1810), Gauss (1829), Poisson (1831), Ostrogradsky (1834), and Jacobi (1837) have been among the contributors. An important general work is that of Sarrus (1842) which was condensed and improved by Cauchy (1844). Other valuable treatises and memoirs have been written by Strauch (1849), Jellett (1850), Hesse (1857), Clebsch (1858), and Carll (1885), but perhaps the most important work of the century is that of Weierstrass. His celebrated course on the theory is epoch-making, and it may be asserted that he was the first to place it on a firm and unquestionable foundation.

The Application of the Infinitesimal Calculus to problems in physics and astronomy was contemporary with the origin of the science. All through the eighteenth century these applications were multiplied, until at its close Laplace and Lagrange had brought the whole range of the study of forces into the realm of analysis. To Lagrange (1773) we owe the introduction of the theory of the potential3 into dynamics, although the name "potential function" and the fundamental memoir of the subject are due to Green (1827, printed in 1828). The name "potential" is due to Gauss (1840), and the distinction between potential and potential function to Clausius. With its development are connected the names of Dirichlet, Riemann, Neumann, Heine, Kronecker, Lipschitz, Christoffel, Kirchhoff, Beltrami, and many of the leading physicists of the century.

It is impossible in this place to enter into the great variety of other applications of analysis to physical problems. Among them are the investigations of Euler on vibrating chords; Sophie Germain on elastic membranes; Poisson, Lamé, Saint-Venant, and Clebsch on the elasticity of three-dimensional bodies; Fourier on heat diffusion; Fresnel on light; Maxwell, Helmholtz, and Hertz on electricity; Hansen, Hill, and Gyldén on astronomy; Maxwell on spherical harmonics; Lord Rayleigh on acoustics; and the contributions of Dirichlet, Weber, Kirchhoff, F. Neumann, Lord Kelvin, Clausius, Bjerknes, MacCullagh, and Fuhrmann to physics in general. The labors of Helmholtz should be especially mentioned, since he contributed to the theories of dynamics, electricity, etc., and brought his great analytical powers to bear on the fundamental axioms of mechanics as well as on those of pure mathematics.

1 Williamson, B., Infinitesimal Calculus, Encyclopædia Britannica, 9th edition; Cantor, M., Geschichte der Mathematik, Vol. III, pp. 150-316; Vivanti, G., Note sur l'histoire de l'infiniment petit, Bibliotheca Mathematica, 1894, p. 1; Mansion, P., Esquisse de l'histoire du calcul infinitésimal, Ghent, 1887. Le deux centième anniversaire de l'invention du calcul différentiel; Mathesis, Vol. IV, p. 163.

2 Carll, L. B., Calculus of Variations, New York, 1885, Chap. V; Todhunter, I., History of the Progress of the Calculus of Variations, London, 1861; Reiff, R., Die Anfänge der Variationsrechnung, Mathematisch-naturwissenschaftliche Mittheilungen, Tübingen, 1887, p. 90.

3 Bacharach, M., Abriss der Geschichte der Potentialtheorie, 1883. This contains an extensive bibliography.