History of Modern Mathematics

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ARTICLE 14: PROBABILITIES AND LEAST SQUARES

The Theory of Probabilities and Errors[1] is, as applied to observations, largely a nineteenth-century development. The doctrine of probabilities dates, however, as far back as Fermat and Pascal (1654). Huygens (1657) gave the first scientific treatment of the subject, and Jakob Bernoulli's Ars Conjectandi (posthumous, 1713) and De Moivre's Doctrine of Chances (1718)[2] raised the subject to the plane of a branch of mathematics. The theory of errors may be traced back to Cotes's Opera Miscellanea (posthumous, 1722), but a memoir prepared by Simpson in 1755 (printed 1756) first applied the theory to the discussion of errors of observation. The reprint (1757) of this memoir lays down the axioms that positive and negative errors are equally probable, and that there are certain assignable limits within which all errors may be supposed to fall; continuous errors are discussed and a probability curve is given. Laplace (1774) made the first attempt to deduce a rule for the combination of observations from the principles of the theory of probabilities. He represented the law of probability of errors by a curve y = □(x), x being any error and y its probability, and laid down three properties of this curve: (1) It is symmetric as to the y-axis; (2) the x-axis is an asymptote, the probability of the error ∞ being 0; (3) the area enclosed is 1, it being certain that an error exists. He deduced a formula for the mean of three observations. He also gave (1781) a formula for the law of facility of error (a term due to Lagrange, 1774), but one which led to unmanageable equations. Daniel Bernoulli (1778) introduced the principle of the maximum product of the probabilities of a system of concurrent errors.

The Method of Least Squares is due to Legendre (1805), who introduced it in his Nouvelles méthodes pour la détermination des orbites des comètes. In ignorance of Legendre's contribution, an Irish-American writer, Adrain, editor of "The Analyst" (1808), first deduced the law of facility of error, , c and h being constants depending on precision of observation. He gave two proofs, the second being essentially the same as Herschel's (1850). Gauss gave the first proof which seems to have been known in Europe (the third after Adrain's) in 1809. To him is due much of the honor of placing the subject before the mathematical world, both as to the theory and its applications.

Further proofs were given by Laplace (1810, 1812), Gauss (1823), Ivory (1825, 1826), Hagen (1837), Bessel (1838), Donkin (1844, 1856), and Crofton (1870). Other contributors have been Ellis (1844), De Morgan (1864), Glaisher (1872), and Schiaparelli (1875). Peters's (1856) formula for r, the probable error of a single observation, is well known.3

Among the contributors to the general theory of probabilities in the nineteenth century have been Laplace, Lacroix (1816), Littrow (1833), Quetelet (1853), Dedekind (1860), Helmert (1872), Laurent (1873), Liagre, Didion, and Pearson. De Morgan and Boole improved the theory, but added little that was fundamentally new. Czuber has done much both in his own contributions (1884, 1891), and in his translation (1879) of Meyer. On the geometric side the influence of Miller and The Educational Times has been marked, as also that of such contributors to this journal as Crofton, McColl, Wolstenholme, Watson, and Artemas Martin.

1 Merriman, M., Method of Least Squares, New York, 1884, p. 182; Transactions of Connecticut Academy, 1877, Vol. IV, p. 151, with complete bibliography; Todhunter, I., History of the Mathematical Theory of Probability, 1865; Cantor, M., Geschichte der Mathematik, Vol. III, p. 316.

2 Eneström, G., Review of Cantor, Bibliotheca Mathematica, 1896, p. 20.

3 Bulletin of New York Mathematical Society, Vol. II, p. 57.