History of Modern Mathematics

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GENERAL TENDENCIES

The opening of the nineteenth century was, as we have seen, a period of profound introspection following a period of somewhat careless use of the material accumulated in the seventeenth century. The mathematical world sought to orientate itself, to examine the foundations of its knowledge, and to critically examine every step in its several theories. It then took up the line of discovery once more, less recklessly than before, but still with thoughts directed primarily in the direction of invention. At the close of the century there came again a period of introspection, and one of the recent tendencies is towards a renewed study of foundation principles. In England one of the leaders in this movement is Russell, who has studied the foundations of geometry (1897) and of mathematics in general (1903). In America the fundamental conceptions and methods of mathematics have been considered by Bôocher in his St. Louis address in 1904,5 and the question of a series of irreducible postulates has been studied by Huntington. In Italy, Padoa and Bureli-Forti have studied the fundamental postulates of algebra, and Pieri those of geometry. In Germany, Hilbert has probably given the most attention to the foundation principles of geometry (1899), and more recently he has investigated the compatibility of the arithmetical axioms (1900). In France, Poincaré has considered the rôle of intuition and of logic in mathematics,6 and in every country the foundation principles have been made the object of careful investigation.

As an instance of the orientation already mentioned, the noteworthy address of Hilbert at Paris in 19007 stands out prominently. This address reviews the field of pure mathematics and sets forth several of the greatest questions demanding investigation at the present time. In the particular line of geometry the memoir which Segré wrote in 1891, on the tendencies in geometric investigation, has recently been revised and brought up to date.8

There is also seen at the present time, as never before, a tendency to co-operate, to exchange views, and to internationalize mathematics. The first international congress of mathematicians was held at Zurich in 1897, the second one at Paris in 1900, and the third at Heidelberg in 1904. The first international congress of philosophy was held at Paris in 1900, the third section being devoted to logic and the history of the sciences

(on this occasion chiefly mathematics), and the second congress was held at Geneva in 1904. There was also held an international congress of historic sciences at Rome in 1903, an international committee on the organization of a congress on the history of sciences being at that time formed. The result of such gatherings has been an exchange of views in a manner never before possible, supplementing in an inspiring way the older form of international communication through published papers.

In the United States there has been shown a similar tendency to exchange opinions and to impart verbal information as to recent discoveries. The American Mathematical Society, founded in 1894,9 has doubled its membership in the past decade,10 and has increased its average of annual papers from 30 to 150. It has also established two sections, one at Chicago (1897) and one at San Francisco (1902). The activity of its members and the quality of papers prepared has led to the publication of the Transactions, beginning with 1900. In order that its members may be conversant with the lines of investigation in the various mathematical centers, the society publishes in its Bul letin the courses in advanced mathematics offered in many of the leading universities of the world. Partly as a result of this activity, and partly because of the large number of American students who have recently studied abroad, a remarkable change is at present passing over the mathematical work done in the universities and colleges of this country. Courses that a short time ago were offered in only a few of our leading universities are now not uncommon in institutions of college rank. They are often given by men who have taken advanced degrees in mathematics, at Göttingen, Berlin, Paris, or other leading universities abroad, and they are awakening a great interest in the modern field. A recent investigation (1903) showed that 67 students in ten American institutions were taking courses in the theory of functions, 11 in the theory of elliptic functions, 94 in projective geometry, 26 in the theory of invariants, 45 in the theory of groups, and 46 in the modern advanced theory of equations, courses which only a few years ago were rarely given in this country. A similar change is seen in other countries, notably in England and Italy, where courses that a few years ago were offered only in Paris or in Germany are now within the reach of university students at home. The interest at present manifested by American scholars is illustrated by the fact that only four countries (Germany, Russia, Austria, and France) had more representatives than the United States, among the 336 regular members at the third international mathematical congress at Heidelberg in 1904.

The activity displayed at the present time in putting the work of the masters into usable form, so as to define clear points of departure along the several lines of research, is seen in the large number of collected works published or in course of publication

in the last decade. These works have usually been published under governmental patronage, often by some learned society, and always under the editorship of some recognized authority. They include the works of Galileo, Fermat, Descartes, Huygens, Laplace, Gauss, Galois, Cauchy, Hesse, Plücker, Grassmann, Dirichlet, Laguerre, Kronecker, Fuchs, Weierstrass, Stokes, Tait, and various other leaders in mathematics. It is only natural to expect a number of other sets of collected works in the near future, for not only is there the remote past to draw upon, but the death roll of the last decade has been a large one. The following is only a partial list of eminent mathematicians who have recently died, and whose collected works have been or are in the course of being published, or may be deemed worthy of publication in the future: Cayley (1895), Neumann (1895), Tisserand (1896), Brioschi (1897), Sylvester (1897), Weierstrass (1897), Lie (1899), Beltrami (1900), Bertrand (1900), Tait (1901), Hermite (1901), Fuchs (1902), Gibbs (1903), Cremona (1903), and Salmon (1904), besides such writers as Frost (1898), Hoppe (1900), Craig (1900), Schlömilch (1901), Everett on the side of mathematical physics (1904), and Paul Tannery, the best of the modern French historians of mathematics (1904).11

It is, of course, impossible to detect with any certainty the present tendencies in mathematics. Judging, however, by the number and nature of the published papers and works of the past few years, it is reasonable to expect a great development in all lines, especially in such modern branches as the theory of groups, theory of functions, theory of invariants, higher geometry, and differential equations. If we may judge from the works in applied mathematics which have recently appeared, we are entering upon an era similar to that in which Laplace labored, an era in which all these modern theories of mathematics shall find application in the study of physical problems, including those that relate to the latest discoveries. The profound study of applied mathematics in France and England, the advanced work in discovery in pure mathematics in Germany and France, and the search for the logical bases for the science that has distinguished Italy as well as Germany, are all destined to affect the character of the international mathematics of the immediate future. Probably no single influence will be more prominent in the internationalizing process than the tendency of the younger generation of American mathematicians to study in England, France, Germany, and Italy, and to assimilate the best that each of these countries has to offer to the world.

5 Bulletin of the American Mathematical Society (N. S.), Vol. XI, p. 115.

6 Compte rendu du deuxième congrès international des mathématiciens tenu à Paris, 1900. Paris, 1902, p. 115.

7 Göttinger Nachrichten, 1900, p. 253; Archiv der Mathematik und Physik, 1901, pp. 44, 213; Bulletin of the American Mathematical Society, 1902, p. 437.

8 Bulletin of the American Mathematical Society (N. S.), Vol. X, p. 443.

9 It was founded as the New York Mathematical Society six years earlier, in 1888.

10 It is now, in 1905, approximately 500.

11 For students wishing to investigate the work of mathematicians who died in the last two decades of the nineteenth century, Eneström's "Bio-bibliographie der 1881-1900 verstorbenen Mathematiker," in the Bibliotheca Mathematica Vol. II (3), p. 326 (1901), will be found valuable.