



# *The Game of Logic*

*by Lewis Carroll*

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## Chapter 1: Part One - *Propositions*

*NEW LAMPS FOR OLD.*

*“Light come, light go.”*

### **1. Propositions.**

*“Some new Cakes are nice.” “No new Cakes are nice.” “All new cakes are nice.”*

*There are three ‘PROPOSITIONS’ for you--the only three kinds we are going to use in this Game: and the first thing to be done is to learn how to express them on the Board.*

*Let us begin with*

*“Some new Cakes are nice.”*

*But before doing so, a remark has to be made--one that is rather important, and by no means easy to understand all in a moment: so please to read this VERY carefully.*

*The world contains many THINGS (such as “Buns”, “Babies”, “Beetles”. “Battledores”. &c.); and these Things possess many ATTRIBUTES (such as “baked”, “beautiful”, “black”, “broken”, &c.: in fact, whatever can be “attributed to”, that is “said to belong to”, any Thing, is an Attribute). Whenever we wish to mention a Thing, we use a SUBSTANTIVE: when we wish to mention an Attribute, we use an ADJECTIVE. People have asked the question “Can a Thing exist without any Attributes belonging to it?” It is a very puzzling question, and I’m not going to try to answer it: let us turn up our noses, and treat it with contemptuous silence, as if it really wasn’t worth noticing. But, if they put it the other way, and ask “Can an Attribute exist without any Thing for it to belong to?”, we may say at once “No: no more than a Baby could go a railway-journey with no one to take care of it!” You never saw “beautiful” floating about in the air, or littered about on the floor, without any Thing to BE beautiful, now did you?*

*And now what am I driving at, in all this long rigmarole? It is this. You may put “is” or “are” between names of two THINGS (for example, “some Pigs are fat Animals”), or between the names of two ATTRIBUTES (for example, “pink is light-red”), and in each case it will make good sense. But, if you put “is” or “are” between the name of a THING and the name of an ATTRIBUTE (for example, “some Pigs are pink”), you do NOT make good sense (for how can a Thing BE an Attribute?) unless you have an understanding with the person to whom you are speaking. And the simplest understanding would, I think, be this--that the Substantive shall be supposed to be repeated at the end of the sentence, so that the sentence, if written out in full, would be “some Pigs are pink (Pigs)”. And now the word “are” makes quite good sense.*

*Thus, in order to make good sense of the Proposition “some new Cakes are nice”, we must suppose it to be written out in full, in the form “some new Cakes are nice (Cakes)”. Now this contains two ‘TERMS’--“new Cakes” being one of them, and “nice (Cakes)” the other. “New Cakes,” being the one we are talking about, is called the ‘SUBJECT’ of the Proposition, and “nice (Cakes)” the ‘PREDICATE’. Also this Proposition is said to be a ‘PARTICULAR’ one, since it does not speak of the WHOLE of its Subject, but only of a PART of it. The other two kinds are said to be ‘UNIVERSAL’, because they speak of the WHOLE of their Subjects--the one denying niceness, and the other asserting it, of the WHOLE class of “new Cakes”. Lastly, if you would like to have a definition of the word ‘PROPOSITION’ itself, you may take this:--“a sentence stating that some, or none, or all, of the Things belonging to a certain class, called its ‘Subject’, are also Things belonging to a certain other class, called its ‘Predicate’”.*

*You will find these seven words--PROPOSITION, ATTRIBUTE, TERM, SUBJECT, PREDICATE, PARTICULAR, UNIVERSAL--charmingly useful, if any friend should happen to ask if you have ever studied Logic. Mind you bring all seven words into your answer, and your friend will go away deeply impressed--‘a sadder and a wiser man’.*

*Now please to look at the smaller Diagram on the Board, and suppose it to be a cupboard, intended for all the Cakes in the world (it would have to be a good large one, of course). And let us suppose all the new ones to be put into the upper half (marked ‘x’), and all the rest (that is, the NOT-new ones) into the lower half (marked ‘x’). Thus the lower half would contain ELDERLY Cakes, AGED Cakes, ANTE-DILUVIAN Cakes--if there are any: I haven’t seen many, myself--and so on. Let us also suppose all the nice Cakes to be put into the left-hand half (marked ‘y’), and all the rest (that is, the not-nice ones) into*

the right-hand half (marked 'y'). At present, then, we must understand x to mean "new", x' "not-new", y "nice", and y' "not-nice."

And now what kind of Cakes would you expect to find in compartment No. 5?

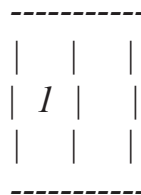
It is part of the upper half, you see; so that, if it has any Cakes in it, they must be NEW: and it is part of the left-hand half; so that they must be NICE. Hence if there are any Cakes in this compartment, they must have the double 'ATTRIBUTE' "new and nice": or, if we use letters, they must be "x y."

Observe that the letters x, y are written on two of the edges of this compartment. This you will find a very convenient rule for knowing what Attributes belong to the Things in any compartment. Take No. 7, for instance. If there are any Cakes there, they must be "x'y", that is, they must be "not-new and nice."

Now let us make another agreement--that a red counter in a compartment shall mean that it is 'OCCUPIED', that is, that there are SOME Cakes in it. (The word 'some,' in Logic, means 'one or more' so that a single Cake in a compartment would be quite enough reason for saying "there are SOME Cakes here"). Also let us agree that a grey counter in a compartment shall mean that it is 'EMPTY', that is that there are NO Cakes in it. In the following Diagrams, I shall put '1' (meaning 'one or more') where you are to put a RED counter, and '0' (meaning 'none') where you are to put a GREY one.

As the Subject of our Proposition is to be "new Cakes", we are only concerned, at present, with the UPPER half of the cupboard, where all the Cakes have the attribute x, that is, "new."

Now, fixing our attention on this upper half, suppose we found it marked like this,



that is, with a red counter in No. 5. What would this tell us, with regard to the class of "new Cakes"?

Would it not tell us that there are *SOME* of them in the *x y*-compartment? That is, that some of them (besides having the Attribute *x*, which belongs to both compartments) have the Attribute *y* (that is, “nice”). This we might express by saying “some *x*-Cakes are *y*-Cakes”, or, putting words instead of letters,

“Some new Cakes are nice (Cakes)”,

or, in a shorter form,

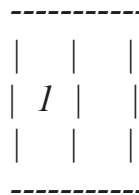
“Some new Cakes are nice”.

At last we have found out how to represent the first Proposition of this Section. If you have not *CLEARLY* understood all I have said, go no further, but read it over and over again, till you *DO* understand it. After that is once mastered, you will find all the rest quite easy.

It will save a little trouble, in doing the other Propositions, if we agree to leave out the word “Cakes” altogether. I find it convenient to call the whole class of Things, for which the cupboard is intended, the ‘UNIVERSE.’ Thus we might have begun this business by saying “Let us take a Universe of Cakes.” (Sounds nice, doesn’t it?)

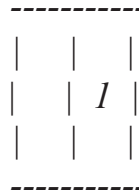
Of course any other Things would have done just as well as Cakes. We might make Propositions about “a Universe of Lizards”, or even “a Universe of Hornets”. (Wouldn’t *THAT* be a charming Universe to live in?)

So far, then, we have learned that



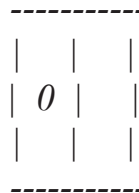
means “some *x* and *y*,” i.e. “some new are nice.”

*I think you will see without further explanation, that*



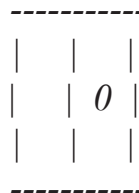
*means “some x are y’,” i.e. “some new are not-nice.”*

*Now let us put a GREY counter into No. 5, and ask ourselves the meaning of*



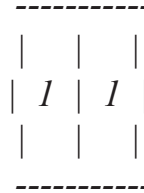
*This tells us that the x y-compartment is EMPTY, which we may express by “no x are y”, or, “no new Cakes are nice”. This is the second of the three Propositions at the head of this Section.*

*In the same way,*



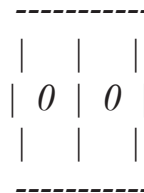
*would mean “no x are y’,” or, “no new Cakes are not-nice.”*

*What would you make of this, I wonder?*



*I hope you will not have much trouble in making out that this represents a DOUBLE Proposition: namely, “some x are y, AND some are y’,” i.e. “some new are nice, and some are not-nice.”*

*The following is a little harder, perhaps:*



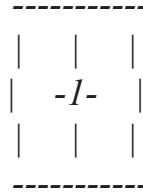
*This means “no x are y, AND none are y’,” i.e. “no new are nice, AND none are not-nice”: which leads to the rather curious result that “no new exist,” i.e. “no Cakes are new.” This is because “nice” and “not-nice” make what we call an ‘EXHAUSTIVE’ division of the class “new Cakes”: i.e. between them, they EXHAUST the whole class, so that all the new Cakes, that exist, must be found in one or the other of them.*

*And now suppose you had to represent, with counters the contradictory to “no Cakes are new”, which would be “some Cakes are new”, or, putting letters for words, “some Cakes are x”, how would you do it?*

*This will puzzle you a little, I expect. Evidently you must put a red counter SOMEWHERE in the x-half of the cupboard, since you know there are SOME new Cakes. But you must not put it into the LEFT-HAND compartment, since you do not know them to be NICE: nor may you put it into the RIGHT-HAND one, since you do not know them to be NOT-NICE.*

*What, then, are you to do? I think the best way out of the difficulty is to place the red counter*

ON THE DIVISION-LINE between the *xy*-compartment and the *xy'*-compartment. This I shall represent (as I always put 'I' where you are to put a red counter) by the diagram



Our ingenious American cousins have invented a phrase to express the position of a man who wants to join one or the other of two parties--such as their two parties 'Democrats' and 'Republicans'--but can't make up his mind WHICH. Such a man is said to be "sitting on the fence." Now that is exactly the position of the red counter you have just placed on the division-line. He likes the look of No. 5, and he likes the look of No. 6, and he doesn't know WHICH to jump down into. So there he sits astride, silly fellow, dangling his legs, one on each side of the fence!

Now I am going to give you a much harder one to make out. What does this mean?



This is clearly a DOUBLE Proposition. It tells us not only that "some *x* are *y*," but also the "no *x* are NOT *y*." Hence the result is "ALL *x* are *y*," i.e. "all new Cakes are nice", which is the last of the three Propositions at the head of this Section.

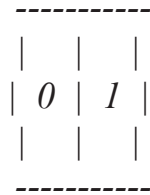
We see, then, that the Universal Proposition

"All new Cakes are nice"

consists of TWO Propositions taken together, namely,

"Some new Cakes are nice," and "No new Cakes are not-nice."

*In the same way*



would mean “all *x* are *y*”, that is,

“All new Cakes are not-nice.”

Now what would you make of such a Proposition as “The Cake you have given me is nice”? Is it Particular or Universal?

“Particular, of course,” you readily reply. “One single Cake is hardly worth calling ‘some,’ even.”

No, my dear impulsive Reader, it is ‘Universal’. Remember that, few as they are (and I grant you they couldn’t well be fewer), they are (or rather ‘it is’) ALL that you have given me! Thus, if (leaving ‘red’ out of the question) I divide my Universe of Cakes into two classes--the Cakes you have given me (to which I assign the upper half of the cupboard), and those you HAVEN’T given me (which are to go below)--I find the lower half fairly full, and the upper one as nearly as possible empty. And then, when I am told to put an upright division into each half, keeping the NICE Cakes to the left, and the NOT-NICE ones to the right, I begin by carefully collecting ALL the Cakes you have given me (saying to myself, from time to time, “Generous creature! How shall I ever repay such kindness?”), and piling them up in the left-hand compartment. AND IT DOESN’T TAKE LONG TO DO IT!

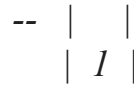
Here is another Universal Proposition for you. “Barzillai Beckalegg is an honest man.” That means “ALL the Barzillai Beckaleggs, that I am now considering, are honest men.” (You think I invented that name, now don’t you? But I didn’t. It’s on a carrier’s cart, somewhere down in Cornwall.)

This kind of Universal Proposition (where the Subject is a single Thing) is called an ‘INDIVIDUAL’ Proposition.



Now let us take "NICE Cakes" as the Subject of Proposition: that is, let us fix our thoughts on the LEFT-HAND half of the cupboard, where all the Cakes have attribute y, that is, "nice."

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Suppose we find it marked like this:



What would that tell us?



I hope that it is not necessary, after explaining the HORIZONTAL oblong so fully, to spend much time over the UPRIGHT one. I hope you will see, for yourself, that this means "some y are x", that is,

"Some nice Cakes are new."

"But," you will say, "we have had this case before. You put a red counter into No. 5, and you told us it meant 'some new Cakes are nice'; and NOW you tell us that it means 'some NICE Cakes are NEW'! Can it mean BOTH?"

The question is a very thoughtful one, and does you GREAT credit, dear Reader! It DOES mean both. If you choose to take x (that is, "new Cakes") as your Subject, and to regard No. 5 as part of a HORIZONTAL oblong, you may read it "some x are y", that is, "some new Cakes are nice": but, if you choose to take y (that is, "nice Cake") as your Subject, and to regard No. 5 as part of an UPRIGHT oblong, THEN you may read it "some y are x", that is, "some nice Cakes are new". They are merely two different ways of expressing the very same truth.

Without more words, I will simply set down the other ways in which this upright oblong might be marked, adding the meaning in each case. By comparing them with the various

*cases of the horizontal oblong, you will, I hope, be able to understand them clearly.*

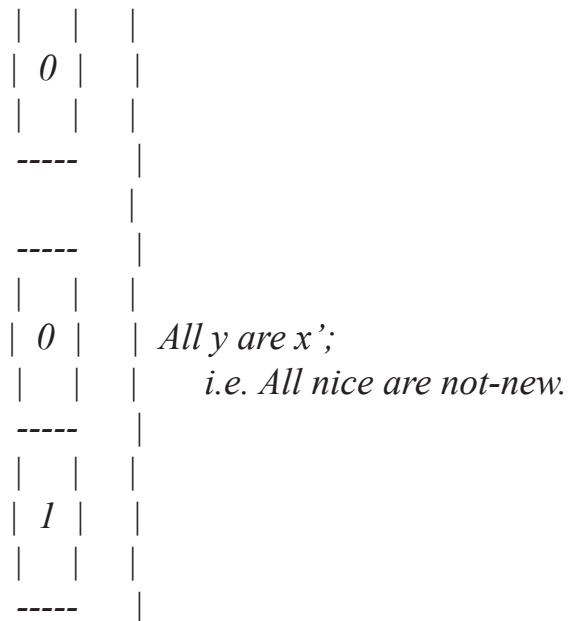
*You will find it a good plan to examine yourself on this table, by covering up first one column and then the other, and ‘dodging about’, as the children say.*

*Also you will do well to write out for yourself two other tables--one for the LOWER half of the cupboard, and the other for its RIGHT-HAND half.*

*And now I think we have said all we need to say about the smaller Diagram, and may go on to the larger one.*

<i>Symbols.</i>	<i>Meanings.</i>
-----             -----	<i>Some y are x’; i.e. Some nice are not-new.</i>
-----       1       -----	
-----       0       -----	<i>No y are x; i.e. No nice are new.</i>
-----             -----	<i>[Observe that this is merely another way of expressing “No new are nice.”]</i>





This may be taken to be a cupboard divided in the same way as the last, but ALSO divided into two portions, for the Attribute *m*. Let us give to *m* the meaning “wholesome”: and let us suppose that all WHOLESOME Cakes are placed INSIDE the central Square, and all the UNWHOLESOME ones OUTSIDE it, that is, in one or other of the four queer-shaped OUTER compartments.

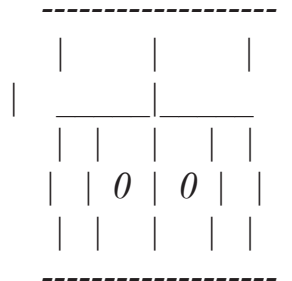
We see that, just as, in the smaller Diagram, the Cakes in each compartment had TWO Attributes, so, here, the Cakes in each compartment have THREE Attributes: and, just as the letters, representing the TWO Attributes, were written on the EDGES of the compartment, so, here, they are written at the CORNERS. (Observe that *m*’ is supposed to be written at each of the four outer corners.) So that we can tell in a moment, by looking at a compartment, what three Attributes belong to the Things in it. For instance, take No. 12. Here we find *x*, *y*’, *m*, at the corners: so we know that the Cakes in it, if there are any, have the triple Attribute, ‘*xy*’*m*’, that is, “new, not-nice, and wholesome.” Again, take No. 16. Here we find, at the corners, *x*’, *y*’, *m*’: so the Cakes in it are “not-new, not-nice, and unwholesome.” (Remarkably untempting Cakes!)

It would take far too long to go through all the Propositions, containing *x* and *y*, *x* and *m*, and *y* and *m* which can be represented on this diagram (there are ninety-six altogether, so I am sure you will excuse me!) and I must content myself with doing two or three, as

*specimens. You will do well to work out a lot more for yourself.*

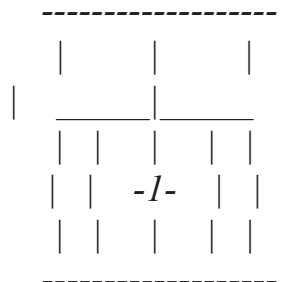
*Taking the upper half by itself, so that our Subject is “new Cakes”, how are we to represent “no new Cakes are wholesome”?*

*This is, writing letters for words, “no x are m.” Now this tells us that none of the Cakes, belonging to the upper half of the cupboard, are to be found INSIDE the central Square: that is, the two compartments, No. 11 and No. 12, are EMPTY. And this, of course, is represented by*



*And now how are we to represent the contradictory Proposition “SOME x are m”? This is a difficulty I have already considered. I think the best way is to place a red counter ON THE DIVISION-LINE between No. 11 and No. 12, and to understand this to mean that ONE of the two compartments is ‘occupied,’ but that we do not at present know WHICH.*

*This I shall represent thus:--*

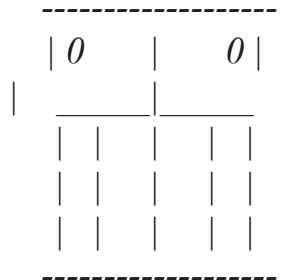


*Now let us express “all x are m.”*

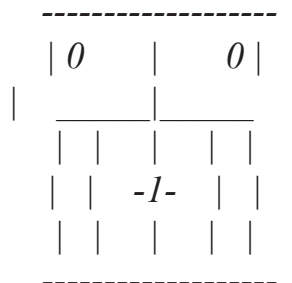
*This consists, we know, of TWO Propositions,*

*“Some x are m,” and “No x are m’.”*

*Let us express the negative part first. This tells us that none of the Cakes, belonging to the upper half of the cupboard, are to be found OUTSIDE the central Square: that is, the two compartments, No. 9 and No. 10, are EMPTY. This, of course, is represented by*

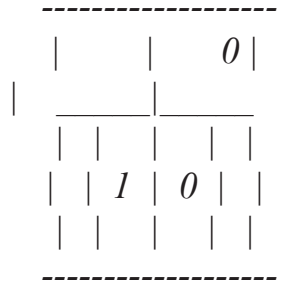


*But we have yet to represent “Some x are m.” This tells us that there are SOME Cakes in the oblong consisting of No. 11 and No. 12: so we place our red counter, as in the previous example, on the division-line between No. 11 and No. 12, and the result is*



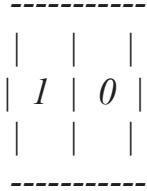
*Now let us try one or two interpretations.*

*What are we to make of this, with regard to x and y?*



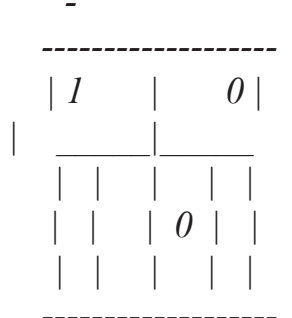
*This tells us, with regard to the xy'-Square, that it is wholly 'empty', since BOTH compartments are so marked. With regard to the xy-Square, it tells us that it is 'occupied'. True, it is only ONE compartment of it that is so marked; but that is quite enough, whether the other be 'occupied' or 'empty', to settle the fact that there is SOMETHING in the Square.*

*If, then, we transfer our marks to the smaller Diagram, so as to get rid of the m-subdivisions, we have a right to mark it*

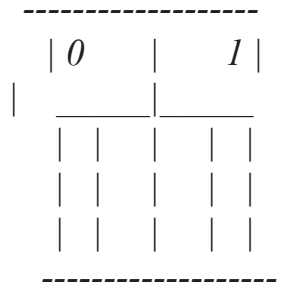


*which means, you know, "all x are y."*

*The result would have been exactly the same, if the given oblong had been marked thus:-*



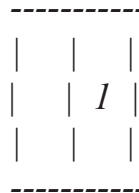
*Once more: how shall we interpret this, with regard to x and y?*



*This tells us, as to the xy-Square, that ONE of its compartments is 'empty'. But this information is quite useless, as there is no mark in the OTHER compartment. If the other compartment happened to be 'empty' too, the Square would be 'empty': and, if it happened to be 'occupied', the Square would be 'occupied'. So, as we do not know WHICH is the case, we can say nothing about THIS Square.*

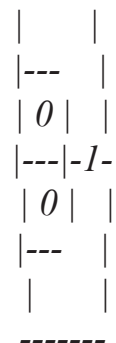
*The other Square, the xy'-Square, we know (as in the previous example) to be 'occupied'.*

*If, then, we transfer our marks to the smaller Diagram, we get merely this:--*



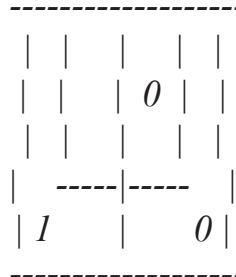
*which means, you know, "some x are y'."*

*These principles may be applied to all the other oblongs. For instance, to represent "all y' are m'" we should mark the RIGHT-HAND UPRIGHT OBLONG (the one that has the attribute y') thus:--*

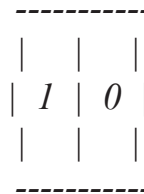




and, if we were told to interpret the lower half of the cupboard, marked as follows, with regard to *x* and *y*,



we should transfer it to the smaller Diagram thus,



and read it "all *x*' are *y*."

Two more remarks about Propositions need to be made.

One is that, in every Proposition beginning with "some" or "all", the ACTUAL EXISTENCE of the 'Subject' is asserted. If, for instance, I say "all misers are selfish," I mean that misers ACTUALLY EXIST.

If I wished to avoid making this assertion, and merely to state the LAW that miserliness necessarily involves selfishness, I should say "no misers are unselfish" which does not assert that any misers exist at all, but merely that, if any DID exist, they WOULD be selfish.

*The other is that, when a Proposition begins with “some” or “no”, and contains more than two Attributes, these Attributes may be re-arranged, and shifted from one Term to the other, “ad libitum.” For example, “some abc are def” may be re-arranged as “some bf are acde,” each being equivalent to “some Things are abcdef”. Again “No wise old men are rash and reckless gamblers” may be re-arranged as “No rash old gamblers are wise and reckless,” each being equivalent to “No men are wise old rash reckless gamblers.”*