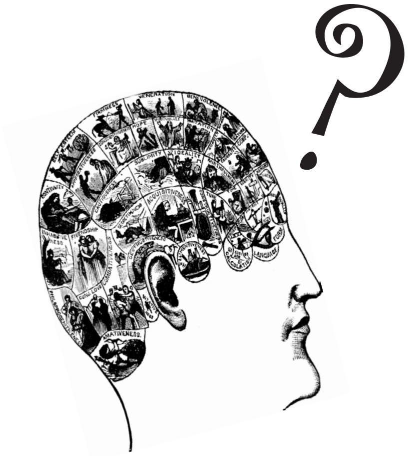

The Game of Logic

by Lewis Carroll

Chapter 1: Part Two - *Syllogisms*

Now suppose we divide our Universe of Things in three ways, with regard to three different Attributes. Out of these three Attributes, we may make up three different couples (for instance, if they were a, b, c, we might make up the three couples ab, ac, bc). Also suppose we have two Propositions given us, containing two of these three couples, and that from them we can prove a third Proposition containing the third couple. (For example, if we divide our Universe for m, x, and y; and if we have the two Propositions given us, “no m are x’” and “all m’ are y”, containing the two couples mx and my, it might be possible to prove from them a third Proposition, containing x and y.)

In such a case we call the given Propositions ‘THE PREMISSES’, the third one ‘THE CONCLUSION’ and the whole set ‘A SYLLOGISM’.

Evidently, ONE of the Attributes must occur in both Premisses; or else one must occur in ONE Premiss, and its CONTRADICTORY in the other.

In the first case (when, for example, the Premisses are “some m are x” and “no m are y’”) the Term, which occurs twice, is called ‘THE MIDDLE TERM’, because it serves as a sort of link between the other two Terms.

In the second case (when, for example, the Premisses are “no m are x’” and “all m’ are y”) the two Terms, which contain these contradictory Attributes, may be called ‘THE MIDDLE TERMS’.

Thus, in the first case, the class of “m-Things” is the Middle Term; and, in the second case, the two classes of “m-Things” and “m’-Things” are the Middle Terms.

The Attribute, which occurs in the Middle Term or Terms, disappears in the Conclusion, and is said to be “eliminated”, which literally means “turned out of doors”.

Now let us try to draw a Conclusion from the two Premisses--

"Some new Cakes are unwholesome;
No nice Cakes are unwholesome."

In order to express them with counters, we need to divide Cakes in THREE different ways, with regard to newness, to niceness, and to wholesomeness. For this we must use the larger Diagram, making x mean "new", y "nice", and m "wholesome". (Everything INSIDE the central Square is supposed to have the attribute m, and everything OUTSIDE it the attribute m', i.e. "not-m".)

You had better adopt the rule to make m mean the Attribute which occurs in the MIDDLE Term or Terms. (I have chosen m as the symbol, because 'middle' begins with 'm'.)

Now, in representing the two Premisses, I prefer to begin with the NEGATIVE one (the one beginning with "no"), because GREY counters can always be placed with CERTAINTY, and will then help to fix the position of the red counters, which are sometimes a little uncertain where they will be most welcome.

Let us express, the "no nice Cakes are unwholesome (Cakes)", i.e. "no y-Cakes are m'-(Cakes)". This tells us that none of the Cakes belonging to the y-half of the cupboard are in its m'-compartments (i.e. the ones outside the central Square). Hence the two compartments, No. 9 and No. 15, are both 'EMPTY'; and we must place a grey counter in EACH of them, thus:--

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 |  --|--  |
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 | | | | | |
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 |0   |    |
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We have now to express the other Premiss, namely, "some new Cakes are unwholesome (Cakes)", i.e. "some x-Cakes are m'-(Cakes)". This tells us that some of the Cakes in the x-half of the cupboard are in its m'-compartments. Hence ONE of the two compartments, No. 9 and No. 10, is 'occupied': and, as we are not told in WHICH of these two compartments to place the red counter, the usual rule would be to lay it on the division-line between them: but, in this case, the other Premiss has settled the matter for us, by declaring No. 9 to be EMPTY. Hence the red counter has no choice, and MUST go into No. 10, thus:--

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|0    |    1|
|   --|--   |
| |   |   | |
|--|-----|--|
| |   |   | |
|   --|--   |
|0    |     |
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And now what counters will this information enable us to place in the SMALLER Diagram, so as to get some Proposition involving x and y only, leaving out m? Let us take its four compartments, one by one.

First, No. 5. All we know about THIS is that its OUTER portion is empty: but we know nothing about its inner portion. Thus the Square MAY be empty, or it MAY have something in it. Who can tell? So we dare not place ANY counter in this Square.

Secondly, what of No. 6? Here we are a little better off. We know that there is SOMETHING in it, for there is a red counter in its outer portion. It is true we do not know whether its inner portion is empty or occupied: but what does THAT matter? One solitary Cake, in one corner of the Square, is quite sufficient excuse for saying "THIS SQUARE IS OCCUPIED", and for marking it with a red counter.

As to No. 7, we are in the same condition as with No. 5--we find it PARTLY 'empty', but we do not know whether the other part is empty or occupied: so we dare not mark this Square.

And as to No. 8, we have simply no information at all.

The result is

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|   | 1 |
|---|---|
|   |   |
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Our 'Conclusion', then, must be got out of the rather meager piece of information that there is a red counter in the xy'-Square. Hence our Conclusion is "some x are y' ", i.e. "some new Cakes are not-nice (Cakes)": or, if you prefer to take y' as your Subject, "some not-nice Cakes are new (Cakes)"; but the other looks neatest.

We will now write out the whole Syllogism, putting the symbol " ∴ " for "therefore", and omitting "Cakes", for the sake of brevity, at the end of each Proposition.

"Some new Cakes are unwholesome;
No nice Cakes are unwholesome
∴ Some new Cakes are not-nice."

And you have now worked out, successfully, your first 'SYLLOGISM'. Permit me to congratulate you, and to express the hope that it is but the beginning of a long and glorious series of similar victories!

We will work out one other Syllogism--a rather harder one than the last--and then, I think, you may be safely left to play the Game by yourself, or (better) with any friend whom you can find, that is able and willing to take a share in the sport.

Let us see what we can make of the two Premisses--

"All Dragons are uncanny;
All Scotchmen are canny."

Remember, I don't guarantee the Premisses to be FACTS. In the first place, I never even saw a Dragon: and, in the second place, it isn't of the slightest consequence to us, as LOGICIANS, whether our Premisses are true or false: all WE have to do is to make out whether they LEAD LOGICALLY TO THE CONCLUSION, so that, if THEY were true, IT would be true also.

You see, we must give up the "Cakes" now, or our cupboard will be of no use to us. We must take, as our 'Universe', some class of things which will include Dragons and Scotchmen: shall we say 'Animals'? And, as "canny" is evidently the Attribute belonging to the 'Middle Terms', we will let m stand for "canny", x for "Dragons", and y for "Scotchmen". So that our two Premisses are, in full,

"All Dragon-Animals are uncanny (Animals);
All Scotchman-Animals are canny (Animals)."

And these may be expressed, using letters for words, thus:--

"All x are m';
All y are m."

The first Premiss consists, as you already know, of two parts:--

"Some x are m'," and "No x are m."

And the second also consists of two parts:--

"Some y are m," and "No y are m'."

Let us take the negative portions first.

We have, then, to mark, on the larger Diagram, first, "no x are m", and secondly, "no y are m'". I think you will see, without further explanation, that the two results, separately, are

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  |   |   |       |0  |   |
 |  --|--  |       |  --|--  |
 |  |0 | 0|  |      | | | | |
|--|--|--|--|      |--|--|--|--|
  | | | | |        | | | | |
 |  --|--  |       |  --|--  |
  |   |   |       |0  |   |
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and that these two, when combined, give us

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|0   |     |
|  --|--   |
| |0 | 0|  |
|--|--|--|--|
| |  |  |  |
|  --|--   |
|0   |     |
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We have now to mark the two positive portions, "some x are m'" and "some y are m".

The only two compartments, available for Things which are xm', are No. 9 and No. 10. Of these, No. 9 is already marked as 'empty'; so our red counter must go into No. 10.

Similarly, the only two, available for ym, are No. 11 and No. 13. Of these, No. 11 is already marked as 'empty'; so our red counter MUST go into No. 13.

The final result is

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|0   |    1|
|  --|--   |
| |0 | 0|  |
|--|--|--|--|
| |1 |  |  |
|  --|--   |
|0   |     |
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And now how much of this information can usefully be transferred to the smaller Diagram?

Let us take its four compartments, one by one.

As to No. 5? This, we see, is wholly 'empty'. (So mark it with a grey counter.)

As to No. 6? This, we see, is 'occupied'. (So mark it with a red counter.)

As to No. 7? Ditto, ditto.

As to No. 8? No information.

The smaller Diagram is now pretty liberally marked:--

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| 0 | 1 |
|---|---|
| 1 |   |
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And now what Conclusion can we read off from this? Well, it is impossible to pack such abundant information into ONE Proposition: we shall have to indulge in TWO, this time.

First, by taking x as Subject, we get "all x are y'", that is,

"All Dragons are not-Scotchmen":

secondly, by taking y as Subject, we get "all y are x'", that is,

"All Scotchmen are not-Dragons".

Let us now write out, all together, our two Premisses and our brace of Conclusions.

"All Dragons are uncanny;
All Scotchmen are canny.
∴ All Dragons are not-Scotchmen;
All Scotchmen are not-Dragons."

Let me mention, in conclusion, that you may perhaps meet with logical treatises in which it is not assumed that any Thing EXISTS at all, by "some x are y" is understood to mean "the Attributes x, y are COMPATIBLE, so that a Thing can have both at once", and "no x are y" to mean "the Attributes x, y are INCOMPATIBLE, so that nothing can have both at once".

In such treatises, Propositions have quite different meanings from what they have in our 'Game of Logic', and it will be well to understand exactly what the difference is.

First take "some x are y". Here WE understand "are" to mean "are, as an actual FACT"--which of course implies that some x-Things EXIST. But THEY (the writers of these other treatises) only understand "are" to mean "CAN be", which does not at all imply that any EXIST. So they mean LESS than we do: our meaning includes theirs (for of course "some x ARE y" includes "some x CAN BE y"), but theirs does NOT include ours. For example, "some Welsh hippopotami are heavy" would be TRUE, according to these writers (since the Attributes "Welsh" and "heavy" are quite COMPATIBLE in a hippopotamus), but it would be FALSE in our Game (since there are no Welsh hippopotami to BE heavy).

Secondly, take "no x are y". Here WE only understand "are" to mean "are, as an actual FACT"--which does not at all imply that no x CAN be y. But THEY understand the Proposition to mean, not only that none ARE y, but that none CAN POSSIBLY be y. So they mean more than we do: their meaning includes ours (for of course "no x CAN be y" includes "no x ARE y"), but ours does NOT include theirs. For example, "no Policemen are eight feet high" would be TRUE in our Game (since, as an actual fact, no such splendid specimens are ever found), but it would be FALSE, according to these writers (since the Attributes "belonging to the Police Force" and "eight feet high" are quite COMPATIBLE: there is nothing to PREVENT a Policeman from growing to that height, if sufficiently rubbed with Rowland's Macassar Oil--which said to make HAIR grow, when rubbed on hair, and so of course will make a POLICEMAN grow, when rubbed on a Policeman).

Thirdly, take "all x are y", which consists of the two partial Propositions "some x are y" and "no x are y'". Here, of course, the treatises mean LESS than we do in the FIRST part, and more than we do in the SECOND. But the two operations don't balance each other--any more than you can console a man, for having knocked down one of his chimneys, by giving him an extra door-step.

If you meet with Syllogisms of this kind, you may work them, quite easily, by the system I have given you: you have only to make 'are' mean 'are CAPABLE of being', and all will go smoothly. For "some x are y" will become "some x are capable of being y", that is, "the Attributes x, y are COMPATIBLE". And "no x are y" will become "no x are capable of being y", that is, "the Attributes x, y are INCOMPATIBLE". And, of course, "all x are y" will become "some x are capable of being y, and none are capable of being y'", that is, "the Attributes x, y are COMPATIBLE, and the Attributes x, y' are INCOMPATIBLE." In using the Diagrams for this system, you must understand a red counter to mean "there may POSSIBLY be something in this compartment," and a grey one to mean "there cannot POSSIBLY be anything in this compartment."