George Peacock

by Alexander Macfarlane

GEORGE PEACOCK[1]
(1791-1858)

George Peacock was born on April 9, 1791, at Denton in the north of England, 14 miles from Richmond in Yorkshire. His father, the Rev. Thomas Peacock, was a clergyman of the Church of England, incumbent and for 50 years curate of the parish of Denton, where he also kept a school. In early life Peacock did not show any precocity of genius, and was more remarkable for daring feats of climbing than for any special attachment to study. He received his elementary education from his father, and at 17 years of age, was sent to Richmond, to a school taught by a graduate of Cambridge University to receive instruction preparatory to entering that University. At this school he distinguished himself greatly both in classics and in the rather elementary mathematics then required for entrance at Cambridge. In 1809 he became a student of Trinity College, Cambridge.

Here it may be well to give a brief account of that University, as it was the alma mater of four out of the six mathematicians discussed in this course of lectures[2].

At that time the University of Cambridge consisted of seventeen colleges, each of which had an independent endowment, buildings, master, fellows and scholars. The endowments, generally in the shape of lands, have come down from ancient times; for example, Trinity College was founded by Henry VIII in 1546, and at the beginning of the 19th century it consisted of a master, 60 fellows and 72 scholars. Each college was provided with residence halls, a dining hall, and a chapel. Each college had its own staff of instructors called tutors or lecturers, and the function of the University apart from the colleges was mainly to examine for degrees. Examinations for degrees consisted of a pass examination and an honors examination, the latter called a tripos. Thus, the mathematical tripos meant the examinations of candidates for the degree of Bachelor of Arts who had made a special study of mathematics. The examination was spread over a week, and those who obtained honors were divided into three classes, the highest class being called wranglers, and the highest man among the wranglers,

senior wrangler. In more recent times this examination developed into what De Morgan called a "great writing race;" the questions being of the nature of short problems. A candidate put himself under the training of a coach, that is, a mathematician who made it a business to study the kind of problems likely to be set, and to train men to solve and write out the solution of as many as possible per hour. As a consequence the lectures of the University professors and the instruction of the college tutors were neglected, and nothing was studied except what would pay in the tripos examination. Modifications have been introduced to counteract these evils, and the conditions have been so changed that there are now no senior wranglers. The tripos examination used to be followed almost immediately by another examination in higher mathematics to determine the award of two prizes named the Smith's prizes. "Senior wrangler" was considered the greatest academic distinction in England.

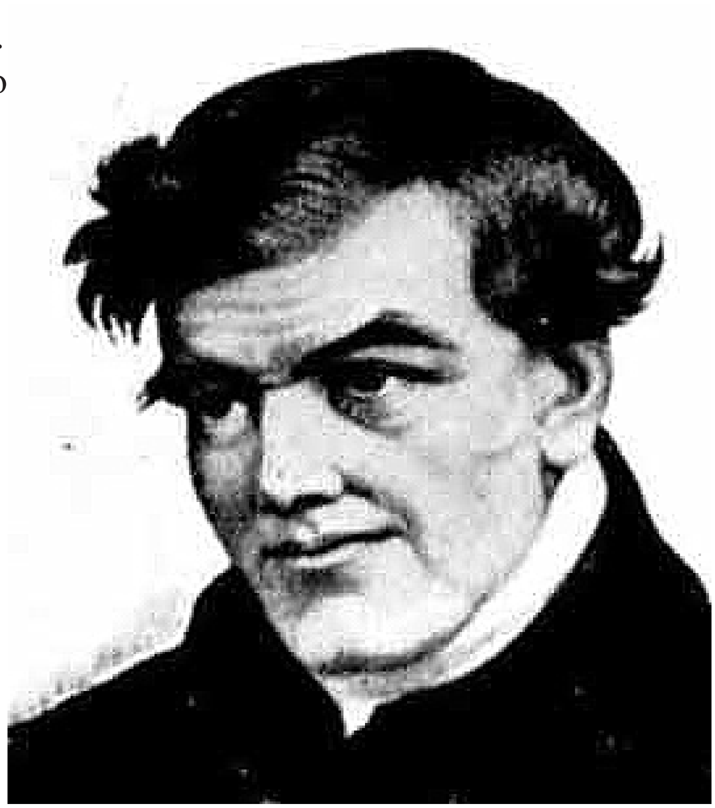
In 1812 Peacock took the rank of second wrangler, and the second Smith's prize, the senior wrangler being John Herschel. Two years later he became a candidate for a fellowship in his college and won it immediately, partly by means of his extensive and accurate knowledge of the classics. A fellowship then meant about £200 a year, tenable for seven years provided the Fellow did not marry meanwhile, and capable of being extended after the seven years provided the Fellow took clerical Orders. The limitation to seven years, although the Fellow devoted himself exclusively to science, cut short and prevented by anticipation the career of many a laborer for the advancement of science. Sir Isaac Newton was a Fellow of Trinity College, and its limited terms nearly deprived the world of the Principia.

The year after taking a Fellowship, Peacock was appointed a tutor and lecturer of his college, which position he continued to hold for many years. At that time the state of mathematical learning at Cambridge was discreditable. How could that be? you may ask; was not Newton a professor of mathematics in that University? did he not write the Principia in Trinity College? had his influence died out so soon? The true reason was he was worshipped too much as an authority; the University had settled down to the study of Newton instead of Nature, and they had followed him in one grand mistake—the ignoring of the differential notation in the calculus. Students of the differential calculus are more or less familiar with the controversy which raged over the respective claims of Newton and Leibnitz to the invention of the calculus; rather over the question whether Leibnitz was an independent inventor, or appropriated the fundamental ideas from Newton's writings and correspondence, merely giving them a new clothing in the form of the differential notation. Anyhow, Newton's countrymen adopted the latter alternative; they clung to the fluxional notation of Newton; and following Newton, they ignored the notation of Leibnitz and everything written in that notation. The Newtonian notation is as follows: If y denotes a fluent, then $\dot{y}$ denotes its fluxion, and $\ddot{y}$ the fluxion of $\dot{y}$ if y itself be considered a fluxion, then denotes its fluent, and y□ the fluent of y□ and so on; a differential is denoted by o. In the notation of Leibnitzis written and so on. The result of this Chauvinism on the part of the British mathematicians of the eighteenth century was that the developments of the calculus were made by the contemporary mathematicians of the Continent, namely, the Bernoullis, Euler, Clairault, Delambre, Lagrange, Laplace, Legendre. At the beginning of the 19th century, there was only one mathematician in Great Britain (namely Ivory, a Scotsman) who was familiar with the achievements of the Continental mathematicians. Cambridge University in particular was wholly given over not merely to the use of the fluxional notation but to ignoring the differential notation. The celebrated saying of Jacobi was then literally true, although it had ceased to be true when he gave it utterance. He visited Cambridge about 1842. When dining as a guest at the high table of one of the colleges he was asked who in his opinion was the greatest of the living mathematicians of England; his reply was “There is none.”

Peacock, in common with many other students of his own standing, was profoundly impressed with the need of reform, and while still an undergraduate formed a league with Babbage and Herschel to adopt measures to bring it about. In 1815 they formed what they called the Analytical Society, the object of which was stated to be to advocate the d’ism of the Continent versus the dot-age of the University. Evidently the members of the new society were armed with wit as well as mathematics. Of these three reformers, Babbage afterwards became celebrated as the inventor of an analytical engine, which could not only perform the ordinary processes of arithmetic, but, when set with the proper data, could tabulate the values of any function and print the results. A part of the machine was constructed, but the inventor and the Government (which was supplying the funds) quarrelled, in consequence of which the complete machine exists only in the form of drawings. These are now in the possession of the British Government, and a scientific commission appointed to examine them has reported that the engine could be constructed. The third reformer—Herschel—was a son of Sir William Herschel, the astronomer who discovered Uranus, and afterwards as Sir John Herschel became famous as an astronomer and scientific philosopher.

The first movement on the part of the Analytical Society was to translate from the French the smaller work of Lacroix on the differential and integral calculus; it was published in 1816. At that time the best manuals, as well as the greatest works on mathematics, existed in the French language. Peacock followed up the translation with a volume containing a copious Collection of Examples of the Application of the Differential and Integral Calculus, which was published in 1820. The sale of both books was rapid, and contributed materially to further the object of the Society. Then high wranglers of one year became the examiners of the mathematical tripos three or four years afterwards. Peacock was appointed an examiner in 1817, and he did not fail to make use of the position as a powerful lever to advance the cause of reform. In his questions set for the examination the differential notation was for the first time officially employed in Cambridge. The innovation did not escape censure, but he wrote to a friend as follows: "I assure you that I shall never cease to exert myself to the utmost in the cause of reform, and that I will never decline any office which may increase my power to effect it. I am nearly certain of being nominated to the office of Moderator in the year 1818-1819, and as I am an examiner in virtue of my office, for the next year I shall pursue a course even more decided than hitherto, since I shall feel that men have been prepared for the change, and will then be enabled to have acquired a better system by the publication of improved elementary books. I have considerable influence as a lecturer, and I will not neglect it. It is by silent perseverance only, that we can hope to reduce the manyheaded monster of prejudice and make the University answer her character as the loving mother of good learning and science." These few sentences give an insight into the character of Peacock: he was an ardent reformer and a few years brought success to the cause of the Analytical Society.

Another reform at which Peacock labored was the teaching of algebra. In 1830 he published a Treatise on Algebra which had for its object the placing of algebra on a true scientific basis, adequate for the development which it had received at the hands of the Continental mathematicians. As to the state of the science of algebra in Great Britain, it may be judged of by the following facts. Baron Maseres, a Fellow of Clare College, Cambridge, and William Frend, a second wrangler, had both written books protesting against the use of the negative quantity. Frend published his Principles of Algebra in 1796, and the preface reads as follows: "The ideas of number are the clearest and most distinct of the human mind; the acts of the mind upon them are equally simple and clear. There cannot be confusion in them, unless numbers too great for the comprehension of the learner are employed, or some arts are used which are not justifiable. The first error in teaching the first principles of algebra is obvious on perusing a few pages only of the first part of Maclaurin's Algebra. Numbers are there divided into two sorts, positive and negative; and an attempt is made to explain the nature of negative numbers by allusion to book debts and other arts. Now when a person cannot explain the principles of a science without reference to a metaphor, the probability is, that he has never thought accurately upon the subject. A number may be greater or less than another number; it may be added to, taken from, multiplied into, or divided by, another number; but in other respects it is very intractable; though the whole world should be destroyed, one will be one, and three will be three, and no art whatever can change their nature. You may put a mark before one, which it will obey; it submits to be taken away from a number greater than itself, but to attempt to take it away from a number less than itself is ridiculous. Yet this is attempted by algebraists who talk of a number less than nothing; of multiplying a negative number into a negative number and thus producing a positive number; of a number being imaginary. Hence they talk of two roots to every equation of the second order, and the learner is to try which will succeed in a given equation; they talk of solving an equation which requires two impossible roots to make it soluble; they can find out some impossible numbers which being multiplied together produce unity. This is all jargon, at which common sense recoils; but from its having been once adopted, like many other figments, it finds the most strenuous supporters among those who love to take things upon trust and hate the colour of a serious thought." So far, Frend. Peacock knew that Argand, Français and Warren had given what seemed to be an explanation not only of the negative quantity but of the imaginary, and his object was to reform the teaching of algebra so as to give it a true scientific basis.



At that time every part of exact science was languishing in Great Britain. Here is the description given by Sir John Herschel: "The end of the 18th and the beginning of the 19th century were remarkable for the small amount of scientific movement going on in Great Britain, especially in its more exact departments. Mathematics were at the last gasp, and Astronomy nearly so—I mean in those members of its frame which depend upon precise measurement and systematic calculation. The chilling torpor of routine had begun to spread itself over all those branches of Science which wanted the excitement of experimental research." To elevate astronomical science the Astronomical Society of London was founded, and our three reformers Peacock, Babbage and Herschel were prime movers in the undertaking. Peacock was one of the most zealous promoters of an astronomical observatory at Cambridge, and one of the founders of the Philosophical Society of Cambridge.

The year 1831 saw the beginning of one of the greatest scientific organizations of modern times. That year the British Association for the Advancement of Science (prototype of the American, French and Australasian Associations) held its first meeting in the ancient city of York. Its objects were stated to be: first, to give a stronger impulse and a more systematic direction to scientific enquiry; second, to promote the intercourse of those who cultivate science in different parts of the British Empire with one another and with foreign philosophers; third, to obtain a more general attention to the objects of science, and the removal of any disadvantages of a public kind which impede its progress. One of the first resolutions adopted was to procure reports on the state and progress of particular sciences, to be drawn up from time to time by competent persons for the information of the annual meetings, and the first to be placed on the list was a report on the progress of mathematical science. Dr. Whewell, the mathematician and philosopher, was a Vice-president of the meeting: he was instructed to select the reporter. He first asked Sir W. R. Hamilton, who declined; he then asked Peacock, who accepted. Peacock had his report ready for the third meeting of the Association, which was held in Cambridge in 1833; although limited to Algebra, Trigonometry, and the Arithmetic of Sines, it is one of the best of the long series of valuable reports which have been prepared for and printed by the Association.

In 1837 he was appointed Lowndean professor of astronomy in the University of Cambridge, the chair afterwards occupied by Adams, the co-discoverer of Neptune, and now occupied by Sir Robert Ball, celebrated for his Theory of Screws. In 1839 he was appointed Dean of Ely, the diocese of Cambridge. While holding this position he wrote a text book on algebra in two volumes, the one called Arithmetical Algebra, and the other Symbolical Algebra. Another object of reform was the statutes of the University; he worked hard at it and was made a member of a commission appointed by the Government for the purpose; but he died on November 8, 1858, in the 68th year of his age. His last public act was to attend a meeting of the Commission.

Peacock's main contribution to mathematical analysis is his attempt to place algebra on a strictly logical basis. He founded what has been called the philological or symbolical school of mathematicians; to which Gregory, De Morgan and Boole belonged. His answer to Maseres and Frend was that the science of algebra consisted of two parts—arithmetical algebra and symbolical algebra— and that they erred in restricting the science to the arithmetical part. His view of arithmetical algebra is as follows: "In arithmetical algebra we consider symbols as representing numbers, and the operations to which they are submitted as included in the same definitions as in common arithmetic; the signs + and – denote the operations of addition and subtraction in their ordinary meaning only, and those operations are considered as impossible in all cases where the symbols subjected to them possess values which would render them so in case they were replaced by digital numbers; thus in expressions such as a + b we must suppose a and b to be quantities of the same kind; in others, like a – b, we must suppose a greater than b and therefore homogeneous with it; in products and quotients, like ab and a/b we must suppose the multiplier and divisor to be abstract numbers; all results whatsoever, including negative quantities, which are not strictly deducible as legitimate conclusions from the definitions of the several operations must be rejected as impossible, or as foreign to the science."

Peacock's principle may be stated thus: the elementary symbol of arithmetical algebra denotes a digital, i.e., an integer number; and every combination of elementary symbols must reduce to a digital number, otherwise it is impossible or foreign to the science. If a and b are numbers, then a + b is always a number; but a – b is a number only when b is less than a. Again, under the same conditions, ab is always a number, but a/b is really a number only when b is an exact divisor of a. Hence we are reduced to the following dilemma: Either a/b must be held to be an impossible expression in general, or else the meaning of the fundamental symbol of algebra must be extended so as to include rational fractions. If the former horn of the dilemma is chosen, arithmetical algebra becomes a mere shadow; if the latter horn is chosen, the operations of algebra cannot be defined on the supposition that the elementary symbol is an integer number. Peacock attempts to get out of the difficulty by supposing that a symbol which is used as a multiplier is always an integer number, but that a symbol in the place of the multiplicand may be a fraction. For instance, in ab, a can denote only an integer number, but b may denote a rational fraction. Now there is no more fundamental principle in arithmetical algebra than that ab = ba; which would be illegitimate on Peacock's principle.

One of the earliest English writers on arithmetic is Robert Record, who dedicated his work to King Edward the Sixth. The author gives his treatise the form of a dialogue between master and scholar. The scholar battles long over this difficulty,—that multiplying a thing could make it less. The master attempts to explain the anomaly by reference to proportion; that the product due to a fraction bears the same proportion to the thing multiplied that the fraction bears to unity. But the scholar is not satisfied and the master goes on to say: “If I multiply by more than one, the thing is increased; if I take it but once, it is not changed, and if I take it less than once, it cannot be so much as it was before. Then seeing that a fraction is less than one, if I multiply by a fraction, it follows that I do take it less than once.” Whereupon the scholar replies, “Sir, I do thank you much for this reason,—and I trust that I do perceive the thing.”

The fact is that even in arithmetic the two processes of multiplication and division are generalized into a common multiplication; and the difficulty consists in passing from the original idea of multiplication to the generalized idea of a tensor, which idea includes compressing the magnitude as well as stretching it. Let m denote an integer number; the next step is to gain the idea of the reciprocal of m, not as 1/m but simply as /m. When m and /n are compounded we get the idea of a rational fraction; for in general m/n will not reduce to a number nor to the reciprocal of a number.

Suppose, however, that we pass over this objection; how does Peacock lay the foundation for general algebra? He calls it symbolical algebra, and he passes from arithmetical algebra to symbolical algebra in the following manner: “Symbolical algebra adopts the rules of arithmetical algebra but removes altogether their restrictions; thus symbolical subtraction differs from the same operation in arithmetical algebra in being possible for all relations of value of the symbols or expressions employed. All the results of arithmetical algebra which are deduced by the application of its rules, and which are general in form though particular in value, are results likewise of symbolical algebra where they are general in value as well as in form; thus the product of am and an which is am+n when m and n are whole numbers and therefore general in form though particular in value, will be their product likewise when m and n are general in value as well as in form; the series for (a + b)n determined by the principles of arithmetical algebra when n is any whole number, if it be exhibited in a general form, without reference to a final term, may be shown upon the same principle to the equivalent series for (a + b)n when n is general both in form and value.”

The principle here indicated by means of examples was named by Peacock the “principle of the permanence of equivalent forms,” and at page 59 of the Symbolical Algebra it is thus enunciated: “Whatever algebraical forms are equivalent when the symbols are general in form, but specific in value, will be equivalent likewise when the symbols are general in value as well as in form.”

For example, let a, b, c, d denote any integer numbers, but subject to the restrictions that b is less than a, and d less than c; it may then be shown arithmetically that

$$(a - b)\ (c - d) = ac + bd - ad - bc.$$

Peacock's principle says that the form on the left side is equivalent to the form on the right side, not only when the said restrictions of being less are removed, but when a, b, c, d denote the most general algebraical symbol. It means that a, b, c, d may be rational fractions, or surds, or imaginary quantities, or indeed operators such as d/dx. The equivalence is not established by means of the nature of the quantity denoted; the equivalence is assumed to be true, and then it is attempted to find the different interpretations which may be put on the symbol.

It is not difficult to see that the problem before us involves the fundamental problem of a rational logic or theory of knowledge; namely, how are we able to ascend from particular truths to more general truths. If a, b, c, d denote integer numbers, of which b is less than a and d less than c, then

$$(a - b)\ (c - d) = ac + bd - ad - bc.$$

It is first seen that the above restrictions may be removed, and still the above equation hold. But the antecedent is still too narrow; the true scientific problem consists in specifying the meaning of the symbols, which, and only which, will admit of the forms being equal. It is not to find some meanings, but the most general meaning, which allows the equivalence to be true. Let us examine some other cases; we shall find that Peacock's principle is not a solution of the difficulty; the great logical process of generalization cannot be reduced to any such easy and arbitrary procedure. When a, m, n denote integer numbers, it can be shown that

$$aman = am+n.$$

According to Peacock the form on the left is always to be equal to the form on the right, and the meanings of a, m, n are to be found by interpretation. Suppose that a takes the form of the incommensurate quantity e, the base of the natural system of logarithms. A number is a degraded form of a complex quantity $p + q\sqrt{-1}$ and a complex quantity is a degraded form of a quaternion; consequently one meaning which may be assigned to m and n is that of quaternion. Peacock's principle would lead us to suppose that emen = em+n, m and n denoting quaternions; but that is just what Hamilton, the inventor of the quaternion generalization, denies. There are reasons for believing that he was mistaken, and that the forms remain equivalent even under that extreme generalization of m and n; but the point is this: it is not a question of conventional definition and formal truth; it is a question of objective definition and real truth. Let the symbols have the prescribed meaning, does or does not the equivalence still hold? And if it does not hold, what is the higher or more complex form which the equivalence assumes?

1 This Lecture was delivered April 12, 1901.—Editors.

2 Dr. Macfarlane's first course included the first six lectures given in this volume.—Editors.