LECTURES ON TEN BRITISH MATHEMATICIANS OF THE NINETEENTH CENTURY

Sir William Rowan Hamilton

BY ALEXANDER MACFARLANE

SIR WILLIAM ROWAN HAMILTON[1]
(1805-1865)

William Rowan Hamilton was born in Dublin, Ireland, on the 3d of August, 1805. His father, Archibald Hamilton, was a solicitor in the city of Dublin; his mother, Sarah Hutton, belonged to an intellectual family, but she did not live to exercise much influence on the education of her son. There has been some dispute as to how far Ireland can claim Hamilton; Professor Tait of Edinburgh in the Encyclopaedia Brittanica claims him as a Scotsman, while his biographer, the Rev. Charles Graves, claims him as essentially Irish. The facts appear to be as follows: His father's mother was a Scotch woman; his father's father was a citizen of Dublin. But the name "Hamilton" points to Scottish origin, and Hamilton himself said that his family claimed to have come over from Scotland in the time of James I. Hamilton always considered himself an Irishman; and as Burns very early had an ambition to achieve something for the renown of Scotland, so Hamilton in his early years had a powerful ambition to do something for the renown of Ireland. In later life he used to say that at the beginning of the century people read French mathematics, but that at the end of it they would be reading Irish mathematics.

Hamilton, when three years of age, was placed in the charge of his uncle, the Rev. James Hamilton, who was the curate of Trim, a country town, about twenty miles from Dublin, and who was also the master of the Church of England school. From his uncle he received all his primary and secondary education and also instruction in Oriental languages. As a child Hamilton was a prodigy; at three years of age he was a superior reader of English and considerably advanced in arithmetic; at four a good geographer; at five able to read and translate Latin, Greek, and Hebrew, and liked to recite Dryden, Collins, Milton and Homer; at eight a reader of Italian and French and giving vent to his feelings in extemporized Latin; at ten a student of Arabic and Sanscrit. When twelve years old he met Zerah Colburn, the American calculating boy, and engaged with him in trials of arithmetical skill, in which trials Hamilton came off with honor, although

Colburn was generally the victor. These encounters gave Hamilton a decided taste for arithmetical computation, and for many years afterwards he loved to perform long operations in arithmetic in his mind, extracting the square and cube root, and solving problems that related to the properties of numbers. When thirteen he received his initiation into algebra from Clairault's Algebra in the French, and he made an epitome, which he ambitiously entitled "A Compendious Treatise on Algebra by William Hamilton."

When Hamilton was fourteen years old, his father died and left his children slenderly provided for. Henceforth, as the elder brother of three sisters, Hamilton had to act as a man. This year he addressed a letter of welcome, written in the Persian language, to the Persian Ambassador, then on a visit to Dublin; and he met again Zerah Colburn. In the interval Zerah had attended one of the great public schools of England. Hamilton had been at a country school in Ireland, and was now able to make a successful investigation of the methods by which Zerah made his lightning calculations. When sixteen, Hamilton studied the Differential Calculus by the help of a French textbook, and began the study of the Mécanique céleste of Laplace, and he was able at the beginning of this study to detect a flaw in the reasoning by which Laplace demonstrates the theorem of the parallelogram of forces. This criticism brought him to the notice of Dr. Brinkley, who was then the professor of astronomy in the University of Dublin, and resided at Dunkirk, about five miles from the centre of the city. He also began an investigation for himself of equations which represent systems of straight lines in a plane, and in so doing hit upon ideas which he afterwards developed into his first mathematical memoir to the Royal Irish Academy. Dr. Brinkley is said to have remarked of him at this time: "This young man, I do not say will be, but is, the first mathematician of his age."

At the age of eighteen Hamilton entered Trinity College, Dublin, the University of Dublin founded by Queen Elizabeth, and differing from the Universities of Oxford and Cambridge in having only one college. Unlike Oxford, which has always given prominence to classics, and Cambridge, which has always given prominence to mathematics, Dublin at that time gave equal prominence to classics and to mathematics. In his first year Hamilton won the very rare honor of optime at his examination in Homer. In the old Universities marks used to be and in some cases still are published, descending not in percentages but by means of the scale of Latin adjectives: optime, valdebene, bene, satis, mediocriter, vix medi, non; optime means passed with the very highest distinction; vix means passed but with great difficulty. This scale is still in use in the medical examinations of the University of Edinburgh.

Before entering college Hamilton had been accustomed to translate Homer into blank verse, comparing his result with the translations of Pope and Cowper; and he had already produced some original poems. In this, his first year he wrote a poem "On college ambition" which is a fair specimen of his poetical attainments.

Oh! Ambition hath its hour
Of deep and spirit-stirring power;
Not in the tented field alone,
Nor peer-engirded court and throne;
Nor the intrigues of busy life;
But ardent Boyhood's generous strife,
While yet the Enthusiast spirit turns
Where'er the light of Glory burns,
Thinks not how transient is the blaze,
But longs to barter Life for Praise.

Look round the arena, and ye spy
Pallid cheek and faded eye;
Among the bands of rivals, few
Keep their native healthy hue:
Night and thought have stolen away
Their once elastic spirit's play.
A few short hours and all is o'er,
Some shall win one triumph more;
Some from the place of contest go
Again defeated, sad and slow.

What shall reward the conqueror then
For all his toil, for all his pain,
For every midnight throb that stole
So often o'er his fevered soul?
Is it the applaudings loud
Or wond'ring gazes of the crowd;
Disappointed envy's shame,
Or hollow voice of fickle Fame?
These may extort the sudden smile,

May swell the heart a little while;
But they leave no joy behind,
Breathe no pure transport o'er the mind,
Nor will the thought of selfish gladness
Expand the brow of secret sadness.

Yet if Ambition hath its hour
Of deep and spirit-stirring power,
Some bright rewards are all its own,
And bless its votaries alone:
The anxious friend's approving eye;
The generous rivals' sympathy;
And that best and sweetest prize
Given by silent Beauty's eyes!
These are transports true and strong,
Deeply felt, remembered long:
Time and sorrow passing o'er
Endear their memory but the more.

The "silent Beauty" was not an abstraction, but a young lady whose brothers were fellow-students of Trinity College. This led to much effusion of poetry; but unfortunately while Hamilton was writing poetry about her another young man was talking prose to her; with the result that Hamilton experienced a disappointment. On account of his self-consciousness, inseparable probably from his genius, he felt the disappointment keenly. He was then known to the professor of astronomy, and walking from the College to the Observatory along the Royal Canal, he was actually tempted to terminate his life in the water.

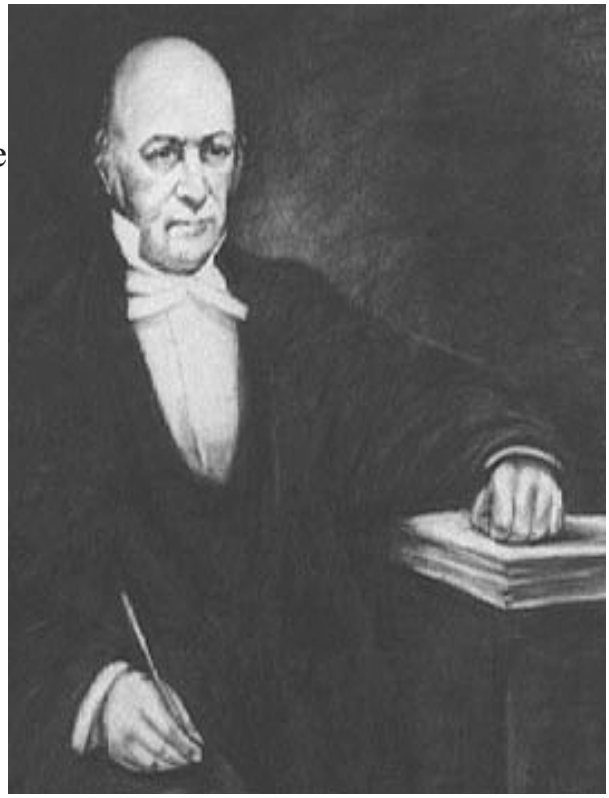
In his second year he formed the plan of reading so as to compete for the highest honors both in classics and in mathematics. At graduation two gold medals were awarded, the one for distinction in classics, the other for distinction in mathematics. Hamilton aimed at carrying off both. In his junior year he received an optime in mathematical physics; and, as the winner of two optimes, the one in classics, the other in mathematics, he immediately became a celebrity in the intellectual circle of Dublin.

In his senior year he presented to the Royal Irish Academy a memoir embodying his research on systems of lines. He now called it a "Theory of Systems of Rays" and it was printed in the Transactions. About this time Dr. Brinkley was appointed to the

bishopric of Cloyne, and in consequence resigned the professorship of astronomy. In the United Kingdom it is customary when a post becomes vacant for aspirants to lodge a formal application with the appointing board and to supplement their own application by testimonial letters from competent authorities. In the present case quite a number of candidates appeared, among them Airy, who afterwards became Astronomer Royal of England, and several Fellows of Trinity College, Dublin. Hamilton did not become a formal candidate, but he was invited to apply, with the result that he received the appointment while still an undergraduate, and not twenty-two years of age. Thus was his undergraduate career signalized much more than by the carrying off of the two gold medals. Before assuming the duties of his chair he made a tour through England and Scotland, and met for the first time the poet Wordsworth at his home at Rydal Mount, in Cumberland. They had a midnight walk, oscillating backwards and forwards between Rydal and Ambleside, absorbed in converse on high themes, and finding it almost impossible to part. Wordsworth afterwards said that Coleridge and Hamilton were the two most wonderful men, taking all their endowments together, that he had ever met.

In October, 1827, he came to reside at the place which was destined to be the scene of his scientific labors. I had the pleasure of visiting it last summer as the guest of his successor. The Observatory is situated on the top of a hill, Dunsink, about five miles from Dublin. The house adjoins the observatory; to the east is an extensive lawn; to the west a garden with stone wall and shaded walks; to the south a terraced field; at the foot of the hill is the Royal Canal; to the southeast the city of Dublin; while the view is bounded by the sea and the Dublin and Wicklow Mountains; a fine home for a poet or a philosopher or a mathematician, and in Hamilton all three were combined.

Settled at the Observatory he started out diligently as an observer, but he found it difficult to stand the low temperatures incident to the work. He never attained skill as an observer, and unfortunately he depended on a very poor assistant. Himself a brilliant computer, with a good observer for assistant, the work of the observatory ought to have flourished. One of the first distinguished visitors at the Observatory was the poet Wordsworth, in commemoration of which one of the shaded walks in the garden was named Wordsworth's walk. Wordsworth advised him to concentrate his powers on science; and, not long after, wrote him as follows: "You send me showers of verses which I receive with much pleasure, as do we all: yet have we fears that this employment may seduce you from the path of science which you seem destined to tread with so much honor to yourself and profit to others. Again and again I must repeat that the composition of verse is infinitely more of an art than men are prepared to believe, and absolute success in it depends upon innumerable minutiæ which it



grieves me you should stoop to acquire a knowledge of. . . Again I do venture to submit to your consideration, whether the poetical parts of your nature would not find a field more favorable to their exercise in the regions of prose; not because those regions are humbler, but because they may be gracefully and profitably trod, with footsteps less careful and in measures less elaborate."

Hamilton possessed the poetic imagination; what he was deficient in was the technique of the poet. The imagination of the poet is kin to the imagination of the mathematician; both extract the ideal from a mass of circumstances. In this connection De Morgan wrote: "The moving power of mathetical invention is not reasoning but imagination. We no longer apply the homely term maker in literal translation of poet ; but discoverers of all kinds, whatever may be their lines, are makers, or, as we mow say, have the creative genius." Hamilton spoke of the Mécanique analytique of Lagrange as a "scientific poem"; Hamilton himself was styled the Irish Lagrange. Engineers venerate Rankine, electricians venerate Maxwell; both were scientific discoverers and likewise poets, that is, amateur poets. The proximate cause of the shower of verses was that Hamilton had fallen in love for the second time. The young lady was Miss de Vere, daughter of an accomplished Irish baronet, and who like Tennyson's Lady Clara Vere de Vere could look back on a long and illustrious descent. Hamilton had a pupil in Lord Adare, the eldest son of the Earl of Dunraven, and it was while visiting Adare Manor that he was introduced to the De Vere family, who lived near by at Curragh Chase. His suit was encouraged by the Countess of Dunraven, it was favorably received by both father and mother, he had written many sonnets of which Ellen de Vere was the inspiration, he had discussed with her astronomy, poetry and philosophy; and was on the eve of proposing when he gave up because the young lady incidentally said to him that "she could not live happily anywhere but at Curragh." His action shows the working of a too self-conscious mind, proud of his own intellectual achievements, and

too much awed by her long descent. So he failed for the second time; but both of these ladies were friends of his to the last.

At the age of 27 he contributed to the Irish Academy a supplementary paper on his Theory of Systems of Rays, in which he predicted the phenomenon of conical refraction; namely, that under certain conditions a single ray incident on a biaxial crystal would be broken up into a cone of rays, and likewise that under certain conditions a single emergent ray would appear as a cone of rays. The prediction was made by Hamilton on Oct. 22nd; it was experimentally verified by his colleague Prof. Lloyd on Dec. 14th. It is not experiment alone or mathematical reasoning alone which has built up the splendid temple of physical science, but the two working together; and of this we have a notable exemplification in the discovery of conical refraction.

Twice Hamilton chose well but failed; now he made another choice and succeeded. The lady was a Miss Bayly, who visited at the home of her sister near Dunsink hill. The lady had serious misgivings about the state of her health; but the marriage took place. The kind of wife which Hamilton needed was one who could govern him and efficiently supervise all domestic matters; but the wife he chose was, from weakness of body and mind, incapable of doing it. As a consequence, Hamilton worked for the rest of his life under domestic difficulties of no ordinary kind.

At the age of 28 he made a notable addition to the theory of Dynamics by extending to it the idea of a Characteristic Function, which he had previously applied with success to the science of Optics in his Theory of Systems of Rays. It was contributed to the Royal Society of London, and printed in their Philosophical Transactions. The Royal Society of London is the great scientific society of England, founded in the reign of Charles II, and of which Newton was one of the early presidents; Hamilton was invited to become a fellow but did not accept, as he could not afford the expense.

At the age of 29 he read a paper before the Royal Irish Academy, which set forth the result of long meditation and investigation on the nature of Algebra as a science; the paper is entitled “Algebra as the Science of Pure Time.” The main idea is that as Geometry considered as a science is founded upon the pure intuition of space, so algebra as a science is founded upon the pure intuition of time. He was never satisfied with Peacock’s theory of algebra as a “System of Signs and their Combinations”; nor with De Morgan’s improvement of it; he demanded a more real foundation. In reading Kant’s Critique of Pure Reason he was struck by the following passage: “Time and space are two sources of knowledge from which various a priori synthetical cognitions can be derived. Of this, pure mathematics gives a splendid example in the

case of our cognitions of space and its various relations. As they are both pure forms of sensuous intuition, they render synthetical propositions a priori possible." Thus, according to Kant, space and time are forms of the intellect; and Hamilton reasoned that, as geometry is the science of the former, so algebra must be the science of the latter. When algebra is based on any unidimensional subject, such as time, or a straight line, a difficulty arises in explaining the roots of a quadratic equation when they are imaginary. To get over this difficulty Hamilton invented a theory of algebraic couplets, which has proved a conundrum in the mathematical world. Some 20 years ago there nourished in Edinburgh a mathematician named Sang who had computed the most elaborate tables of logarithms in existence—which still exist in manuscript. On reading the theory in question he first judged that either Hamilton was crazy, or else that he (Sang) was crazy, but eventually reached the more comforting alternative. On the other hand, Prof. Tait believes in its soundness, and endeavors to bring it down to the ordinary comprehension.

We have seen that the British Association for the Advancement of Science was founded in 1831, and that its first meeting was in the ancient city of York. It was a policy of the founders not to meet in London, but in the provincial cities, so that thereby greater interest in the advance of science might be produced over the whole land. The cities chosen for the place of meeting in following years were the University towns: Oxford, Cambridge, Edinburgh, Dublin. Hamilton was the only representative of Ireland present at the Oxford meeting; and at the Oxford, Cambridge, and Edinburgh meetings he not only contributed scientific papers, but he acquired renown as a scientific orator. In the case of the Dublin meeting he was chief organizer beforehand, and chief orator when it met. The week of science was closed by a grand dinner given in the library of Trinity College; and an incident took place which is thus described by an American scientist:

"We assembled in the imposing hall of Trinity Library, two hundred and eighty feet long, at six o'clock. When the company was principally assembled, I observed a little stir near the place where I stood, which nobody could explain, and which, in fact, was not comprehended by more than two or three persons present. In a moment, however, I perceived myself standing near the Lord Lieutenant and his suite, in front of whom a space had been cleared, and by whom was Professor Hamilton, looking very much embarrassed. The Lord Lieutenant then called him by name, and he stepped into the vacant space. 'I am,' said his Excellency, 'about to exercise a prerogative of royalty, and it gives me great pleasure to do it, on this splendid public occasion, which has brought together so many distinguished men from all parts of the empire, and from all

parts even of the world where science is held in honor. But, in exercising it, Professor Hamilton, I do not confer a distinction. I but set the royal, and therefore the national mark on a distinction already acquired by your genius and labors.' He went on in this way for three of four minutes, his voice very fine, rich and full; his manner as graceful and dignified as possible; and his language and allusions appropriate and combined into very ample flowing sentences. Then, receiving the State sword from one of his attendants, he said, 'Kneel down, Professor Hamilton'; and laying the blade gracefully and gently first on one shoulder, and then on the other, he said, 'Rise up, Sir William Rowan Hamilton.' The Knight rose, and the Lord Lieutenant then went up, and with an appearance of great tact in his manner, shook hands with him. No reply was made. The whole scene was imposing, rendered so, partly by the ceremony itself, but more by the place in which it passed, by the body of very distinguished men who were assembled there, and especially by the extraordinarily dignified and beautiful manner in which it was performed by the Lord Lieutenant. The effect at the time was great, and the general impression was that, as the honor was certainly merited by him who received it, so the words by which it was conferred were so graceful and appropriate that they constituted a distinction by themselves, greater than the distinction of knighthood. I was afterwards told that this was the first instance in which a person had been knighted by a Lord Lieutenant either for scientific or literary merit."

Two years after another great honor came to Hamilton—the presidency of the Royal Irish Academy. While holding this office, in the year 1843, when 38 years old, he made the discovery which will ever be considered his highest title to fame. The story of the discovery is told by Hamilton himself in a letter to his son: "On the 16th day of October, which happened to be a Monday, and Council day of the Royal Irish Academy, I was walking in to attend and preside, and your mother was walking with me along the Royal Canal, to which she had perhaps driven; and although she talked with me now and then, yet an undercurrent of thought was going on in my mind, which gave at last a result, whereof it is not too much to say that I felt at once the importance. An electric circuit seemed to close; and a spark flashed forth, the herald (as I foresaw immediately) of many long years to come of definitely directed thought and work, by myself if spared, and at all events on the part of others, if I should even be allowed to live long enough distinctly to communicate the discovery. Nor could I resist the impulse—unphilosophical as it may have been—to cut with a knife on a stone of Brougham Bridge, as we passed it, the fundamental formula with the symbols i, j, k; namely,

$$i2 = j2 = k2 = ijk = -1,$$

which contains the solution of the problem, but of course as an inscription has long since mouldered away. A more durable notice remains, however, in the Council Book of the Academy for that day, which records the fact that I then asked for and obtained leave to read a paper on Quaternions, at the first general meeting of the session, which reading took place accordingly on Monday the 13th of November following."

Last summer Prof. Joly and I took the walk here described. We started from the Observatory, walked down the terraced field, then along the path by the side of the Royal Canal towards Dublin until we came to the second bridge spanning the canal. The path of course goes under the Bridge, and the inner side of the Bridge presents a very convenient surface for an inscription. I have seen this incident quoted as an example of how a genius strikes on a discovery all of a sudden. No doubt a problem was solved then and there, but the problem had engaged Hamilton's thoughts and researches for fifteen years. It is rather an illustration of how genius is patience, or a faculty for infinite labor. What was Hamilton struggling to do all these years? To emerge from Flatland into Space; in other words, Algebra had been extended so as to apply to lines in a plane; but no one had been able to extend it so as to apply to lines in space. The greatness of the feat is made evident by the fact that most analysts are still crawling in Flatland. The same year in which he discovered Quaternions the Government granted him a pension of £200 per annum for life, on account of his scientific work.

We have seen how Hamilton gained two optimes, one in classics, the other in physics, the highest possible distinction in his college course; how he was appointed professor of astronomy while yet an undergraduate; how he was a scientific chief in the British Association at 27; how he was knighted for his scientific achievements at 30; how he was appointed president of the Royal Irish Academy at 32; how he discovered Quaternions and received a Government pension at 38; can you imagine that this brilliant and successful genius would fall a victim to intemperance? About this time at a dinner of a scientific society in Dublin he lost control of himself, and was so mortified that, on the advice of friends he resolved to abstain totally. This resolution he kept for two years; when happening to be a member of a scientific party at the castle of Lord Rosse, an amateur astronomer then the possessor of the largest telescope in existence, he was taunted for sticking to water, particularly by Airy the Greenwich astronomer. He broke his good resolution, and from that time forward the craving for alcoholic stimulants clung to him. How could Hamilton with all his noble aspirations

fall into such a vice? The explanation lay in the want of order which reigned in his home. He had no regular times for his meals; frequently had no regular meals at all, but resorted to the sideboard when hunger compelled him. What more natural in such condition than that he should refresh himself with a quaff of that beverage for which Dublin is famous—porter labelled X3? After Hamilton's death the dining-room was found covered with huge piles of manuscript, with convenient walks between the piles; when these literary remains were wheeled out and examined, china plates with the relics of food upon them were found between the sheets of manuscript, plates sufficient in number to furnish a kitchen. He used to carry on, says his eldest son, long trains of algebraical and arithmetical calculations in his mind, during which he was unconscious of the earthly necessity of eating; "we used to bring in a 'snack' and leave it in his study, but a brief nod of recognition of the intrusion of the chop or cutlet was often the only result, and his thoughts went on soaring upwards."

In 1845 Hamilton attended the second Cambridge meeting of the British Association; and after the meeting he was lodged for a week in the rooms in Trinity College which tradition points out as those in which Sir Isaac Newton composed the Principia. This incident was intended as a compliment and it seems to have impressed Hamilton powerfully. He came back to the Observatory with the fixed purpose of preparing a work on Quaternions which might not unworthily compare with the Principia of Newton, and in order to obtain more leisure for this undertaking he resigned the office of president of the Royal Irish Academy. He first of all set himself to the preparation of a course of lectures on Quaternions, which were delivered in Trinity College, Dublin, in 1848, and were six in number. Among his hearers were George Salmon, now well known for his highly successful series of manuals on Analytical Geometry; and Arthur Cayley, then a Fellow of Trinity College, Cambridge. These lectures were afterward expanded and published in 1853, under the title of Lectures on Quaternions, at the expense of Trinity College, Dublin. Hamilton had never had much experience as a teacher; the volume was criticised for diffuseness of style, and certainly Hamilton sometimes forgot the expositor in the orator. The book was a paradox—a sound paradox, and of his experience as a paradoxer Hamilton wrote: "It required a certain capital of scientific reputation, amassed in former years, to make it other than dangerously imprudent to hazard the publication of a work which has, although at bottom quite conservative, a highly revolutionary air. It was part of the ordeal through which I had to pass, an episode in the battle of life, to know that even candid and friendly people secretly or, as it might happen, openly, censured or ridiculed me, for what appeared to them my monstrous innovations." One of these monstrous

innovations was the principle that ij is not = ji but = –ji; the truth of which is evident from the diagram. Critics said that he held that 3 x 4 is not = 4 x 3; which proceeds on the assumption that only numbers can be represented by letter symbols.

Soon after the publication of the Lectures, he became aware of its imperfection as a manual of instruction, and he set himself to prepare a second book on the model of Euclid's Elements. He estimated that it would fill 400 pages and take two years to prepare; it amounted to nearly 800 closely printed pages and took seven years. At times he would work for twelve hours on a stretch; and he also suffered from anxiety as to the means of publication. Trinity College advanced £200, he paid £50 out of his own pocket, but when illness came upon him the expense of paper and printing had mounted up to £400. He was seized by an acute attack of gout, from which, after several months of suffering, he died on Sept. 2, 1865, in the 61st year of his age.

It is pleasant to know that this great mathematician received during his last illness an honor from the United States, which made him feel that he had realized the aim of his great labors. While the war between the North and South was in progress, the National Academy of Sciences was founded, and the news which came to Hamilton was that he had been elected one of ten foreign members, and that his name had been voted to occupy the specially honorable position of first on the list. Sir William Rowan Hamilton was thus the first foreign associate of the National Academy of Sciences of the United States.

As regards religion Hamilton was deeply reverential in nature. He was born and brought up in the Church of England, which was then the established Church in Ireland. He lived in the time of the Oxford movement, and for some time he sympathized with it; but when several of his friends, among them the brother of Miss De Vere, passed over into the Roman Catholic Church, he modified his opinion of the movement and remained Protestant to the end.

The immense intellectual activity of Hamilton, especially during the years when he was engaged on the enormous labor of writing the Elements of Quaternions, made him a recluse, and necessarily took away from his power of attending to the practical affairs of life. Some said that however great a master of pure time he might be he was not a master of sublunary time. His neighbors also took advantage of his goodness of

heart. Surrounding the house there is an extensive lawn affording good pasture, and on it Hamilton pastured a cow. A neighbor advised Hamilton that his cow would be much better contented by having another cow for company and bargained with Hamilton to furnish the companion provided Hamilton paid something like a dollar per month.

Here is Hamilton's own estimate of himself. "I have very long admired Ptolemy's description of his great astronomical master, Hipparchus, as □□□□ □□□□π□□□□ □□□ □□□□□□□□□□; a labor-loving and truth-loving man. Be such my epitaph."

Hamilton's family consisted of two sons and one daughter. At the time of his death, the Elements of Quaternions was all finished excepting one chapter. His eldest son, William Edwin Hamilton, wrote a preface, and the volume was published at the expense of Trinity College, Dublin. Only 500 copies were printed, and many of those were presented. In consequence it soon became a scarce book, and as much as $35.00 has been paid for a copy. A new edition, in two volumes, is now being published by Prof. Joly, his successor in Dunsink Observatory.

1 This Lecture was delivered April 16, 1901.—Editors.