Chapter 6

WILLIAM KINGDON CLIFFORD[1]

(1845-1879)

William Kingdon Clifford was born at Exeter, England, May 4, 1845. His father was a well-known and active citizen and filled the honorary office of justice of the peace; his mother died while he was still young. It is believed that Clifford inherited from his mother not only some of his genius, but a weakness in his physical constitution. He received his elementary education at a private school in Exeter, where examinations were annually held by the Board of Local Examinations of the Universities of Oxford and Cambridge; at these examinations Clifford gained numerous distinctions in widely different subjects. When fifteen years old he was sent to King's College, London, where he not only demonstrated his peculiar mathematical abilities, but also gained distinction in classics and English literature.

When eighteen, he entered Trinity College, Cambridge; the college of Peacock, De Morgan, and Cayley. He already had the reputation of possessing extraordinary mathematical powers; and he was eccentric in appearance, habits and opinions. He was reported to be an ardent High Churchman, which was then a more remarkable thing at Cambridge than it is now. His undergraduate career was distinguished by eminence in mathematics, English literature and gymnastics. One who was his companion in gymnastics wrote: "His neatness and dexterity were unusually great, but the most remarkable thing was his great strength as compared with his weight, as shown in some exercises. At one time he would pull up on the bar with either hand, which is well known to be one of the greatest feats of strength. His nerve at dangerous heights was extraordinary." In his third year he won the prize awarded by Trinity College for declamation, his subject being Sir Walter Raleigh; as a consequence he was called on to deliver the annual oration at the next Commemoration of Bene-

[1] This Lecture was delivered April 23, 1901.—Editors.

factors of the College. He chose for his subject, Dr. Whewell, Master of the College, eminent for his philosophical and scientific attainments, whose death had occurred but recently. He treated it in an original and unexpected manner; Dr. Whewell's claim to admiration and emulation being put on the ground of his intellectual life exemplifying in an eminent degree the active and creating faculty. "Thought is powerless, except it make something outside of itself; the thought which conquers the world is not contemplative but active. And it is this that I am asking you to worship to-day."

To obtain high honors in the Mathematical Tripos, a student must put himself in special training under a mathematican, technically called a coach, who is not one of the regular college instructors, nor one of the University professors, but simply makes a private business of training men to pass that particular examination. Skill consists in the rate at which one can solve and more especially write out the solution of problems. It is excellent training of a kind, but there is no time for studying fundamental principles, still less for making any philosophical investigations. Mathematical insight is something higher than skill in solving problems; consequently the senior wrangler has not always turned out the most distinguished mathematician in after life. We have seen that De Morgan was fourth wrangler. Clifford also could not be kept to the dust of the race-course; but such was his innate mathematical insight that he came out second wrangler. Other instances of the second wrangler turning out the better mathematician are Whewell, Sylvester, Kelvin, Maxwell.

In 1868, when he was 23 years old, he was elected a Fellow of his College; and while a resident fellow, he took part in the eclipse expedition of 1870 to Italy, and passed through the experience of a shipwreck near Catania on the coast of the island of Sicily. In 1871 he was appointed professor of Applied Mathematics and Mechanics in University College, London; De Morgan's college, but not De Morgan's chair. Henceforth University College was the centre of his labors.

He was now urged by friends to seek admission into the Royal Society of London. This is the ancient scientific society of England, founded in the time of Charles II, and numbering among its first presidents Sir Isaac Newton. About the middle of the nineteenth century the admission of new members was restricted to fifteen each year; and from applications the Council recommends fifteen names which are posted up, and subsequently balloted for by the Fellows. Hamilton and De Morgan never applied. Clifford did not apply immediately, but he became a Fellow a few years later. He joined the London Mathematical Society—for it met in University College—and he became one of its leading spirits. Another metropolitan Society in which he took much interest was the Metaphysical Society; like Hamilton, De Morgan, and Boole, Clifford was a scientific philosopher.

In 1875 Clifford married; the lady was Lucy, daughter of Mr. John Lane, formerly of Barbadoes. His home in London became the meeting-point of a numerous body of friends, in which almost every possible variety of taste and opinion was represented, and many of whom had nothing else in common. He took a special delight in amusing children, and for their entertainment wrote a collection of fairy tales called *The Little People*. In this respect he was like a contemporary mathematician, Mr. Dodgson—"Lewis Carroll"—the author of *Alice in Wonderland*. A children's party was one of Clifford's greatest pleasures. At one such party he kept a waxwork show, children doing duty for the figures; but I daresay he drew the line at walking on all fours, as Mr. Dodgson was accustomed to do. A children's party was to be held in a house in London and it happened that there was a party of adults held simultaneously in the neighboring house; to give the children a surprise Dodgson resolved to walk in on all fours; unfortunately he crawled into the parlor of the wrong house!

Clifford possessed unsurpassed power as a teacher. Mr. Pollock, a fellow student, gives an instance of Clifford's theory of what teaching ought to be, and his constant way of carrying it out in his discourses and conversations on mathematical and scientific subjects. "In the analytical treatment of statics there occurs a proposition called Ivory's Theorem concerning the attractions of an ellipsoid. The textbooks demonstrate it by a formidable apparatus of coordinates and integrals, such as we were wont to call a *grind*. On a certain day in the Long Vacation of 1866, which Clifford and I spent at Cambridge, I was not a little exercised by the theorem in question, as I suppose many students have been before and since. The chain of symbolic proof seemed artificial and dead; it compelled the understanding, but failed to satisfy the reason. After reading and learning the proposition one still failed to see what it was all about. Being out for a walk with Clifford, I opened my perplexities to him; I think that I can recall the very spot. What he said I do not remember in detail; which is not surprising, as I have had no occasion to remember anything about Ivory's Theorem these twelve years. But I know that as he spoke he appeared not to be working out a question, but simply telling what he saw. Without any diagram or symbolic aid he described the geometrical conditions on which the solution depended, and they seemed to stand out visibly in space. There were no longer consequences to be deduced, but real and evident facts which only required to be seen."

Clifford inherited a constitution in which nervous energy and physical strength were unequally balanced. It was in his case specially necessary to take good care of his health, but he did the opposite; he would frequently sit up most of the night working or talking. Like Hamilton he would work twelve hours on a stretch; but, unlike Hamilton, he had laborious professional duties demanding his personal attention at the same time. The consequence was that five years after his appointment to the chair in University College, his health broke down; indications of pulmonary disease appeared. To recruit his health he spent six months in Algeria and Spain, and came back to his professional duties again. A year and a half later his health broke down a second time, and he was obliged to leave again for the shores of the Mediterranean. In the fall of 1878 he returned to England for the last time, when the winter came he left for the Island of Madeira; all hope of recovery was gone; he died March 3, 1879 in the 34th year of his age.

On the title page of the volume containing his collected mathematical papers I find a quotation, "If he had lived we might have known something." Such is the feeling one has when one looks at his published works and thinks of the shortness of his life. In his lifetime there appeared *Elements of Dynamic, Part I.* Posthumously there have appeared *Elements of Dynamic, Part II; Collected Mathematical Papers; Lectures and Essays; Seeing and Thinking; Common Sense of the Exact Sciences.* The manuscript of the last book was left in a very incomplete state, but the design was filled up and completed by two other mathematicians.

In a former lecture I had occasion to remark on the relation of Mathematics to Poetry—on the fact that in mathematical investigation there is needed a higher power of imagination akin to the creative instinct of the poet. The matter is discussed by Clifford in a discourse on "Some of the conditions of mental development," which he delivered at the Royal Institution in 1868 when he was 23 years of age. This institution was founded by Count Rumford, an American, and is located in London. There are Professorships of Chemistry, Physics, and Physiology; its professors have included Davey, Faraday, Young, Tyndall, Rayleigh, Dewar. Their duties are not to teach the elements of their science to regular students, but to make investigations, and to lecture to the members of the institution, who are in general wealthy and titled people.

In this discourse Clifford said "Men of science have to deal with extremely abstract and general conceptions. By constant use and familiarity, these, and the relations between them, become just as real and external as the ordinary objects of experience, and the perception of new relations among them is so rapid, the correspondence of the mind to external circumstances so great, that a real scientific sense is developed, by which things are perceived as immediately and truly as I see you now. Poets and painters and musicians also are so accustomed to put outside of them the idea of beauty, that it becomes a real external existence, a thing which they see with spiritual eyes and then describe to you, but by no means create, any more than we seem to create the ideas of table and forms and light, which we put together long ago. There is no scientific discoverer, no poet, no painter, no musician, who will not tell you that he found ready made his discovery or poem or picture—that it came to him from outside, and that he did not consciously create it from within. And there is reason to think that these senses or insights are things which actually increase among mankind. It is certain, at least, that the scientific sense is immensely more developed now than it was three hundred years ago; and though it may be impossible to find any absolute standard of art, yet it is acknowledged that a number of minds which are subject to artistic training will tend to arrange themselves under certain great groups and that the members of each group will give an independent and yet consentient testimony about artistic questions. And this arrangement into schools, and the definiteness of the conclusions reached in each, are on the increase, so that here, it would seem, are actually two new senses, the scientific and the artistic, which the mind is now in the process of forming for itself."

Clifford himself wrote a good many poems, but only a few have been published. The following verses were sent to George Eliot, the novelist, with a presentation copy of *The Little People*:

Baby drew a little house,
    Drew it all askew;
Mother saw the crooked door
    And the window too.

Mother heart, whose wide embrace
    Holds the hearts of men,
Grows with all our growing hopes,
    Gives them birth again,

Listen to this baby-talk:
    'Tisn't wise or clear;
But what baby-sense it has
    Is for you to hear.

An amusement in which Clifford took pleasure even in his maturer years was the flying of kites. He made some mathematical investigations in the subject, anticipating, as it were, the interest which has been taken in more recent years in the subject of motion through the atmosphere. Clifford formed a project of writing a series of textbooks on Mathematics beginning at the very commencement of each subject and carrying it on rapidly to the most advanced stages. He began with the *Elements of Dynamic*, of which three books were printed in his lifetime, and a fourth book, in a supplementary volume, after his death. The work is unique for the clear ideas given of the science; ideas and principles are more prominent than symbols and formulae. He takes such familiar words as *spin, twist, squirt, whirl*, and gives them an exact meaning. The book is an example of what he meant by scientific insight, and from its excellence we can imagine what the complete series of textbooks would have been.

In Clifford's lifetime it was said in England that he was the only mathematician who could discourse on mathematics to an audience composed of people of general culture and make them think that they understood the subject. In 1872 he was invited to deliver an evening lecture before the members of the British Association, at Brighton; he chose for his subject "The aims and instruments of scientific thought." The main theses of the lecture are *First*, that scientific thought is the application of past experience to new circumstances by means of an observed order of events. *Second*, this order of events is not theoretically or absolutely exact, but only exact enough to correct experiments by. As an instance of what is, and what is not scientific thought, he takes the phenomenon of double refraction. "A mineralogist, by measuring the angles of a crystal, can tell you whether or no it possesses the property of double refraction without looking through it. He requires no scientific thought to do that. But Sir William Rowan Hamilton, knowing these facts and also the explanation of them which Fresnel had given, thought about the subject, and he predicted that by looking through certain crystals in a particular direction we should see not two dots but a continuous circle. Mr. Lloyd made the experiment, and saw the circle, a result which had never been even suspected. This has always been considered one of the most signal instances of scientific thought in the domain of physics. It is most distinctly an application of experience gained under certain circumstances to entirely different circumstances."

In physical science there are two kinds of law—distinguished as "empirical" and "rational." The former expresses a relation which is sufficiently true for practical purposes and within certain limits; for example, many of the formulas used by engineers. But a rational law states a connection which is accurately true, without any modification of limit. In the theorems of geometry we have examples of scientific exactness; for example, in the theorem that the sum of the three interior angles of a plane triangle is equal to two right angles. The equality is one not of approximation, but of exactness. Now the philosopher Kant pointed to such a truth and said: We know that it is true not merely here and now, but everywhere and for all time; such knowledge cannot be gained by experience; there must be some other source of such knowledge. His solution was that space and time are forms of the sensibility; that truths about them are not obtained by empirical induction, but by means of intuition; and that the characters of necessity and universality distinguished these truths from other truths. This philosophy was accepted by Sir William Rowan Hamilton, and to him it was not a barren philosophy, for it served as the starting point of his discoveries in algebra which culminated in the discovery of quaternions.

This philosophy was admired but not accepted by Clifford; he was, so long as he lived, too strongly influenced by the philosophy which has been built upon the theory of evolution. He admits that the only way of escape from Kant's conclusions is by denying the theoretical exactness of the proposition referred to. He says, "About the beginning of the present century the foundations of geometry were criticised independently by two mathematicians, Lobatchewsky and Gauss, whose results have been extended and generalized more recently by Riemann and Helmholtz. And the conclusion to which these investigations lead is that, although the assumptions which were very properly made by the ancient geometers are practically exact—that is to say, more exact than experiment can be—for such finite things as we have to deal with, and such portions of space as we can reach; yet the truth of them for very much larger things, or very much smaller things, or parts of space which are at present beyond our reach, is a matter to be decided by experiment, when its powers are considerably increased. I want to make as clear as possible the real state of this question at present, because it is often supposed to be a question of words or metaphysics, whereas it is a very distinct and simple question of fact. I am supposed to know that the three angles of a rectilinear triangle are exactly equal to two right angles. Now suppose that three points are taken in space, distant from one another as far as the Sun is from α Centauri, and that the shortest distances between these points are drawn so as to form a triangle. And suppose the angles of this triangle to be very accurately measured and added together; this can at present be done so accurately that the error shall certainly be less than one minute, less therefore than the five-thousandth part of a right angle. Then I do not know that this sum would differ at all from two right angles; but also I do not know that the difference would be less than ten degrees or the ninth part of a right angle."

You will observe that Clifford's philosophy depends on the validity of Lobatchewsky's ideas. Now it has been shown by an Italian mathematician, named Beltrami, that the plane geometry of Lobatchewsky corresponds to trigonometry on a surface called the *pseudosphere*. Clifford and other followers of Lobatchewsky admit Beltrami's interpretation, an interpretation which does not involve any paradox about geometrical space, and which leaves the trigonometry of the plane alone as a different thing. If that interpretation is true, the Lobatchewskian plane triangle is after all a triangle on a special surface, and the *straight* lines joining the points are not the shortest absolutely, but only the *shortest* with respect to the surface, whatever that may mean. If so, then Clifford's argument for the empirical nature of the proposition referred to fails; and nothing prevents us from falling back on Kant's position, namely, that there is a body of knowledge characterized by absolute exactness and possessing universal application in time and space; and as a particular case thereof we believe that the sum of the three angles of Clifford's gigantic triangle is precisely two right angles.

Trigonometry on a spherical surface is a generalized form of plane trigonometry, from the theorems of the former we can deduce the theorems of the latter by supposing the radius of the sphere to be infinite. The sum of the three angles of a spherical triangle is greater than two right angles; the sum of the angles of a plain triangle is equal to two right angles; we infer that there is another surface, complementary to the sphere, such that the angles of any triangle on it are less than two right angles. The complementary surface to which I refer is not the pseudosphere, but the equilateral hyperboloid. As the plane is the transition surface between the sphere and the equilateral hyperboloid, and a triangle on it is the transition triangle between the spherical triangle and the equilateral hyperboloidal triangle, the sum of the angles of the plane triangle must be exactly equal to two right angles.

In 1873, the British Association met at Bradford; on this occasion the evening discourse was delivered by Maxwell, the celebrated physicist. He chose for his subject "Molecules." The application of the method of spectrum-analysis assures the physicist that he can find out in his laboratory truths of universal validity in space and time. In fact, the chief maxim of physical science, according to Maxwell is, that physical changes are independent of the conditions of space and time, and depend only on conditions of configuration of bodies, temperature, pressure, etc. The address closed with a celebrated passage in striking contrast to Clifford's address: "In the heavens we discover by their light, and by their light alone, stars so distant from each other that no material thing can ever have passed from one to another; and yet this light, which is to us the sole evidence of the existence of these distant worlds, tells us also that each of them is built up of molecules of the same kinds as those which are found on earth. A molecule of hydrogen, for example, whether in Sirius or in Arcturus, executes its vibrations in precisely the same time. No theory of evolution can be formed to account for the similarity of molecules, for evolution necessarily implies continuous change, and the molecule is incapable of growth or decay, of generation or destruction. None of the processes of Nature since the time when Nature began, have produced the slightest difference in the properties of any molecule. We are therefore unable to ascribe either the existence of the molecules or the identity of their properties to any of the causes which we call natural. On the other hand, the exact equality of each molecule to all others of the same kind gives it, as Sir John Herschel has well said, the essential character of a manufactured article, and precludes the idea of its being eternal and self-existent."

What reply could Clifford make to this? In a discourse on the "First and last catastrophe" delivered soon afterwards, he said "If anyone not possessing the great authority of Maxwell, had put forward an argument, founded upon a scientific basis, in which there occurred assumptions about what things can and what things cannot have existed from eternity, and about the exact similarity of two or more things established by experiment, we would say: 'Past eternity; absolute exactness; won't do'; and we should pass on to another book. The experience of all scientific culture for all ages during which it has been a light to men has shown us that we never do get at any conclusions of that sort. We do not get at conclusions about infinite time, or infinite exactness. We get at conclusions which are as nearly true as experiment can show, and sometimes which are a great deal more correct than direct experiment can be, so that we are able actually to correct one experiment by deductions from another, but we never get at conclusions which we have a right to say are absolutely exact."

Clifford had not faith in the exactness of mathematical science nor faith in that maxim of physical science which has built up the new astronomy, and extended all the bounds of physical science. Faith in an exact order of Nature was the characteristic of Faraday, and he was by unanimous consent the greatest electrician of the nineteenth century. What is the general direction of progress in science? Physics is becoming more and more mathematical; chemistry is becoming more and more physical, and I daresay the biological sciences are moving in the same direction. They are all moving towards exactness; consequently a true philosophy of science will depend on the principles of mathematics much more than upon the phenomena of biology. Clifford, I believe, had he lived longer, would have changed his philosophy for a more mathematical one. In 1874 there appeared in *Nature* among the letters from correspondents one to the following effect:

An anagram: The practice of enclosing discoveries in sealed packets and sending them to Academies seems so inferior to the old one of Huyghens, that the following is sent you for publication in the old conservated form:

$$A^8C^3DE^{12}F^4GH^6J^6L^3M^3N^5O^6PR^4S^5T^{14}U^6V^2WXY^2.$$

This anagram was explained in a book entitled *The Unseen Universe*, which was published anonymously in 1875; and is there translated, "Thought conceived to affect the matter of another universe simultaneously with this may explain a future state." The book was evidently a work of a physicist or physicists, and as physicists were not so numerous then as they are now, it was not difficult to determine the authorship from internal evidence. It was attributed to Tait, the professor of physics at Edinburgh University, and Balfour Stewart, the professor of physics at Owens College, Manchester. When the fourth edition appeared, their names were given on the title page.

The kernel of the book is the above so-called discovery, first published in the form of an anagram. Preliminary chapters are devoted to a survey of the beliefs of ancient peoples on the subject of the immortality of the soul; to physical axioms; to the physical doctrine of energy, matter, and ether; and to the biological doctrine of development; in the last chapter we come to the unseen universe. What is meant by the *unseen universe*? Matter is made up of molecules, which are supposed to be vortex-rings of an imperfect fluid, namely, the luminiferous ether; the luminous ether is made up of much smaller molecules, which are vortex-rings in a second ether. These smaller molecules with the ether in which they float are the unseen universe. The authors see reason to believe that the unseen universe absorbs energy from the visible universe and *vice versa*. The soul is a frame which is made of the refined molecules and exists in the unseen universe. In life it is attached to the body. Every thought we think is accompanied by certain motions of the coarse molecules of the brain, these motions are propagated through the visible universe, but a part of each motion is absorbed by the fine molecules of the soul. Consequently the soul has an organ of memory as well as the body; at death the soul with its organ of memory is simply set free from association with the coarse molecules of the body. In this way the authors consider that they have shown the physical possibility of the immortality of the soul.

The curious part of the book follows: the authors change their possibility into a theory and apply it to explain the main doctrines of Christianity; and it is certainly remarkable to find in the same book a discussion of Carnot's heat-engine and extensive quotations from the apostles and prophets. Clifford wrote an elaborate review which he finished in one sitting occupying twelve hours. He pointed out the difficulties to which the main speculation, which he admitted to be ingenious, is liable; but his wrath knew no bounds when he proceeded to consider the application to the doctrines of Christianity; for from being a High Churchman in youth he became an agnostic in later years; and he could not write on any religious question without using language which was offensive even to his friends.

The *Phaedo* of Plato is more satisfying to the mind than the *Unseen Universe* of Tait and Stewart. In it, Socrates discusses with his friends the immortality of the soul, just before taking the draught of poison. One argument he advances is, How can the works of an artist be more enduring than the artist himself? This is a question which comes home in force when we peruse the works of Peacock, De Morgan, Hamilton, Boole, Cayley and Clifford.