Chapter 9

THOMAS PENYNGTON KIRKMAN[1]

(1806-1895)

Thomas Penyngton Kirkman was born on March 31, 1806, at Bolton in Lancashire. He was the son of John Kirkman, a dealer in cotton and cotton waste; he had several sisters but no brother. He was educated at the Grammar School of Bolton, where the tuition was free. There he received good instruction in Latin and Greek, but no instruction in geometry or algebra; even Arithmetic was not then taught in the headmaster's upper room. He showed a decided taste for study and was by far the best scholar in the school. His father, who had no taste for learning and was succeeding in trade, was determined that his only son should follow his own business, and that without any loss of time. The schoolmaster tried to persuade the father to let his son remain at school; and the vicar also urged the father, saying that if he would send his son to Cambridge University, he would guarantee for sixpence that the boy would win a fellowship. But the father was obdurate; young Kirkman was removed from school, when he was fourteen years of age, and placed at a desk in his father's office. While so engaged, he continued of his own accord his study of Latin and Greek, and added French and German.

After ten years spent in the counting room, he tore away from his father, secured the tuition of a young Irish baronet, Sir John Blunden, and entered the University of Dublin with the view of passing the examinations for the degree of B.A. There he never had instruction from any tutor. It was not until he entered Trinity College, Dublin, that he opened any mathematical book. He was not of course abreast with men who had good preparation. What he knew of mathematics, he owed to his own study, having never had a single hour's

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instruction from any person. To this self-education is due, it appears to me, both the strength and the weakness to be found in his career as a scientist. However, in his college course he obtained honors, or premiums as they are called, and graduated as a moderator, something like a wrangler.

Returning to England in 1835, when he was 29 years old, he was ordained as a minister in the Church of England. He was a curate for five years, first at Bury, afterwards at Lymm; then he became the vicar of a newly-formed parish—Croft with Southworth in Lancashire. This parish was the scene of his life's labors. The income of the benefice was not large, about £200 per annum; for several years he supplemented this by taking pupils. He married, and property which came to his wife enabled them to dispense with the taking of pupils. His father became poorer, but was able to leave some property to his son and daughters. His parochial work, though small, was discharged with enthusiasm; out of the roughest material he formed a parish choir of boys and girls who could sing at sight any four-part song put before them. After the private teaching was over he had the leisure requisite for the great mathematical researches in which he now engaged.

Soon after Kirkman was settled at Croft, Sir William Rowan Hamilton began to publish his quaternion papers and, being a graduate of Dublin University, Kirkman was naturally one of the first to study the new analysis. As the fruit of his meditations he contributed a paper to the *Philosophical Magazine* "On pluquaternions and homoid products of sums of n squares." He proposed the appellation "pluquaternions" for a linear expression involving more than three imaginaries (the i, j, k of Hamilton), "not dreading" he says, "the pluperfect criticism of grammarians, since the convenient barbarism is their own." Hamilton, writing to De Morgan, remarked "Kirkman is a very clever fellow," where the adjective has not the American colloquial meaning but the English meaning.

For his own education and that of his pupils he devoted much attention to mathematical mnemonics, studying the *Memoria Technica* of Grey. In 1851 he contributed a paper on the subject to the Literary and Philosophical Society of Manchester, and in 1852 he published a book, *First Mnemonical Lessons in Geometry, Algebra, and Trigonometry*, which is dedicated to his former pupil, Sir John Blunden. De Morgan pronounced it "the most curious crochet I ever saw," which was saying a great deal, for De Morgan was familiar with many quaint books in mathematics. In the preface he says that much of the distaste for mathematical study springs largely from the difficulty of retaining in the memory the previous results and reasoning. "This difficulty is closely connected with the unpronounceableness of the formulæ; the memory of the tongue and the ear are not easily turned to account; nearly everything depends on the thinking faculty or on the practice of the eye alone. Hence many, who see hardly anything formidable in the study of a language, look upon mathematical acquirements as beyond their power, when in truth they are very far from being so. My object is to enable the learner to 'talk to himself,' in rapid, vigorous and suggestive syllables, about the matters which he must digest and remember. I have sought to bring the memory of the vocal organs and the ear to the assistance of the reasoning faculty and have never scrupled to sacrifice either good grammar or good English in order to secure the requisites for a useful *mnemonic*, which are smoothness, condensation, and jingle."

As a specimen of his mnemonics we may take the cotangent formula in spherical trigonometry:

$$\cot A \sin C + \cos b \cos C = \cot a \sin b$$

To remember this formula most masters then required some aid to the memory; for instance the following: If in any spherical triangle four parts be taken in succession, such as $AbCa$, consisting of two means bC and two extremes Aa, then the product of the cosines of the two means is equal to the sine of the mean side × cotangent of the extreme side minus sine of the mean angle × cotangent of the extreme angle, that is

$$\cos b \cos C = \sin b \cot a - \sin C \cot A.$$

This is an appeal to the reason. Kirkman, however, proceeds on the principle of appealing to the memory of the ear, of the tongue, and of the lips altogether; a true *memoria technica*. He distinguishes the large letter from the small by calling them *Ang, Bang, Cang* (*ang* from angle in contrast to side). To make the formula more euphoneous he drops the s from cos and the n from sin. Hence the formula is

cot *Ang* si *Cang* and co *b* co *Cang* are cot *a* si *b*

which is to be chanted till it becomes perfectly familiar to the ear and the lips. The former rule is a hint offered to the judgment; Kirkman's method is something to be taught by rote. In his book Kirkman makes much use of verse, in the turning of which he was very skillful.

In the early part of the nineteenth century a publication named the *Lady's and Gentlemen's Diary* devoted several columns to mathematical problems. In 1844 the editor offered a prize for the solution of the following question: "Determine the number of combinations that can be made out of n symbols, each combination having p symbols, with this limitation, that no combination of q symbols which may appear in any one of them, may be repeated in any other." This is a problem of great difficulty; Kirkman solved it completely for the special case of $p = 3$ and $q = 2$ and printed his results in the second volume of the *Cambridge and Dublin Mathematical Journal*. As a chip off this work he published in the *Diary* for 1850 the famous problem of the fifteen schoolgirls as follows: "Fifteen young ladies of a school walk out three abreast for seven days in succession; it is required to arrange them daily so that no two shall walk abreast more than once." To form the schedules for seven days is not difficult; but to find all the possible schedules is a different matter. Kirkman found all the possible combinations of the fifteen young ladies in groups of three to be 35, and the problem was also considered and solved by Cayley, and has been discussed by many later writers; Sylvester gave 91 as the greatest number of days; and he also intimated that the principle of the puzzle was known to him when an undergraduate at Cambridge, and that he had given it to fellow undergraduates. Kirkman replied that up to the time he proposed the problem he had neither seen Cambridge nor met Sylvester, and narrated how he had hit on the question.

The Institute of France offered several times in succession a prize for a memoir on the theory of the polyedra; this fact together with his work in combinations led Kirkman to take up the subject. He always writes *polyedron* not *polyhedron*; for he says we write *periodic* not *perihodic*. When Kirkman began work nothing had been done beyond the very ancient enumeration of the five regular solids and the simple combinations of crystallography. His first paper, "On the representation and enumeration of the polyedra," was communicated in 1850 to the Literary and Philosophical Society of Manchester. He starts with the well-known theorem $P + S = L + 2$, where P is the number of points or summits, S the number of plane bounding surfaces and L the number of linear edges in a geometrical solid. "The question—how many n-edrons are there?—has been asked, but it is not likely soon to receive a definite answer. It is far from being a simple question, even when reduced to the narrower compass—how many n-edrons are there whose summits are all trihedral"? He enumerated and constructed the fourteen 8-edra whose faces are all triangles.

In 1858 the French Institute modified its prize question. As the subject for the *concours* of 1861 was announced: "Perfectionner en quelque point important la théorie géométrique des polyèdres," where the indefiniteness of the question indicates the very imperfect state of knowledge on the subject. The prize offered was 3000 francs. Kirkman appears to have worked at it with a view of competing, but he did not send in his memoir. Cayley appears to have intended to compete. The time was prolonged for a year, but there was no award and the prize was taken down. Kirkman communicated his results to the Royal Society through his friend Cayley, and was soon elected a Fellow. Then he contributed directly an elaborate paper entitled "Complete theory of the Polyedra." In the preface he says, "The following memoir contains a complete solution of the classification and enumeration of the P-edra Q-acra. The actual construction of the solids is a task impracticable from its magnitude, but it is here shown that we can enumerate them with an accurate account of their symmetry to any values of P and Q." The memoir consisted of 21 sections; only the two introductory sections, occupying 45 quarto pages, were printed by the Society, while the others still remain in manuscript. During following years he added many contributions to this subject.

In 1858 the French Academy also proposed a problem in the Theory of Groups as the subject for competition for the grand mathematical prize in 1860: "Quels peuvent être les nombres de valeurs des fonctions bien définies qui contiennent un nombre donné de lettres, et comment peut on former les fonctions pour lesquelles il existe un nombre donné de valeurs?" Three memoirs were presented, of which Kirkman's was one, but no prize was awarded. Not the slightest summary was vouchsafed of what the competitors had added to science, although it was confessed that all had contributed results both new and important; and the question, though proposed for the first time for the year 1860, was withdrawn from competition contrary to the usual custom of the Academy. Kirkman contributed the results of his investigation to the Manchester Society under the title "The complete theory of groups, being the solution of the mathematical prize question of the French Academy for 1860." In more recent years the theory of groups has engaged the attention of many mathematicians in Germany and America; so far as British contributors are concerned Kirkman was the first and still remains the greatest.

In 1861 the British Association met at Manchester; it was the last of its meetings which Sir William Rowan Hamilton attended. After the meeting Hamilton visited Kirkman at his home in the Croft rectory, and that meeting was no doubt a stimulus to both. As regards pure mathematics they were probably the two greatest in Britain; both felt the loneliness of scientific work, both were metaphysicians of penetrating power, both were good versifiers if not great poets. Of nearly the same age, they were both endowed with splendid physique; but the care which was taken of their health was very different; in four years Hamilton died but Kirkman lived more than 30 years longer.

About 1862 the *Educational Times*, a monthly periodical published in London, began to devote several columns to the proposing and solving of mathematical problems, taking up the work after the demise of the *Diary*. This matter was afterwards reprinted in separate volumes, two for each year. In these reprints are to be found many questions proposed by Kirkman; they are generally propounded in quaint verse, and many of them were suggested by his study of combinations. A good specimen is "The Revenge of Old King Cole"

> "Full oft ye have had your fiddler's fling,
> For your own fun over the wine;
> And now" quoth Cole, the merry old king,
> "Ye shall have it again for mine.
> My realm prepares for a week of joy
> At the coming of age of a princely boy—
> Of the grand six days procession in square,
> In all your splendour dressed,
> Filling the city with music rare
> From fiddlers five abreast," etc.

The problem set forth by this and other verses is that of 25 men arranged in five rows on Monday. Shifting the second column one step upward, the third two steps, the fourth three steps, and the fifth four steps gives the arrangement for Tuesday. Applying the same rule to Tuesday gives Wednesday's array, and similarly are found those for Thursday and Friday. In none of these can the same two men be found in one row. But the rule fails to work for Saturday, so that a special arrangement must be brought in which I leave to my hearers to work out. This problem resembles that of the fifteen schoolgirls.

Monday

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

Tuesday

A	G	M	S	Y
F	L	R	X	E
K	Q	W	D	J
P	V	C	I	O
U	B	H	N	T

Wednesday

A	L	W	I	T
F	Q	C	N	Y
K	V	H	S	E
P	B	M	X	J
N	G	R	D	O

Thursday

A	Q	H	X	O
F	V	M	D	T
K	B	R	I	Y
P	G	W	N	E
U	L	C	S	J

The Rev. Kirkman became at an early period of his life a broad churchman. About 1863 he came forward in defense of the Bishop of Colenso, a mathematician, and later he contributed to a series of pamphlets published in aid of the cause of "Free Enquiry and Free Expression." In one of his letters to me Kirkman writes as follows: "*The Life of Colenso* by my friend Rev. Sir George Cox, Bart., is a most charming book; and the battle of the Bishops against the lawyers in the matter of the vacant see of Natal, to which Cox is the bishop-elect, is exciting. Canterbury refuses to ask, as required, the Queen's mandate to consecrate him. The Natal churchmen have just petitioned the Queen to make the Primate do his duty according to law. Natal was made a See with perpetual succession, and is endowed. The endowment has been lying idle since Colenso's death in 1883; and the bishops who have the law courts dead against them here are determined that no successor to Colenso shall be consecrated. There is a Bishop of South African Church there, whom they thrust in while Colenso lived, on pretense that Colenso was excommunicate. We shall soon see whether the lawyers or the bishops are to win." It was Kirkman's own belief that his course in this matter injured his chance of preferment in the church; he never rose above being rector of Croft.

While a broad churchman the Rev. Mr. Kirkman was very vehement against the leaders of the materialistic philosophy. Two years after Tyndall's Belfast address, in which he announced that he could discern in matter the promise and potency of every form of life, Kirkman published a volume entitled *Philosophy without Assumptions*, in which he criticises in very vigorous style the materialistic and evolutional philosophy advocated by Mill, Spencer, Tyndall, and Huxley. In ascribing everything to matter and its powers or potencies he considers that they turn philosophy upside down. He has, he writes, first-hand knowledge of himself as a continuous person, endowed with will; and he infers that there are will forces around; but he sees no evidence of the existence of matter. Matter is an assumption and forms no part of his philosophy. He relies on Boscovich's theory of an atom as simply the center of forces. Force he understands from his knowledge of will, but any other substance he does not understand. The obvious difficulty in this philosophy is to explain the belief in the existence of other conscious beings—other will forces. Is it not the *great* assumption which everyone is obliged to make; verified by experience, but still in its nature an assumption? Kirkman tries to get over this difficulty by means of a syllogism, the major premise of which he has to manufacture, and which he presents to his reason for adoption or rejection. How can a universal proposition be easier to grasp than the particular case included in it? If the mind doubts about an individual case, how can it be sure about an infinite number of such cases? It is a *petitio principii*.

As a critic of the materialistic philosophy Kirkman is more successful. He criticises Herbert Spencer on free will as follows: "The short chapter of eight pages on Will cost more philosophical toil than all the two volumes on Psychology. The author gets himself in a heat, he runs himself into a corner, and brings himself dangerously to bay. Hear him: 'To reduce the general question to its simplest form; psychical changes either conform to law, or they do not. If they do not conform to law, this work, in common with all other works on the subject, is sheer nonsense; no science of Psychology is possible. If they do conform to law, there cannot be any such thing as free will.' Here we see the horrible alternative. If the assertors of free will refuse to commit suicide, they must endure the infinitely greater pang of seeing Mr. Spencer hurl himself and his books into that yawning gulf, a sacrifice long devoted, and now by pitiless Fate consigned, to the abysmal gods of nonsense. Then pitch him down say I. Shall I spare him who tells me that my movements in this orbit of conscious thought and responsibility are made under 'parallel conditions' with those of yon driven moon? Shall I spare him who has juggled me out of my Will, my noblest attribute; who has hocuspocused me out of my subsisting personality; and then, as a refinement of cruelty, has frightened me out of the rest of my wits by forcing me to this terrific alternative that either the testimony of this Being, this Reason and this Conscience is one ever-thundering lie, or else he, even he, has talked nonsense? He has talked nonsense, I say it because I have proved it. And every man must of course talk nonsense who begins his philosophy with abstracts in the clouds instead of building on the witness of his own self-consciousness. 'If they do conform to law,' says Spencer, 'there cannot be any such thing as free will.' The force of this seems to depend on his knowledge of 'law.' When I ask, What does this writer know of law—definite working law in the Cosmos?—the only answer I can get is—Nothing, except a very little which he has picked up, often malappropriately, as we have seen, among the mathematicians. When I ask—What does he know *about* law?—there is neither beginning nor end to the reply. I am advised to read his books *about* law, and to master the differentiations and integrations of the coherences, the correlations, the uniformities, and universalities which he has established in the abstract over all space and all time by his vast experience and miraculous penetration. I have tried to do this, and have found all pretty satisfactory, except the lack of one thing—something like proof of his competence to decide all that scientifically. When I persist in my demand for such proof, it turns out at last—that he knows by heart the whole Hymn Book, the Litanies, the Missal, and the Decretals of the Must-be-ite religion! 'Conform to law.' Shall I tell you what he means by that? Exactly ninety-nine hundredths of his meaning under the word *law* is *must be.*"

Kirkman points out that the kind of proof offered by these philosophers is a bold assertion of *must-be-so.* For instance he mentions Spencer's evolution of consciousness out of the unconscious: "That an effectual adjustment may be made they (the separate impressions or constituent changes of a complex correspondence to be coordinated) *must be* brought into relation with each other. But this implies some center of communication common to them all, through which they severally pass; and as they *cannot* pass through it simultaneously, they *must* pass through it in succession. So that as the external phenomena responded to become greater in number and more complicated in kind, the variety and rapidity of the changes to which this common center of communication is subject *must* increase, there *must* result an unbroken series of those changes, there *must* arise a consciousness."

The paraphrase which Kirkman gave of Spencer's definition of Evolution commended itself to such great minds as Tait and Clerk-Maxwell. Spencer's definition is: "Evolution is a change from an indefinite incoherent homogeneity to a definite coherent heterogeneity, through continuous differentiations and integrations." Kirkman's paraphrase is "Evolution is a change from a nohowish untalkaboutable all-likeness, to a somehowish and in-general-talkaboutable not-all-likeness, by continuous somethingelseifications and sticktogetherations." The tone of Kirkman's book is distinctly polemical and full of sarcasm. He unfortunately wrote as a theologian rather than as a mathematician. The writers criticised did not reply, although they felt the edge of his sarcasm; and they acted wisely, for they could not successfully debate any subject involving exact science against one of the most penetrating mathematicians of the nineteenth century.

We have seen that Hamilton appreciated Kirkman's genius; so did Cayley, De Morgan, Clerk-Maxwell, Tait. One of Tait's most elaborate researches was the enumeration and construction of the knots which can be formed in an endless cord—a subject which he was induced to take up on account of its bearing on the vortex theory of atoms. If the atoms are vortex filaments their differences in kind, giving rise to differences in the spectra of the elements, must depend on a greater or less complexity in the form of the closed filament, and this difference would depend on the knottiness of the filament. Hence the main question was "How many different forms of knots are there with any given small number of crossings?" Tait made the investigation for three, four, five, six, seven, eight crossings. Kirkman's investigations on the polyedra were much allied. He took up the problem and, with some assistance from Tait, solved it not only for nine but for ten crossings. An investigation by C. N. Little, a graduate of Yale University, has confirmed Kirkman's results.

Through Professor Tait I was introduced to Rev. Mr. Kirkman; and we discussed the mathematical analysis of relationships, formal logic, and other subjects. After I had gone to the University of Texas, Kirkman sent me through Tait the following question which he said was current in society: "Two boys, Smith and Jones, of the same age, are each the nephew of the other; how many legal solutions?" I set the analysis to work, wrote out the solutions, and the paper is printed in the *Proceedings* of the Royal Society of Edinburgh. There are four solutions, provided Smith and Jones are taken to be mere arbitrary, names; if the convention about surnames holds there are only two legal solutions. On seeing my paper Kirkman sent the question to the *Educational Times* in the following improved form:

> Baby Tom of baby Hugh
> The nephew is and uncle too;
> In how many ways can this be true?

Thomas Penyngton Kirkman died on February 3, 1895, having very nearly reached the age of 89 years. I have found only one printed notice of his career, but all his writings are mentioned in the new German Encyclopædia of Mathematics. He was an honorary member of the Literary and Philosophical Societies of Manchester and of Liverpool, a Fellow of the Royal Society, and a foreign member of the Dutch Society of Sciences at Haarlem. I may close by a quotation from one of his letters: “What I have done in helping busy Tait in knots is, like the much more difficult and extensive things I have done in polyedra or groups, not at likely to be talked about intelligently by people so long as I live. But it is a faint pleasure to think it will one day win a little praise.”