CHAPTER XIX.

Of Immediate Inference as applied to Complex Propositions.

Section 701. So far we have treated of inference, or reasoning, whether mediate or immediate, solely as applied to simple propositions. But it will be remembered that we divided propositions into simple and complex. I t becomes incumbent upon us therefore to consider the laws of inference as applied to complex propositions. Inasmuch however as every complex proposition is reducible to a simple one, it is evident that the same laws of inference must apply to both.

Section 702. We must first make good this initial statement as to the essential identity underlying the difference of form between simple and complex propositions.

Section 703. Complex propositions are either Conjunctive or Disjunctive (Section 214).

Section 704. Conjunctive propositions may assume any of the four forms, A, E, I, O, as follows--

  (A) If A is B, C is always D.
  (E) If A is B, C is never D.
  (I) If A is B, C is sometimes D.
  (O) If A is B, C is sometimes not D.

Section 705. These admit of being read in the form of simple propositions, thus--

  (A) If A is B, C is always D = All cases of A being B are cases of C being D. (Every AB is a CD.)

  (E) If A is B, C is never D = No cases of A being B are cases of C being D. (No AB is a CD.)

  (I) If A is B, C is sometimes D = Some cases of A being B are cases of C being D. (Some AB's are CD's.)

  (O) If A is B, C is sometimes not D = Some cases of A being B are not cases of C being D. (Some AB's are not CD's.)

Section 706. Or, to take concrete examples,

  (A) If kings are ambitious, their subjects always suffer.

= All cases of ambitious kings are cases of subjects suffering.

(E) If the wind is in the south, the river never freezes.
= No cases of wind in the south are cases of the river freezing.

(I) If a man plays recklessly, the luck sometimes goes against him.
= Some cases of reckless playing are cases of going against one.

(O) If a novel has merit, the public sometimes do not buy it.
= Some cases of novels with merit are not cases of the public buying.

Section 707. We have seen already that the disjunctive differs from the conjunctive proposition in this, that in the conjunctive the truth of the antecedent involves the truth of the consequent, whereas in the disjunctive the falsity of the antecedent involves the truth of the consequent. The disjunctive proposition therefore

  Either A is B or C is D

may be reduced to a conjunctive

  If A is not B, C is D,

and so to a simple proposition with a negative term for subject.

  All cases of A not being B are cases of C being D.
  (Every not-AB is a CD.)

Section 708. It is true that the disjunctive proposition, more than any other form, except U, seems to convey two statements in one breath. Yet it ought not, any more than the E proposition, to be regarded as conveying both with equal directness. The proposition 'No A is B' is not considered to assert directly, but only implicitly, that 'No B is A.' In the same way the form 'Either A is B or C is D' ought to be interpreted as meaning directly no more than this, 'If A is not B, C is D.' It asserts indeed by implication also that 'If C is not D, A is B.' But this is an immediate inference, being, as we shall presently see, the contrapositive of the original. When we say 'So and so is either a knave or a fool,' what we are directly asserting is that, if he be not found to be a knave, he will be found to be a fool. By implication we make the further statement that, if he be not cleared of folly, he will stand condemned of knavery. This inference is so immediate that it seems indistinguishable from the former proposition: but since the two members of a complex proposition play the part of subject and predicate, to say that the two statements are identical would amount to asserting that the same proposition can have

two subjects and two predicates. From this point of view it becomes clear that there is no difference but one of expression between the disjunctive and the conjunctive proposition. The disjunctive is merely a peculiar way of stating a conjunctive proposition with a negative antecedent.

Section 709. Conversion of Complex Propositions.

  A / If A is B, C is always D.
    \.'. If C is D, A is sometimes B.

  E / If A is B, C is never D.
    \.'. If C is D, A is never B.

  I / If A is S, C is sometimes D.
    \.'. If C is D, A is sometimes B.

Section 710. Exactly the same rules of conversion apply to conjunctive as to simple propositions.

Section 711. A can only be converted per accidens, as above.

The original proposition

  'If A is B, C is always D'

is equivalent to the simple proposition

  'All cases of A being B are cases of C being D.'

This, when converted, becomes

  'Some cases of C being D are cases of A being B,'

which, when thrown back into the conjunctive form, becomes

  'If C is D, A is sometimes B.'

Section 712. This expression must not be misunderstood as though it contained any reference to actual existence. The meaning might be better conveyed by the form

  'If C is D, A may be B.'

But it is perhaps as well to retain the other, as it serves to emphasize the fact that formal logic is concerned only with the

connection of ideas.

Section 713. A concrete instance will render the point under discussion clearer. The example we took before of an A proposition in the conjunctive form--

  'If kings are ambitious, their subjects always suffer'

may be converted into

  'If subjects suffer, it may be that their kings are ambitious,'

i.e. among the possible causes of suffering on the part of subjects is to be found the ambition of their rulers, even if every actual case should be referred to some other cause. It is in this sense only that the inference is a necessary one. But then this is the only sense which formal logic is competent to recognise. To judge of conformity to fact is no part of its province. From 'Every AB is a CD' it follows that ' Some CD's are AB's' with exactly the same necessity as that with which 'Some B is A' follows from 'All A is B.' In the latter case also neither proposition may at all conform to fact. From 'All centaurs are animals' it follows necessarily that 'Some animals are centaurs': but as a matter of fact this is not true at all.

Section 714. The E and the I proposition may be converted simply, as above.

Section 715. O cannot be converted at all. From the proposition

  'If a man runs a race, he sometimes does not win it,'

it certainly does not follow that

  'If a man wins a race, he sometimes does not run it.'

Section 716. There is a common but erroneous notion that all conditional propositions are to be regarded as affirmative. Thus it has been asserted that, even when we say that 'If the night becomes cloudy, there will be no dew,' the proposition is not to be regarded as negative, on the ground that what we affirm is a relation between the cloudiness of night and the absence of dew. This is a possible, but wholly unnecessary, mode of regarding the proposition. It is precisely on a par with Hobbes's theory of the copula in a simple proposition being always affirmative. It is true that it may always be so represented at the cost of employing a negative term; and the same is the case here.

Section 717. There is no way of converting a disjunctive proposition except by reducing it to the conjunctive form.

Section 718. Permutation of Complex Propositions.

(A) If A is B, C is always D.
.'. If A is B, C is never not-D. (E)

(E) If A is B, C is never D.
.'. If A is B, C is always not-D. (A)

(I) If A is B, C is sometimes D.
.'. If A is B, C is sometimes not not-D. (O)

(O) If A is B, C is sometimes not D.
.'. If A is B, C is sometimes not-D. (I)

Section 719.

(A) If a mother loves her children, she is always kind to them.
.'. If a mother loves her children, she is never unkind to them. (E)

(E) If a man tells lies, his friends never trust him.
.'. If a man tells lies, his friends always distrust him. (A)

(I) If strangers are confident, savage dogs are sometimes friendly.
.'. If strangers are confident, savage dogs are sometimes not unfriendly. (O)

(O) If a measure is good, its author is sometimes not popular.
.'. If a measure is good, its author is sometimes unpopular. (I)

Section 720. The disjunctive proposition may be permuted as it stands without being reduced to the conjunctive form.

Either A is B or C is D.
.'. Either A is B or C is not not-D.

Either a sinner must repent or he will be damned.
.'. Either a sinner must repent or he will not be saved.

Section 721. Conversion by Negation of Complex Propositions.

(A) If A is B, C is always D.

.'. If C is not-D, A is never B. (E)

(E) If A is B, C is never D.
   .'. If C is D, A is always not-B. (A)

(I) If A is B, C is sometimes D.
   .'. If C is D, A is sometimes not not-B. (O)

(O) If A is B, C is sometimes not D.
   .'. If C is not-D, A is sometimes B. (I)

(E per acc.) If A is B, C is never D.
   .'. If C is not-D, A is sometimes B. (I)

(A per ace.) If A is B, C is always D.
   .'. If C is D, A is sometimes not not-D. (O)

Section 722.

(A) If a man is a smoker, he always drinks.
   .'. If a man is a total abstainer, he never smokes. (E)

(E) If a man merely does his duty, no one ever thanks him.
   .'. If people thank a man, he has always done more than his duty. (A)

(I) If a statesman is patriotic, he sometimes adheres to a party.
   .'. If a statesman adheres to a party, he is sometimes not unpatriotic. (O)

(O) If a book has merit, it sometimes does not sell.
   .'. If a book fails to sell, it sometimes has merit. (I)

(E per acc.) If the wind is high, rain never falls.
   .'. If rain falls, the wind is sometimes high. (I)

(A per acc.) If a thing is common, it is always cheap.
   .'. If a thing is cheap, it is sometimes not uncommon. (O)

Section 723. When applied to disjunctive propositions, the distinctive features of conversion by negation are still discernible. In each of the following forms of inference the converse differs in quality from the convertend and has the contradictory of one of the original terms (Section 515).

Section 724.

(A) Either A is B or C is always D.
   .'. Either C is D or A is never not-B. (E)

(E) Either A is B or C is never D.
   .'. Either C is not-D or A is always B. (A)

(I) Either A is B or C is sometimes D.
   .'. Either C is not-D or A is sometimes not B. (O)

(O) Either A is B or C is sometimes not D.
   .'. Either C is D or A is sometimes not-B. (I)

Section 725.

(A) Either miracles are possible or every ancient historian is
   untrustworthy.
   .'. Either ancient historians are untrustworthy or miracles are
   not impossible. (E)

(E) Either the tide must turn or the vessel can not make the port.
   .'. Either the vessel cannot make the port or the tide must
   turn. (A)

(1) Either he aims too high or the cartridges are sometimes bad.
   .'. Either the cartridges are not bad or he sometimes does not
   aim too high. (0)

(O) Either care must be taken or telegrams will sometimes not be
   correct.
   .'. Either telegrams are correct or carelessness is sometimes
   shown. (1)

Section 726. In the above examples the converse of E looks as if it had
undergone no change but the mere transposition of the
alternative. This appearance arises from mentally reading the E as an
A proposition: but, if it were so taken, the result would be its
contrapositive, and not its converse by negation.

Section 727. The converse of I is a little difficult to grasp. It becomes
easier if we reduce it to the equivalent conjunctive--

 'If the cartridges are bad, he sometimes does not aim too high.'

Here, as elsewhere, 'sometimes' must not be taken to mean more than
'it may be that.'

Section 728. Conversion by Contraposition of Complex Propositions.

As applied to conjunctive propositions conversion by contraposition assumes the following forms--

 (A) If A is B, C is always D.
   .'. If C is not-D, A is always not-B.

 (O) If A is B, C is sometimes not D.
   .'. If C is not-D, A is sometimes not not-B.

 (A) If a man is honest, he is always truthful.
   .'. If a man is untruthful, he is always dishonest.

 (O) If a man is hasty, he is sometimes not malevolent.
   .'. If a man is benevolent, he is sometimes not unhasty.

Section 729. As applied to disjunctive propositions conversion by contraposition consists simply in transposing the two alternatives.

 (A) Either A is B or C is D.
   .'. Either C is D or A is B.

For, when reduced to the conjunctive shape, the reasoning would run thus--

 If A is not B, C is D.
 .'. If C is not D, A is B.

which is the same in form as

 All not-A is B.
 .'. All not-B is A.

Similarly in the case of the O proposition

 (O) Either A is B or C is sometimes not D.
   .'. Either C is D or A is sometimes not B.

Section 730. On comparing these results with the converse by negation of each of the same propositions, A and 0, the reader will see that they differ from them, as was to be expected, only in being permuted. The validity of the inference may be tested, both here and in the case of conversion by negation, by reducing the disjunctive proposition to the conjunctive, and so to the simple form, then performing the process as

in simple propositions, and finally throwing the converse, when so obtained, back into the disjunctive form. We will show in this manner that the above is really the contrapositive of the 0 proposition.

(O) Either A is B or C is sometimes not D.

= If A is not B, C is sometimes not D.

= Some cases of A not being B are not cases of C being D. (Some A is not B.)

= Some cases of C not being D are not cases of A being B. (Some not-B is not not-A.)

= If C is not D, A is sometimes not B.

= Either C is D or A is sometimes not B.