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Eulogy of Leonhard Euler

To understand the life of a great man, who has exemplified his century by enlightening the world, is to eulogize the human spirit. He who has taken it upon himself to paint this interesting canvas will be hard pressed to perform his task if he has not added to a perfect understanding of the progress of sciences, all the necessary attributes of style that this type of eulogy requires and one which is said to be incompatible with the study of the abstract sciences. Even though one is forgiven on the one hand from the necessity to embellish his subject as it is great enough on its own, the biographer who is attached to the facts cannot dispense with the necessary obligation to arrange them tastefully, present them clearly and paint them colorfully. He must show how nature gives birth to a great man and he must deconstruct the circumstances which apply to his development and by doing so expose through the details of the literary works of the scientist for who he is constructing the eulogy, that which he has done for the sciences by not forgetting to examine his state prior to this period and to establish the point of departure.

While assuming the responsibility to present the canvas of the immortal Euler's life, I have felt all of these obligations. I have noticed that it will be all the more difficult to fulfill them with dignity, despite the enormity of my own shortcomings which are increased by the pain that Mr. Euler's death has caused me. I sense the reappearance at this moment the limitations and that an academic discourse will not allow me to complete all of the tasks of the biographer. I will therefore do nothing more than a light sketch on the life of this man, and by doing so, provide the materials and to feel sufficiently strong to write a eulogy worthy of him, and so I will be content to have placed some flowers on the grave of my dear and illustrious master.

Leonhard Euler. Professor of Mathematics, member of the Imperial Academy of Sciences of Saint Petersburg, former Director of the Royal Academy of Arts and Sciences of Prussia, Foreign associate of the Royal Academy of Science of Paris, of the Royal Society of London, etc, was born in Basel on 4/15 April 1707 of Paul Euler, then the pastor of Reihen and of Marguerite Brucker, who was of a distinguished family whose name was well recognized in the republic of letters of which there were several scientists who shared the same name.

He passed the first years of his life in Reihen, and it is within this country setting where progress and life's temptations come slowly, and was marked by his parents' lifestyle for which he became accustomed. His untrammeled character and the purity of his spirit, marked the very first moments which distinguished him for the rest of his life, which are more than likely responsible for having placed him in a position and provided him the wherewithal to have a long and brilliant career which immortalized his name.

At the first moment that his father provided him with instructions, he added mathematics, which he loved and that he had studied with the great Jakob Bernoulli. Pointing his son in the direction of an ecclesiastical vocation, he did not realize the fact that mathematics, which he taught only as an entertaining subject but that it would become in time, the object of the most serious and opinionated applications. However, the seed which he had planted in the soul of this young Geometer did not wait to grow deep roots. Even though he was too well organized to show an exclusive talent for the mathematical sciences, it was only by delivering himself wholly that his genius sensed its potential.

Happily his father no longer thought of dissuading him from the studies he himself liked so much and for which he felt so strongly the influence on the development of cognitive faculties as well as their good use in all branches of our knowledge, so as to seriously undermine his son's intentions. The early indications of Euler's budding genius had all the time to develop and he accomplished this with the determination that advertises superior talent as the precursor of his greatness.

He was sent to Basel to follow philosophy courses, and while he was there he regularly attended his professor's courses at the university and his prodigious memory allowed him to access with ease everything that was not geometry so that he might dedicate the rest of the time to his favorite subject. He possessed a predilection for mathematics and of a spirit that made the great progress and curiosity necessary so to remain avid for the ensuing classes. It did not take long before he was noticed by Johann Bernoulli the greatest of the living geometers. He soon distinguished himself from his fellow students and since Bernoulli was not able to provide all that his young mathematician asked of him, he was told him to bring all the problems that he encountered when studying and every Saturday he would help him work through them. This instilled an excellent process; but only one that can succeed with an extremely talented genius which Mr. Euler possessed. He was destined to exceed his teacher who at the time was unsurpassed in mathematics.

In 1723 Euler received his Master of Arts degree with a thesis based on a comparative analysis of Newtonian and Cartesian philosophies; and so to conform to his father's request, Leonhard Euler went on to the study of theology and oriental languages. These studies would eventually prove of importance in Euler's journey; even though they seemed unnecessary, they were soon to prove useful. Since his father granted him permission to continue in his mathematical pursuits, nothing now would divert his attention. He threw himself into these studies and redoubled his efforts. He continued to consult with Mr. Johann Bernoulli and struck up a strong friendship with his two sons Nikolaus me and Daniel I. It is this bond, based on a similarity of interests that prompted the Saint Petersburg Academy to hire him.

In Russia, Catherine I had finished the project that her husband Peter the Great had started; which was to erect an academy of sciences in his capital. The two young Bernoulli were called in 1725 and when they left Basel they promised Mr. Euler, who passionately wished to follow them, that

they would find a situation for him. When they wrote to Euler in the following year they had found a position and they advised him to apply his mathematical talents to learning physiology.

A great talent never fails. To become a physiologist, Euler needed only to apply himself. He signed up for medical studies and sat through classes with an impatience to enter into a brilliant career.

These studies far from tightening all the springs of his active yet vast mind, left him sufficiently free to compose a dissertation concerning the nature and propagation of sound as well the most efficient way in which to mast ships which the Paris Academy judged worthy of an accessit in 1727. This study is the theses that he defended when applying for the Physics Chair at the University of Basel. This allows us to see that Mr. Euler turned his attention to navigation at an early age; a subject that he enriched with many new discoveries. Happily for our Academy, fate which had as much to do in deciding municipal seats in Basel as at the university was not favorable to him and a few days after the rejection he left his country for Saint Petersburg where he found a stage more suited to the important role that he was to play in the republic of letters. His departure was the response that the Academy and his compatriots, Hermann and the Bernoulli had waited for.

He was appointed as an adjunct professor in mathematics without there being any further discussion concerning physiology. By being sincere to him, neither the influence of his father nor the little riches that it offered would he allow himself to renounce his intentions. At first he enriched the first volumes of the Commentarii with a number of mémoires which only created a rivalry between him and Daniel Bernoulli, a rivalry that always existed without altering their friendship and without plummeting into jealousy, which is unworthy in the heart of a generous person and which tarnishes the sparkle of the most wonderful virtues.

At the time when Mr. Euler entered into mathematics, nothing could be more discouraging. A mediocre talent simply could not expect to make a name for it and it was best to choose another career or to distinguish one brilliantly. The memory of the recently deceased great men that had been part of the past century and the beginning of ours was still particularly fresh in our minds. Hardly had Newton and Leibniz altered the face of geometry when they died and we had not yet forgotten the important services that the discoveries of Huyghens, Bernoulli, Moivre, Tschirnhausen, Taylor, Fermat and so many other mathematicians had provided to all the branches of mathematics.

After such a brilliant period what could Mr. Euler expect? Could he hope that Nature which is not generous with her gifts might provide him with a miracle after having placed so many mathematical heads together at one time' He felt especially inspired by Nature and what she had done for him; so much so that he entered into this career with the absolute assurance that only the knowledge of a decided inspiration can provide, and he showed to all that his predecessors had not exhausted all the riches of geometry and analysis.

Effectively, infinitary analysis was too close to its infancy and insufficiently far from the arms of its creators, that it should so much as attained any sort of perfection. Mechanics, dynamics and especially hydrodynamics and the science of the motion of heavenly bodies all experienced improvements from this new form of calculation; but all was difficulty especially when it was necessary to know perfectly what concerned the knowledge of nature and the properties of numbers,

Fermat's works which he had so successfully produced were lost and all the profound research that went with it. Artillery and navigation were reduced to vague principles created from a heap of observations, often contradictory rather than a well founded theory. The irregularities that existed in the motion of the heavenly bodies and especially the complications in the forces that influence the moon never ceased to bring despair to the geometers. Practical astronomy struggled against the imperfections of telescopes and it can be said that hardly any rules existed for their construction. At one time or another Mr. Euler turned his attention to these different subjects, he perfected integral calculus; was the inventor of a new type of calculus of sines; he simplified analytical operations; with the help of these powerful tools and the astonishing facility with which he knew to manipulate the most intractable expressions, he found a new way to spread light onto all the parts of the mathematical sciences.

Shortly after his reception at the Academy, Mr. Euler was on the verge of embracing a different position than that to which his disposition was most inclined. The death of Empress Catherine I threatened the dissolution of an institution whose newness made it vulnerable. It was looked upon as an Academy that annually cost considerable amounts of money without seeming to offer any applicable utility. A focused direction had not yet been established. One must envision an intellectual society whose purpose was to collect all useful discoveries, perfect them and disperse the findings. The academicians felt the necessity to accept the consequences of this reality and Mr. Euler decided to enter into the Navy. Admiral de Sievers, found that a man of Euler's qualities was a treasure for the fledgling navy and offered him a lieutenancy on the spot with the promise of quick advancement through the ranks.

Happily the circumstances changed in favor of the academy in 1730 when Messrs Hermann and Bulffinger left their positions to return to their respective countries, at which time Mr. Euler received the position of professor of Physics which he filled until the departure of his friend Daniel Bernoulli to whose position he was named in 1733.

The large number of mémoires that Mr. Euler had presented to the Academy point to his extraordinary fertility and underscores his abilities to deal with very difficult question as well as his ability to apply them. There is an example of this doggedness which is most striking. In 1735 when given a deadline to produce the calculations to some problems, to which other mathematicians had dragged out endlessly over months, Mr. Euler focused himself and in three days the work was done much to the surprise of the Academy. But this work was costly as it provoked a very high fever which placed him on death's doorstep. He was nursed back to health but not without having lost the sight in his right eye which was caused by an abscess during his illness. The loss of such a precious organ would have been a strong motivator to better manage ones health in order to maintain the eyesight in the remaining eye; however he was not inclined to slow down. He would give up his food before work, and now work became a perpetual habit.

The great revolution that the discovery of differential and integral calculus had provided for in all of the branches of the mathematical sciences, did not neglect to change Mechanics entirely. Newton, Bernoulli, Hermann and Euler himself had successively enriched this sublime and essential part of mixed mathematics with an infinity of new discoveries. However at this time there were no

complete works on the science of motion, with the exception of two or three that Mr. Euler felt were insufficient. He saw, as through a veil, that the philosophical principles of Newton and Hermann's Phoronomia which was the very best that had appeared on the subject, hid through synthesis, the methods by which these great men were able to enrich Mechanics with so many important discoveries. In order to unearth these discoveries he used all of the analytical resources that were within his grasp and which allowed him to answer question that those before him had dared not attempt. He aligned his discoveries with those of other mathematicians, edited them in a systematic order and the Academy published them in 1736.

The clarity of his ideas, the precision of their exposure, and they are in abundance in the Mechanics of Mr. Euler; the order of the arrangements are the qualities that any author who aspires to be a classic must attain. Obscurity and disorder are not faults for which we will label Mr. Euler, he who has found to enlighten and clarify in his deepest research. This work determined his reputation and fixed his position among the greatest of the living mathematicians. That is to say a great deal since Johann Bernoulli was still alive Barely embarked on his career, it is only a truly independent genius who bursts out so rapidly and to be sized-up next to a man resplendent in glory from so many discoveries which were done at the expenses of the English and French mathematicians who had dared to measure up to him.

I have already mentioned that from the moment of his entrance to the Academy, Mr. Euler enriched the Commentarii with a quantity of mémoires that all bear the imprint of his genius. It is there that we find remarkably the full measure of the theory of curves: tautochrones, brachistochrone, trajectories and the very deep research in integral calculus, on the nature of numbers, concerning series, the motion of heavenly bodies, the attraction of spheroid-elliptical bodies and on an infinity of subjects of which one hundredth part would suffice in making the reputation of anyone else. His superiority in analysis provided the necessary recognition, however what truly made his glory was the solution to the isoperimetric problem, so famous by its controversy between the two Bernoulli brothers, Johann and Jakob each of whom pretended to have found the solution but neither knew of it in its entirety. The sheer number and worth of all these mémoires is astonishing and one cannot imagine how a single man could have accomplished so many works, of which the detail alone is frightening.

One senses that with such an industrious man, that he would not have had the time or inclination to take part in any dissipations or liaisons that are part of a great reputation and would have been forgiven due to his age and temperament. One of the principal relaxations that Mr. Euler permitted himself was music and even then he did not abandon his geometric spirit. While delivering himself to the pleasurable sensations provided by harmony, he deepened the direction and in the middle of the chords he calculated the proportions. It can be forwarded that Euler did this for his relaxation and that during the moments when his spirit best searched for that seclusion, he composed his essay of the new theory of music which was published in 1739. A profound work filled with new ideas of those presented from a new point of view however it was not have popular success due to the reason that there was too much geometry for the musician and too much music for the geometer. However, independent of its theory and built in part on the first fundamentals established by Pythagoras, there

are a number of discoveries that the composer and the instrument manufacturer could use, and elsewhere the types and styles of music are treated and presented with the clarity and precision which characterize all of Mr. Euler's works.

While seeking out the fountainhead of pleasurable harmonies, Mr. Euler proceeded from the principle that the perception of any perfection brings forth the feeling of pleasure, and since order is one of the perfections which provides the soul with pleasant feelings, then all the pleasure that a beautiful music allows us to enjoy consists of the perception of the relationships that sounds have between themselves and relative to the length of time between the notes as in relation to the frequency of the vibrations in the air which produces them. It is this metaphysical principle, both modified and applied to the entirety of music which Mr. Euler's system is evidence.

The principle of insufficiency has been burdened long enough, and to sit is not within the power of this geometer when submitting the qualities of our souls relative to the rigors of his calculations it becomes difficult to show its value. However, if we should allow this, we become obliged to agree that it is impossible to make better use or to reason with greater subtlety or depth. Furthermore considering all of the arguments against this principle, were they intractable would only do little wrong to the work itself. It would be similar to a building perfect in all its details being built on shaky ground; one would admire the proficiency of the architect but would complain of the fact that it was not built on more solid ground.

Prior to the publication of this treatise on the theory of music, Mr. Euler had already published a treatise on Arithmetic. A number of academicians had been instructed at the request of the head of the Academy to write elementary works, and our geometer did not feel that it was a work beneath his talents but rather felt ennobled by its goal which was aimed at public education. The pleasure which he took towards all of the extra duties and the zeal with which he executed them attracted several commissions and among others to inspect the geography department that the Senate conferred upon him in 1740.

Mr. Euler had a new occasion to deploy all of the force of his genius, when the academy of Paris which had already crowned his 1738 mémoire in physics on the nature and the properties of fire; in 1740 he proposed the question of sea tides which was an extremely important question but one which demanded frightening calculations and an understanding of the entire system of the world. His piece on this subject, crowned in 1740 is a chef-d'oeuvre of analysis and geometry. He was not awarded, it is true, and the whole prize, but rather shared it with Daniel Bernoulli and Colin Maclaurin who were not exactly unworthy rivals. The Academy did not often see such brilliant competition, and perhaps received on such a short list of questions the three prize mémoires as a result of those questions. Mr. Euler's is especially remarkable by its clarity and the way in which he explains the effects of the action of the sun and the moon to the exclusion of all other forces on the sea which occurs due to the shape of the earth and how she is altered by the action of these two forces. It is full of his thoughtfulness and he sees this motion on the sea as oscillations. He compensated for the water's inertia which he was forced to consider as inconsequential at the beginning with fortunate integrations which the consideration of this reciprocal motion demanded and finally through his wisdom in the explanation of the principal phenomenon of the tides according to the theory.

If there is anything which can contribute to increase the confidence that we should experience due to the sublime research of Mr. Euler on this subject after having found them in alignment with the experiments, it is without doubt that the marvelous link between this mémoire and that of Mr. Bernoulli. Having started with quite different principles, one having adopted for example the theory of vortices which the other rejected, they arrived at the same conclusion, they even concurred on certain points, which among other was the determination of the tides beneath the polar ice caps. It is in such a way that the truth appears to spread to reveal itself to her confidants, through any which way that they seek her out.

Mr. Euler met regularly with other geometers, and in particular Mr. Bernoulli because of their mutual interest in the study of number theory. Mr. Bernoulli possessed the greater ability in physical principals combined with a patience to help in the solution of the problems which calculations brought about by experiments conducted with the utmost focus and manipulation. Mr. Euler was not susceptible to making experiments often. Sure of his instinct to know true from false and his ability to approximate after combinations and the analogies, his hypotheses were sometimes too bold. However his superiority in analysis placed him well above Mr. Bernoulli and every one else, especially when it was necessary to simplify expressions and transform them into practical applications and recognize in their final formulas, the nature of the result.

There are scientists who owe their reputations to their correspondence and there are others to whom the reward of a great reputation is their correspondence. Mr. Euler did not lack receiving letters. If there was anything to share by the scientists of the most illuminated nations, they rushed to be in contact with him. The exchange of letters that he maintained with Johann Bernoulli began in 1727, and the Nestor of mathematics did not feel that he diminished his reputation by submitting his work from his former disciple for his thoughts.

We arrive at a remarkable period in Mr. Euler's life. The many and brilliant successes of his works which had spread his name throughout Europe, attracted a proposal in 1741 on behalf of the Prussian minister Count Mardefeld. The old Royal society, founded by Leibniz appeared to take on new force through Frederick II as soon as he ascended the throne. He had already conceived of a project worthy of himself through the revival of the academy of sciences, by resuscitating the old establishment. For this reason he called Mr. Euler to his service. The precarious state of our own academy under the Regency, made the extremely advantageous proposals even more so. Mr. Euler surrendered to the invitation of the King and left St. Petersburg with his family in June of 1741 to provide the brilliance to an academy which would be born under the auspices of a crowned philosopher.

Having arrived in Berlin, he was first flattered by the welcome of the king who wrote to him from the midst of his battles from his bivouac at Reichenbach. However, he found the old Society practically lifeless. Wars, which are always devastating to the sciences, had postponed the very generous intentions of the king. In the meanwhile Euler had organized a new literary society, made up in part of members of the Royal Society and in part of others men of letters. Mr. Euler was also a member invested into the last volume of the Mélanges de Berlin, five mémoires which are perhaps the best of what is contained in this collection. They were succeeded with astonishing rapidity and filled

the gaps of the scarce research in the Mémoires which the Academy, from its re-establishment in 1744 took care to publish in a regular annual volume.

This prodigious quantity of papers, especially the ones concerned with what is most profound and deep in mathematics, were to provide an idea of the tone that ruled throughout the letters of these two men and it is certainly the case of Mr. Bernoulli who commented very early of Mr. Euler's genius, it is sufficient here to provide the end of one of the letters selected by chance amongst those of 1739:

De caetero gratissimum mihi fuit intelligere, quod ad admirationem usque Tibi placuerint quae scripsi de oscillationibus verticalibus, propter simplicitatem expressionis et insignem usum, quem praestare possunt in explicadis navium ponderibus; maluissem autem , ut ipse quoque calculum fecisses ex Tuo ingenio, quo mihi patuisset anon in ratiocinando erraverim. Nam ingenue fateor, me Tuis luminibus plus fidere quam meis. Quae uberius afferes, Vir excel de Isoperimericis, credo equidem Te omnia probe ruminasse atque ad veritatis trutinam expendisse, ita ut vix quicquam restet, quod acerrimam Tuam sagacitatem subterfugere potuerit: etc.

Always filled with new and important vistas, often filled with sublime truths and sometimes the most important discoveries, this should astonish us all the more since Mr. Euler continued to supply papers to the Saint Petersburg Academy which began the payment of a pension to him in 1742 and the Commentarii are half-filled with his amazing productivity. To watch his productions come so rapidly one over the other, one might say that the most labor intensive calculations and his most sublime meditations cost him nothing to have simply written them down. Posterity will be hard pressed to establish that the life of one man produced this body of work, the list of which is included at the end of this eulogy.

Having dealt with the importance of the isoperimetric problem, Mr. Euler recognized the usefulness that this research would have in pure analysis as well as in problems in Physics. He had remarked that all of the curved lines that these problems furnished are endowed with a type of greater or lesser and that one can find a number of them by using the isoperimetric method. He was as bold as to offer that every effect could be determined through the method of maxima or minima, which is to say by the study of efficiency, insofar as it is always possible to note the maximum and the minimum affect which nature has generated. Mr. Daniel Bernoulli had also employed this way to determine the shape of a curved elastic blade; however without recognizing the general equation for an elastic curve in his equation he did not pursue its development. He wrote to Mr. Euler of this conjecture that the trajectories that are described around one or more centers of force could be determined by the same method. Mr. Euler revisited this important topic and in 1744 published a complete treatise on isoperimetrics where we can say that he mined the riches of this sublime analysis and he established the first basis for the calculus of variations, when he considered curves which differ in infinitely small distances for a determined curve.

During the same year which was also the year of the reestablishment of the Academy and his nomination as the Director of the Mathematics Class, Mr. Euler published his theory on the motion of planets and comets, a subject which he would further enrich in the future with a number of new discoveries.

The theory of magnetism which took the Paris Academy prize in 1744 is too well known to have to say a great deal about it. Euler began with Descartes' idea that all magnetic phenomenons are elicited from the circulation of imperceptible conduits throughout the corpuscular magnetic body. Mr. Euler had imagined that the magnetic body possessed pores which formed continuous piping, parallel and bristling, similar to veins or valves and so narrow so as to only allow passage for the most subtle parts of the aether, of which the elasticity pushes the relaxed parts into the magnet's pores. Then the force causes it to bend onto itself at the exit only to return again and form a type of vortex. Thought this ingenious idea which was developed after much thought, Mr. Euler was able to explain magnetic phenomenon. The hypothesis was proved out by experiments and these conformed to natural laws which in turn ensured its ultimate probability.

During the same year the king asked Euler his advice concerning the best treatise on Artillery. In England there had appeared a work by Robins on the principles of artillery. The author had attacked Mr. Euler with a truly vulgar critique of his Mechanica which he did not understand. Mr. Euler praised this work to the King and offered to translate it while adding changes and necessary improvements. These additions contain nothing less that a complete theory of the motion of projectiles and nothing has appeared in space in the 38 years since its publications which is superior to what Mr. Euler produced in this very difficult topic of physical mathematics. Without overstatement, the value of this important work was generally recognized. The now deceased minister Turgot had the work translated into French and introduced into the French Artillery schools and at nearly the same time a translation into English appeared which has received all of the finest detail that modern typography could produce. At the same time this work contained all possible merit to the original author Mr. Robins, who Mr. Euler corrected with method and modesty and corrected the mistakes of this adversary, by providing his work with a great reputation. I will abstain from all commentary concerning the nobility of this move, so worthy of a great man. Who could possibly deny him admiration?

One is under the impression that after having found the causes of flames, gravity, electricity and magnetism in aether, and after having found what little resitance this subtle fluid offers in the restriction of the motion of celestial bodies, Mr. Euler could hardly be satisfied by Newton's explanation of the source of light. The examination of this system precedes the new theory of light and color that Mr. Euler published 1746.

In this work he shows how Mr. Newton's vacuum hypothesis is contrary to the material diffusions of the sun and the fixed stars. Since the rays, which intersect in every which way would fill space and offer an opposing resistance to the celestial bodies which the aether, for which this great man denied its existence based on this fact, and shows as well that it is impossible that material particles could move with such inconceivable speed without causing mutual trouble within their paths. He has calculated the loss of solar material and finds that within a few seconds this enormous mass will be dissipated into rays. Finally, he finds another objection as strong as the first, as the composition of transparent bodies which, to provide for a free passage to material rays, should they themselves be destitute of all matter, that is to say, ceasing to have substance.

Descartes had theorized that light came to us in the same way as sound. Effectively, on reflection it would be difficult not to recognize a marked similarity between the senses of sound and sight; that both extend themselves to further distances than our other senses and that sound and light arrive to us by straight lines and that one or the other can be bent. Mr. Euler had seized upon this similarity and followed its parallel by allowing us to see that light is born from of vibratory movement in the aether and sound is produced by a similar movement in the air. As there are differences in colors so to do those of sound depend on the frequency of vibrations and that sounds while passing through the proper bodies for its transmission, can change their direction and a type of refraction occurs as for light rays. This principle, which was derived from practical experimentation, provided all the evidence that was necessary and conclusive. Mr. Euler was in a position to explain, in the easiest terms and those closest to nature, every phenomenon of light and sight and even the different refrangibility that the Newtonian system cannot explain, gushes from Mr. Euler, that the phenomenon could be deduced a priori were they not proved by experiment.

At the same time that he assaulted the system of diffusion, Wolffian philosophy was at its zenith. All that was spoken about were monads and sufficient reason. The extension to which Wolff and his followers gave to this latter principle only provoked amusement to Euler; however the monadic system was a ingenious device for which its dismantlement would prove to be a discovery in the yes of the friends of truth and Euler was only accustomed to admitting an opinion until after having fully investigated its original principles. Euler made it apparent in his Thoughts on the Elements of Bodies, that the smallest particles could possibly be as small as anything that we could imagine by being either infinitely small or nothing at all. That the elements of matter for which the force of inertia is a general property as is space and impenetrability, cannot be endowed with the power of continuous change anymore that Epicurious's atoms, and therefore all of the conclusions concerning the diversity of these forces taken from the principle of indiscernibilty collapse into themselves. It was after having dismantled a system of ideas, which has since followed the fate of some ideas which are big without being true, that Mr. Euler substituted, for the properties that Leibniz and Wolff have attributed to monads, the force of inertia by producing this distillation of matter that Leibniz had already recognized, to the principle for all the changes which occur in this world. He employed the same principle subsequently to explain the principles of collision and pressure and he used it to prove that it is not possible to attribute the facility of thinking to matter.

This campaign against the monads attracted criticism to Mr. Euler, today it is as forgotten as is the system which tried to prevent their destruction. The topic is rarely spoken of except when an example concerning misdirection is needed to show how vulnerable the human spirit becomes when it is exposed and when it is guided only by the imagination.

In what concerns the principle of inertia, Mr. Euler allowed for all forces, the concept is comprehensive and conforms to the simplicity which natures applies to all of its laws. Even though the notion is purely metaphysical, the effects are from geometry, they can be calculated and that all we can expect from a hypothesis is that it is not contrary to the phenomena that it must explain.

This would be an appropriate spot to discuss the great number of other philosophical discoveries by Mr. Euler, where one finds with as much pleasure as admiration, that most pure physics is united

to the most sublime geometry. However, the limits of this eulogy obliges us to move silently through his research into comets tails, the aurora borealis and the zodiacal lights, on the propagation of sound and light, on space and time, on the origin of force as we have omitted the details in the same way that was necessary to omit parts of the mathematical mémoires, so as only to occupy ourselves with Mr. Euler's great works who never left the analytical heights to the physical world with out spreading the light of knowledge. Happy and fertile in the discoveries of important truths in the exact sciences, he did no less when explaining the truths in natural philosophy. Daring to offer suppositions that calculations could justify everything; he was circumspect towards hypotheses which he did not agree with. However, he made sublime and brilliant ones and the world will pronounce itself on the merits of some and posterity will state itself on the merit of others. The historian will have done his duty when he will have said what is the most important in these hypotheses.

From philosophy we return to geometry. Of all useful knowledge that the combined forces of analysis and geometry can elevate to a certain degree of perfection, navigation was the only one that had yet to enjoy the fruits of universal advancement in the physico-mathematical sciences. Hydrography and parts of navigation including nautical astronomy had been studied by geometers jointly with nautical astronomers. Unless we wish to include the imperfect essays by Huyghens and Renau concerning the speed and maneuvering of ships. Mr. Euler was the first who dared to conceive of and execute the project to make navigation a complete science. In a paper on floating bodies published in the Mémoires of the Paris Academy of Sciences and Beaux-Arts of April 1735 and forwarded to the Académie of Saint Petersburg through its author Mr. de Lacroix had initially provided Mr. Euler with the idea. His research on concerning the equilibrium of ships would provide Mr. Euler with the way in which to return stability to a determined value, and the success of this first test encouraged him to provided an in depth treatise of naval science, and he composed the great work that our academy published in 1749. Contained within it is the systematic ordering concerning the theory of equilibrium and the motion of floating bodies and that which the resistance of fluids has to offer as most difficult and sublime.

However these general principles are insufficient. In navigation, it pertains to floating bodies of determined shape. It is necessary to calculate not only the resistance and force, it is necessary to know what to diminish and what to increase as far as it is possible while still guaranteeing that the ship will receive the necessary support from the water to arch and balance, to bring about the shape which unites all possible advantages and which allows the ships to fulfill all its requisites so to be able to achieve its destination.

Furthermore, independently of what this theory can teach us concerning the construction of ships and their maneuvering in general, it is necessary that it instructs us also concerning the ways to reconcile the different properties that a well-constructed ship must possess. There are those properties which are obtained solely through sacrifice, for example the greatest stability and the most efficient course were not compatible. It is therefore of the utmost importance to know how much one must sacrifice for one advantage to succeed in obtaining the others, insofar as the utility of each type of ship requires. These are the instructions contained with the second part of Mr. Euler's work where he has assembled everything for the pilot as well as the builder could possibly hope for concerning

the perfecting of the theory. By writing this he has enriched this interesting part of mathematics with several ingenious and novel approaches. Which have been included in to the collections of the Saint Petersburg, Paris and Berlin Academies as well as principally in the two mémoires in the way in which to compensate for the wind and the effects of pitching and rolling of which the latter was awarded the prize of the Paris Academy in 1759.

Naval architecture which by the omission of informed principles had been obliged and at the mercy to routine developments and even a lengthy experience could not prevent the many mistakes in construction of ship and in their masting, saw themselves suddenly transformed by a complete theory, which other arts did not have the advantage of receiving only after successive false starts and by minor almost negligible adjustments.

However this theory was written in a language which was not familiar to the people of the trade, it presumed mathematical knowledge which could not be expected of wither the builder or the pilot. Practical application could not be successfully extracted as the fruit of Mr. Euler's important discoveries, unless a way could be found to detach the deep thoughts, the difficult and complicated research. Euler sensed the obstacle and after frequent meetings with Admiral Knowles that he had after his return to Saint Petersburg, he was determined to separate from the theory all that was non-essential to naval science and everything that was not easily grasped and in 1773 he published this complete theory concerning the construction and navigation of ships which was within the grasp of everyone who applied themselves to navigation.

Until then never had the work of a mathematician had such brilliant success. Firstly a new edition was issued in Paris and if it was introduced into the naval academies and the King rewarded Mr. Euler with a payment of 6000 pounds as which many discoveries have served France well as well as all the other enlightened nations as told by the Paris editors.

There appeared at the same time a translation in Italian, English and Russian of this exceptional work, and Mr. Euler received on the occasion of the latter a gift of 2000 rubles from our Sovereign.

We have assembled here the principal works of our geometer which are all part of the same object and even though this last part was completed well after his return to Saint Petersburg, it becomes interesting to see within a single glimpse the extent of his contribution to navigation that is to say of the most useful and applicable knowledge known to man.

In 1749 the King entrusted Euler to visit the Finow canal between the Havel and Oder rivers and to correct certain problems that existed. While reading through the fifty-four letters that the King wrote to him between 1741 and 1777, amongst which there are letters in the King's own handwriting and I have noted that more often that not his particularly brilliant solutions were used. He had provided real and immediate solutions to problems concerning the salt works at Schönebeck, the fountain pumps at Sans-Souci and various financial projects. The king often confided in the most trusted way concerning the affairs of the Berlin Academy and the University of Halle.

The time had come in which to assemble in a systematic and contained work the entire body of the important discoveries that Mr. Euler had made in infinitesimal analysis during a thirty year period and which were found spread thinly among the academic collections. Having conceived of this project it became necessary prior to its execution to prepare the world so that it might be able to understand

these sublime lessons with a preliminary work where one would find all the necessary notions that this study demands.

To this effect he prepared his Introductio to the analysis of the infinitely small into which he mined the entire doctrine of functions, either algebraic, or transcendental while showing their transformation, their resolution and their development.

He gathered together everything that he found to be useful and interesting concerning the properties of infinite series and their summations; He opened a new road in which to treat exponential quantities and he deduced the way in which to furnish a more concise and fulsome way for logarithms and their usage. He showed a new algorithm which he found for circular quantities, for which its introduction provided for an entire revolution in the science of calculations, and after having found the utility in the calculus of sine, for which he is truly the author, and the recurrent series, he provides for in the second part the general theory of curves with their divisions and sub-divisions and in a supplement the theory of solids and their surfaces while showing how their measurement leads to the equations with three variables and he ends finally this important work by developing the idea of curves with double curvature which provides for the consideration of the intersection of curved lined surfaces.

In succession to this Introductio came about his lessons in differential calculus as well as those on integral calculus published by our Academy which Mr. Euler never hesitated to regard as the legitimate owners of these important works. The foremost quality of the former work which travels concerning the part of infinitesimal calculus already perfected by its inventors Newton and Leibniz and the Bernoulli, is consistent with the point of view where Mr. Euler had already envisaged the true principles in the systematic order in which he has exposed them and with the methodology which exists and the clarity with which he has shown the utility of the calculus in relation to the doctrine of series and to the theory of the maxima and the minima. His discoveries are linked to those of the first inventors, however the traces of genius of which the essence is discovery are indelibly marked even within the subjects where these faculties could not be exercised by trying to perfect in some way the inventions of others, to provide for a better understanding and simplicity of known principles or to withdraw new knowledge. Who cannot know these characteristics within Mr. Euler's work? He is everywhere, yet the details would extend beyond the scope of this eulogy.

The first steps concerning the origins of integral calculus are lost in differential calculus but are far from the perfection that the latter attained. There are no general rules as in the decomposition of large numbers to follow. If these rules are ever found, posterity will provide Mr. Euler the reward for having prepared the way for which he alone is responsible for the great number of difficult integrations which he achieved. His glory is due to the fact that he pushed back the limits of this sublime calculus far beyond the reach of the primary discoverers. If Newton could come back he would be surprised at the extreme hurdles that this astonishing man conquered.

The third volume of his Integral Calculus contains a new form of calculus which has enriched infinitesimal analysis: the calculus of variations. I have already mentioned that the isoperimetric problem had provided Euler with the first notion. It was eventually seized by Mr. de la Grange, the very worthy successor to Mr. Euler at the Berlin Academy and he was eventually able to disengage

the calculus of variations from its geometrical origins and made it into a problem of analysis and made it possible to resolve problems through this new genre of calculus that Mr. Euler has since perfected he named it the calculus of variations since the relations between the variable quantities is in itself variable.

We have already noted that Mr. Euler's genius was too extensive to be contained within the boundaries of mathematics. He felt that everything which possessed a relation was within his capacity to be measured and to be submitted to calculations. We will now see how closely physics, optics and astronomy owe to his theories of light and colors.

The examination of Newtonian theory provided Euler with the occasion to conduct research concerning the different refrangibility of light rays and the poor effect that the dispersion of colors produced in refractive telescopes. These telescopes were almost entirely abandoned due to this defect. Considering the marvelous structure of the eye, allowed Mr. Euler to imagine a certain combination of various transparent bodies which would remedy this defect. In 1747 he proposed lenses composed of two lenses and the cavity between them was filled with water.

This theory was criticized by Dolland, the famous English optician who stated Newton's authority in these matters. However, Euler did not take long to indicate the incorrectness of these principles. Having conducted some experiments concerning the meniscus whose cavity could be filled with different liquids, confirmed his conclusions and Mr. Dolland who had found two different types of glass in the meanwhile which were capable of examining in greater detail finally admitted to Euler's conjecture in 1757 by the invention of achromatic glasses which made considerable impression in both astronomy and dioptics.

Mr. Dolland's success which so substantially borrowed from a theory that he had first criticized, prompted Mr. Euler to push his research further into the field of dioptic instrumentation and to remedy the defects which are present due to the aberration of light rays brought about as the result of the spherical shape of the glass, and finally provided general rules for the manufacture of telescopes and microscopes. He was entirely convinced of its correctness due to the experiments and by producing glasses according to his theory.

It is due to this controversy with Dolland that we were rewarded with one of the most important discoveries found this century. It has provided astronomers with a very great service by allowing them to see the sky full of new phenomena and by facilitating the work of observations sufficient to explain it. Messrs Euler and d'Alembert who had delved into this difficult material and mined all that its analysis could provide both profoundly and sublimely made it possible to see that Mr. Bernoulli's solution taken from the Taylorian troichoides is not general and certainly not sufficient to explain it.

This controversy which continued for a long time with all of the necessary hard work that was necessarily expended by these great men, gave birth to a number of great mémoires and did not end until Mr. Bernoulli's death.

Another controversy which did not last as long but one which possessed all the more bitterness was one which in 1751 Mr. Koenig attacked Mr. de Maupertuis' priority of the principle of least action. However, since this controversy was not part of a discovery made by Mr. Euler himself, it is

sufficient to mention on his behalf and benefit that he passionately took the side of his friend Mr. de Maupertuis and that a number of excellent mémoires issued from the hand of he who never made any others and have their origins from this dispute.

The solution to the important problem regarding the precession of the equinox and the nutation of the earth's axis that Mr. d'Alembert had been the first to solve, promoted Mr. Euler to publish his research on these matters in the fifth volume of the Berlin mémoires which is the same volume where there appears the felicitous unraveling of the controversy between Leibniz and Bernoulli concerning the logarithms of negative and imaginary numbers.

The problem concerning the precession of the equinoxes engaged Mr. Euler in the investigation into the motion of rotating solid bodies. Insofar as the rotating axis is variable motion in which mechanical principles were not fully sufficient. It was necessary to begin with the problem concerning the precession of the equinoxes which engaged Mr. Euler in the investigation into the motion of rotating solid bodies.

Insofar as the rotating axis is variable, the theory of motion in which mechanical principles were apparent was not fully sufficient. It was necessary to begin with the first principles of motion, and to see whether it would be possible to deduce general rules concerning the determination of motion of a solid body whose rotating axis is mobile.

He accomplished this and discovered a new mechanical principle, with which he was able to treat, in all its generality, the problem involved in the motion of solid bodies.

This research coalesced to produce a new view on the entire science of motion which deserved to be exposed in its entire expanse. In his great work on Mechanics, Mr. Euler only treated the motion of infinitely small bodies, however he reserved the most difficult yet essential section, that of the motion of solid bodies for a separate work which appeared in 1765 and which may be regarded as the complete treatise on Mechanics, since it contains as a form of introduction, the entire principle concerning the motion of mass points, treated in a new fashion and preferable to that which the author had followed previously. Following these principles one finds all of the important discoveries which he had done on the motion of solid bodies. It was through these discoveries which placed him in a position to bring the degree of perfection to the theory of motion of celestial bodies and through that to provide such great services to astronomy and to navigation.

During his entire stay in Berlin, Mr. Euler never ceased to provide very important services to the Saint Petersburg Imperial Academy, either by offering the greater and most important part of his literary works, by looking after its economic interests or by assuming the education of his students.

He never stopped his affiliation to all of these descriptions and it must be known that even the Russian court and army thought of him by awarding him safeguards and a thorough indemnification of all his losses that he suffered during the last war at his country home during the bivouacking of the Russian troops in Berlin.

With a marked predilection for the country where he spent his first years as an adult and to the body from where he saw his celebrity grow, Mr. Euler nourished the desire to return. With the accession of Catherine the Great to the Russian throne, the brilliance of her reign as wise as it was sweet was supportive and had filled the world with universal admiration, and the protection

which she awarded to the sciences and to those who cultivated them provided a new impetus to the Academy which contributed to reassert Mr. Euler in his resolution to end his days at the service of this incomparable princess born to make the happiness of her subjects and the adoration of the universe.

During the month of May 1766 his wishes were nearly granted. Prince Dolgoruky, the Russian minister in Berlin granted him, according to the wishes of the Empress, all of the conditions that he had requested for himself and his family which were extremely advantageous. It was not without extremely difficult proceedings that he obtained his leave and those of his two sons from Berlin. The King refused to discharge the youngest to accompany his father.

The following June, Mr. Euler left Berlin where for 25 years he enjoyed the consideration of his eminent merit. The Princes of the Royal household and particularly the Margraf of Brandenburg-Schwendt regretted his departure and the way in which they displayed this was extremely flattering.

On the day prior to his departure, Prince Adam Czartorinsky invited Euler by the name of the King of Poland to take the road to Warsaw where he spent ten days with all the amenities of a generous prince onto whom this philosopher knew to enjoy without seeking them out.

Once again he saw Saint Petersburg, after his long absence, 17 July 1766. He was immediately presented to Her Imperial Majesty and his first wish to be granted from the Empress was the release of his youngest son which was easily obtained by such a potent intermediary.

Hardly had he moved into his new home, for which Her Imperial Majesty presented him with a gift of 8000 rubles, that he fell gravely ill, from which he recovered, except that he lost his sight. A cataract which had formed on the left eye deprived him entirely of the sight which too much strain had spoiled.

What misfortune for a man to who work had become a need, for which his spirit always on edge with some new discovery, finds itself all of a sudden unable to pursue his works. It might have been anyone except Mr. Euler. Now his prodigious and astonishing memory, enforced by the concentration of all of its forces disengaged from external distraction supplemented what might have been a reason to end the literary career of such a great man.

A young tailor who he had brought from Berlin as a servant and who possessed no talents for mathematics, became the scribe to whom Euler dictated his Elements of Algebra, generally admired as much for the circumstances under which they were composed, as well as for the supreme degree of clarity and method throughout. His spirit of invention traces itself throughout this purely elementary work. One unique feature is that there is a continuously threaded theory of Diophantine analysis and soon after the edition appeared in Russian and French.

The arrival of Mr. Krafft placed Euler in a position to execute a project that he had tossed around in his head; to combine in one unique body of work that which he had accomplished in the space of thirty years, which was the perfection of the theory of optical instrumentation. He applied himself with his usual industriousness and published three great volumes on Dioptics in 1769, 1770, and 1771.

The first volume contained the general theory of this new science since one cannot say that it existed before Mr. Euler. The excessive length that was necessary to provide for glasses prior to the discovery of composed objective and the confusion of images had obliged the astronomers to abandon these entirely telescopes and to limit themselves to the use of reflexive telescopes. It was extremely

complicated to calculate the construction of these glasses and even though this problem should be categorized as a problem in elementary geometry and that it only requires a little knowledge of infinitesimal analysis, the progress had not been made. It is only since Mr. Euler had started to study these matters that this science made progress. The second and third volumes of his work contain the rules for the best process to manufacture glasses, catoptrical telescopes and microscopes. The calculation concerning light ray aberrations, brought about due to the sphericty of the glass, is a masterpiece of analysis of the highest order. One is obliged to admire the great art with which Mr. Euler has known to utilize this analysis to reconcile all of the possible advantages for all types of instruments; those concerning image clarity, the greatest field of vision, the shortest possible length concerning all of the magnifications and for the number of eyecups that are to be used. All types of optical instruments are examined and calculated in this work with the greatest simplicity that had not been seen in similar works either by the complication of the calculations or due to the number of elements which are included.

During the same time that the Academy was publishing this work its presses were occupied in printing the Letters to a Princess of Germany, Integral Calculus, Elements of Algebra, the calculation concerning the comet of 1769, the sun's eclipse and the passage of Venus, all in the same year, the new lunar theory and that of navigation, not withstanding the huge number of mémoires which are found in the Commentarii of this period.

Hardly had the first of these works appeared that the first translation was completed by M. Rumovsky. A new edition came out in Paris and a German translation in Leipzig. As for its contents, it is sufficient to say that as it is within the grasp of a general readership and even to the reach of the gentle sex, it certainly contributed to spreading the name of its illustrious author and to make him worthy to those who could judge him only based on the Letters of a Princess of Germany.

The year 1769 will forever be distinguished in the history of the progress of the sciences by the felicitous competition of those on earth who encouraged the astronomers to be in a position to take advantage of the passage of Venus across the sun's disk. The Russian Empress, the Kings of France, England and Spain sent their astronomers to all the corners of the world to observe this phenomenon, so rare and so important to aid in establishing the dimensions of the solar system. There were ten astronomers animated by the glory of taking part in this event and encouraged by the protection of our august Sovereign, dispersed themselves throughout the vast Russian empire while Mr. Euler thought of new ways to interpret the various parts of their observations to determine the true parallax of the sun, and consequentially the distances from all the planets. He found a particularly elegant one to calculate, not only the observations of passage, but also those of the eclipse of the sun which followed soon after the first phenomenon and from which it was able to determine the geographic location from where the best observations could be taken. The calculations of all these observations was completed by Mr. Lexell from Euler's method, as a result it can be said that once again it is to Mr. Euler that astronomy is grateful due to the perfection that she evinced from the exact determination of the sun's parallax.

The research on the moon had occupied a considerable part of his time. He had already published the 1746 lunar tables and in 1753 they were accompanied by a theory of motions which the late Mr.

Mayer was able to use and which are still in usage by today's astronomers and which allowed him to win the longitude prize. The English parliament at the same time paid Mr. Euler a gratification of 300 pounds to reward Mr. Euler for having provided Mr. Mayer with the theories with which he was able to contribute to the important question of longitude.

Furthermore, the Academy of Paris which, since it provided Mr. Euler with his associate membership, had crowned three of his mémoires concerning the inequalities in the motion of the planets, chosen as the subject for its prize of 1770 and 1772, the perfection of lunar motion and Mr. Euler with help from his son who had already shared the 1761 prize on the masting of ships, were rewarded with the one and the other.

In his last mémoire he had found a solution as to how to understand the number of eccentricities in lunar motion, which he had not been able to determine in his first theory due to the complicated calculations and the incomplete method that was available to him at that time. He had the courage to recast the entire theory with Messrs J.A. Euler, Krafft and Lexell and to follow through with his research until he completed the reconstruction of new tables which appeared jointly with the great work of 1772. Instead of being stopped in his tracks as in the past due to the inability to integrate the three differential equations of the second degree that the mechanical principles furnish, he regrouped them at first into the three coordinates which determine the location of the moon, he then distributed all the inequalities of the moon into separate classes based on whether they depended either on the mean length of the sun from the moon or concerning the eccentricity or the parallax or the inclination of the lunar orbit. All these methods, artfully executed and accompanied with full effect the calculations, that the world's premier analyst was capable of imagining, exceeded beyond all expectations. One is astonished at the vastness of the calculations and the wealth of the resources used to shorten and facilitate their application to true lunar motion.

The patient and focused mindset that this enormous work required, will surprise us even more when we are reminded of the circumstances and at what time this work was done. Deprived of his sight and obligated to make most of the immense calculations by the sole intermediary of his memory and imagination, his household in disarray due to a fire which consumed a great deal of his family's possessions, reduced to the necessity to leave a house in ruins where very nook and cranny was known to him, where the habit of his movements had supplanted the necessity of sight, burdened by the troubles that such sad and sudden upheavals that the re-establishment of a new home would occasion. Mr. Euler was still in a state to compose a work that of its self would immortalize him, as though it had been done in completely opposite circumstances.

I am not aware of any strength, of nothing that better represents the heroic self than this spiritual equanimity and unshakeable courage in the middle of such a reversal of fortune.

A few months after this unfortunate accident, Her Imperial Majesty's generosity lightened the weight of this burden with a present of 6000 rubles. Mr. Euler had a cataract operation by Baron de Wentzel the famous oculist and this operation returned his sight to him and to his entire family's relief. However this joy was short lived since he neglected the necessary precautions, by rushing his

recovery and lost his sight after terrible pain and suffering which may well have been mitigated had he learned to manage the use of his sight.

He was reduced once again to the necessity of using the sight of others. His sons, the professor and the lieutenant-colonel and Messrs Krafft and Lexell continued to lend theirs alternatively either in the execution of his great works or for the great number of mémoires that are found in the last volumes of the Novi Commentarii, to which I dare not speak for fear of abusing the patience of this assembly.

I will stop for a moment to speak of equilibrium and the motion of fluids and the ultimate improvement of achromatic lenses.

Since Daniel Bernoulli's Hydrodynamica, the perfection of calculus which in Mr. Euler's hands became enriched from day to day and even more applicable to the most difficult questions in the realm of physical mathematics which so improved that it was only a matter of time to see it applied to this essential part of the science of motion. Mr. Euler made the wait worthwhile in the four great mémoires on equilibrium and fluid motion which plumbs all that the complete theory of hydrodynamics could have as profound and abstract.

This theory is infinitely fertile with its applications of general principles and contains very satisfactory explanations of a considerable number of natural phenomena. In considering, for example the disturbance to the equilibrium of the air, produced by the differences in heat density, Mr. Euler then goes about to explain the general cause of the winds and in particular the monsoons or the periodic winds of India. While considering the equilibrium of fluids attracted to one or more centers of force, he determines the shape of the earth and the state of equilibrium of the fluids which surround it, which brought about the explanation of the phenomenon of the tides. After having dealt with the state of equilibrium, he established the way in which to reduce the theory of fluid motion to two second degree differential equations and he applied the general principles to water motion in containers, pumps and to pipes of equal and unequal size. The research that he conducted into air motion finally led him to the theory of the propagation of sound and to that of the sound from flutes.

Such are the various and sundry subjects to which he gave depth in his theory of hydrodynamics. If one has been able to write on this thorny issue of mixed mathematics, it is because what Mr. Euler has contributed is so superior to that which we had that it would be preferable that it be removed from the Commentarii and that it be made into a separate work for those who would wish to study this essential part of mechanics.

While composing his work in Dioptics, Mr. Euler had neglected to mention the distance that must exist between the lenses in his theory of perfect lenses which only added to the confusion that these lenses are supposed to correct since these lenses must be of a certain thickness which cannot be neglected in their calculation. The Mémoires concerning the lenses and their application to all types of glasses have been inserted into volume XVIII of the Novi Commentarii and have the purpose to correct this deficiency. Within this work is found the expression of the ways in which to make these instruments of shorter length with a greater field of vision, advantages which were impossible to provide for in the glasses prior to the last adjustments to the requisite calculations. It is with these principles contained within the mémoires that Mr. Euler had me calculate, as part of the addition, the necessary instructions

for the optical manufacturers that the Academy published in 1774 and one to which the German translation is found in the proceedings to the Dioptrica made by Mr. Klugel of Helmstedt.

The general fault which is involved with various death benefits established in Germany and the complaints that were leveled at the tontines in being too favorable to either the brokers or the invested parties, allowed for Mr. Euler to predict what these investments would do based on the assessment of the most accurate information gleaned from imperfect mortuary tables. These researches allowed for the necessary clarity concerning the death benefits for widows which appeared in 1776. Everything that the calculations of probability can provide is found in this important topic.

Mr. Euler had told Count Orlov on more than one occasion that he would provide the Academy with sufficient mémoires to fill the Acta for at least twenty years after his death. He was a man to keep his promise. His blindness and infirmities which come with old age, the extraordinary number of his discoveries could not weaken his passion for work, destroy the efficiency of his organization could exhaust his fertile genius. In the space of seven years, Mr. Golovine more than seventy mémoires that Mr. Golovine worked on more than 70 mémoires and there were over 250 others for which I had done the calculations.

The oldest of these mémoires was separated from the rest and have been published as a collection under the title of the Opuscules analytiques which has appeared this year.

Amongst this great number of mémoires, there is not one that does not contain some new discovery or some ingenious view that points in that direction. One finds the most felicitous integrations and a multitude of contrivances and refinements of the most sublime analysis, truly deep research on the nature and the properties of numbers, the ingenious proofs of a numbers of Fermat's theorems, the solution to a number of very difficult problems concerning equilibrium and the motion of solid bodies both flexible and elastic and the unraveling of a number of apparent paradoxes. Everything that the theory of the motion of celestial bodies and their mutual attraction and eccentricities, which is most abstract and thorny, is contained within, they are as close to perfection as the manipulations that the greatest geometer could contribute to this perfection. There is not one branch of the mathematical sciences that is not grateful to him in this regard.

Such are the works of Mr. Euler and such are his claims to immortality. It is sure that his name will not perish except with the sciences themselves. Placed in to posterity with the illustrious names of Descartes, Galileo, Leibniz, and Newton and with all the rest of the great persons who have honored humanity with their genius and his name will live well after all those personalities and the frivolity of our century will be forever buried in the night of eternal forgetfulness.

Few scientists have written as much as Mr. Euler and no geometer has ever encompassed as many topics at the same time, none has equaled him, either the number or in the richness of the variety of his discoveries.

Upon reflection of all the good that those men who are born to extend the limits of our knowledge can provide to humanity whilst considering how rare these talents are to which nature has appeared to reserved the rights to illuminate the world, one cannot but hope that they are exempt from the general law that human nature submits to sooner or later where they have gone where

none other has carried their careers. Finally, Mr. Euler's career was one that was very long and very distinguished, and we have been consoled by seeing that he was exempt from the ordinary due to an unprecedented application, this passion of being which he maintained to the very end and which distinguished him his entire life and which we discover still through his last works.

During the first days of last September, he seemed inconvenienced with a bout of vertigo, which did not stop him from calculating the motion of aerostatic balloons, which from the little information gleaned from the public papers he was able to arrive at a very complicated integration from the calculation. This vertigo was the advance warnings of his death which came on September 7. On that very same day while at the dinner table he spoke of the discovery of the new planet with Mr. Lexell who had come to see him. He spoke of other topics with his usual incisive intellect, and he had been playing with one of his grandsons when he was struck with apoplexy while having a cup of tea. His last words were, "I am dying" and he lapsed into unconsciousness. He died a few hours later at the age of 76 years, 5 months and 3 days.

The dean of our Academy had died, who for fifty-six years had been its glory and ornament. He witnessed the birth and growth of this Academy; he saw it on the verge of collapse and alternately its rebirth. So great was the influence of this illustrious member on the academic works, that despite what he did for her during his stay in Berlin, there are telltale signs of the period of his departure and return, as though his presence alone was enough to bring it back to life. He had the ultimate consolation to see before his death the dawning of the beautiful days under the wise and enlightened direction of her Excellency, Princess Dashkova who had made this possible and his satisfaction was proportionate to the affection that he always had for this Academy.

Mr. Euler was in possession of a strong and lasting constitution. After the numerous aliments that his health would have suffered due to the severity of his illnesses, he certainly would have succumbed earlier to the effects of an excess of work had he not be born with a robust constitution.

His last days were calm and peaceful. With the exception of some illnesses which come with old age; he enjoyed a robust health which placed him in a position that allowed him to give that to his studies what usually old age provides to rest. By doing so he consecrated the rest of his life to those studies which he had dedicated as a life to science. He enjoyed the glory as the fruit of his genius and the esteem of the public as the fruit of his virtues and the sweetness that was his reward in the bosom of his family.

His possessed what is called erudition to a very important degree. All that is left to us by the great writer of ancient Rome, he read; old mathematical manuscripts were perfectly known to him; the history of all the ages and nations were found in his head and he knew to quote whatever he knew without mistakes. He was familiar with medicine, botany and chemistry; he knew much more than was expected of a scientist whose science was other than the ones he knew so well.

I encountered foreigners who were attracted to him because of his fame, and much more than his celebrity, the fact that the public appeal due to the many virtues which do not often accompany literary merit. I saw these people leave with surprise mixed with admiration. They were unable to understand how a man who for more than half a century, who had been preoccupied in publishing his discoveries in physics and mathematics could have retained so many facts, much of which was

unimportant and useless to the pursuit of his studies. It was the effect of a prodigious memory which loses nothing from a lecture he has heard, and Euler was capable of reciting the Aeneid without interruption from beginning to end and to indicated the beginning and ending line of each page of the edition, and this was also something which he retained in his older years.

It is more than likely the same place that this mental suppleness came from which make us take on unnoticeably the accent from where we are living and to lose that from whence we came. Mr. Euler always possessed his Swiss accent. It amused him to remind me of certain provincial expressions, and idiomatic inversions common to our way of speaking or to use in his conversations words that I had forgotten their meaning and use.

Nothing was equal however; to his ability to drop his calculations and deep thoughts without the slightest hint of displeasure and after having involved himself in the frivolity of ordinary conversation take back up where he left off. The ability to disengage his scientific profile, to disguise his superiority and to place himself at the common level is rare but one that Mr. Euler, it should be noted, possessed. His mood was always on an even-keel, a sweet, natural happiness, a good-natured sarcasm; a story-teller both innocent and simple made his conversation pleasant and coveted.

The great depth of his liveliness that he always possessed and without which this great activity that we have grown to admire could not have existed, sometimes got the best of him. He sparked easily, but his anger passed as quickly as it appeared and he never kept a grudge against anyone. He was righteous and his correctness was irreproachable.

Sworn enemy to all injustice and if he noticed some infraction committed somewhere, he had the frankness to censure and the courage to attack openly irrespective of who it was. Recent examples of what I have just said are still fresh in everyone's mind.

He was entirely imbued with respect for religion and his piety was sincere and his devotion was full of fervor. He fulfilled with the greatest detail all the duties of a Christian. He loved everyone, and if he felt stirrings of indignations it was against those enemies of religion, especially against the declared apostles of atheism that he made a stand in the defense of the Revelation against the objections of atheists in a work which was published in Berlin in 1747.

He was a good husband, good friend, good citizen and loyal in all of his relations to society. Everything points us in justifying our regrets and to prove to the world how much our pain in having lost him is legitimate.

Mr. Euler married twice. In 1733 he married Mlle Catherine Gsell, the daughter of a Swiss painter that Peter I had taken into his service in Holland and a sister to the famous president de Loen. The care of his household obligated him to remarry after the death of his wife his choice for his second spouse fell on Mlle Salome Abigail Gsell in 1776 the sister-in-law of his first wife, the daughter of Marie Graff and the grand-daughter of Sybille Merian, each known for their drawings of insects from Surinam.

Of the thirteen children from his first marriage, eight died at an early age and of the three sons and two daughters who followed him from Berlin, there were only the sons who survived him. The eldest who has for a very long time followed in the footsteps of his illustrious father, is equally

famous due to the prizes that he was awarded by the Academies of Saint Petersburg, Paris, Munich and Göttingen.

His second son, doctor to the Court of Her Imperial Majesty and College councilor enjoys a reputation justly deserved as much by his knowledge as by the zeal that he places in the exercise of his salutary art. The youngest is a lieutenant-colonel in the artillery and director of the armaments factory at Sisterbeck and is also known to the science community due to his astronomical observations, having been one of those sent from the Academy to observe the transit of Venus. The eldest daughter died in 1781 having been married to Mr. de Bell, Major of the Chief of Staff and the youngest was married to Mr. le baron de Dehlen and died on her property in the Duchy of Juliers in 1780. These five children gave him thirty-eight grand-children, of which twenty-six are still alive.

I know of no greater heart-rending spectacles than the ones that I enjoyed so many times while watching this venerable old patriarch encircled by his numerous family, thrilled only to make his golden years happy and to comfort him in his last days with all sorts of cares and attentions.

Ladies and Gentlemen I have attempted in vain to paint touching scenes of domestic happiness which many among you and myself included have been witnesses. Especially you, Gentlemen who can glory in the fact that he was your Master. Here are five of us, and is there a scientist who could boast of having united in the same assembly as many disciples. What is there left to witness to the world except our tender and eternal gratitude and to prove by that which I have never been able to express but fleetingly in this eulogy that our illustrious Master was worthy of admiration as much for his rare virtues as for the astonishing force of his genius. Weep for him and the sciences that have progressed so far and with the Academy which has never suffered such a great loss and with his family of which he was their honor and support. My tears will mix with yours and the memories of the goodness which he did for me in particular will never be erased from my memory.

O my dear friends and colleagues how I have seen, by this brief stroke of the pen written by the heart, the warm tears! I have only been able to touch your hand after the pain stifled my voice, but I will never forget the memory of the mark of your sincere affliction and I hereby give public notice of the sensitivity of spirit and love that you have shown on this occasion to our dear and incomparable Master.