CHAPTER V

THE CLASSIFICATION OF PROPOSITIONS

Section 1. Logicians classify Propositions according to Quantity, Quality, Relation and Modality.

As to Quantity, propositions are either Universal or Particular; that is to say, the predicate is affirmed or denied either of the whole subject or of a part of it—of *All* or of *Some S*.

All S is P (that is, *P* is predicated of *all S*).
Some S is P (that is, *P* is predicated of *some S*).

An Universal Proposition may have for its subject a singular term, a collective, a general term distributed, or an abstract term.

(1) A proposition having a singular term for its subject, as *The Queen has gone to France*, is called a Singular Proposition; and some Logicians regard this as a third species of proposition with respect to quantity, distinct from the Universal and Particular; but that is needless.

(2) A collective term may be the subject, as *The Black Watch is ordered to India*. In this case, as well as in singular propositions, a predication is made concerning the whole subject as a whole.

(3) The subject may be a general term taken in its full denotation, as *All apes are sagacious*; and in this case a Predication is made concerning the whole subject distributively; that is, of each and everything the subject stands for.

(4) Propositions whose subjects are abstract terms, though they may seem to be formally Singular, are really as to their meaning distributive Universals; since whatever is true of a quality is true of whatever thing has that quality so far as that quality is concerned. *Truth will prevail* means that *All true propositions are accepted at last* (by sheer force of being true, in spite of interests, prejudices, ignorance and indifference). To bear this in mind may make one cautious in the use of abstract terms.

In the above paragraphs a distinction is implied between

Singular and Distributive Universals; but, technically, every term, whether subject or predicate, when taken in its full denotation (or universally), is said to be 'distributed,' although this word, in its ordinary sense, would be directly applicable only to general terms. In the above examples, then, 'Queen,' 'Black Watch,' 'apes,' and 'truth' are all distributed terms. Indeed, a simple definition of the Universal Proposition is 'one whose subject is distributed.'

A Particular Proposition is one that has a general term for its subject, whilst its predicate is not affirmed or denied of everything the subject denotes; in other words, it is one whose subject is not distributed: as *Some lions inhabit Africa*.

In ordinary discourse it is not always explicitly stated whether predication is universal or particular; it would be very natural to say *Lions inhabit Africa*, leaving it, as far as the words go, uncertain whether we mean *all* or *some* lions. Propositions whose quantity is thus left indefinite are technically called 'preindesignate,' their quantity not being stated or designated by any introductory expression; whilst propositions whose quantity is expressed, as *All foundling-hospitals have a high death-rate*, or *Some wine is made from grapes*, are said to be 'predesignate.' Now, the rule is that preindesignate propositions are, for logical purposes, to be treated as particular; since it is an obvious precaution of the science of proof, in any practical application, *not to go beyond the evidence*. Still, the rule may be relaxed if the universal quantity of a preindesignate proposition is well known or admitted, as in *Planets shine with reflected light*—understood of the planets of our solar system at the present time. Again, such a proposition as *Man is the paragon of animals* is not a preindesignate, but an abstract proposition; the subject being elliptical for *Man according to his proper nature*; and the translation of it into a predesignate proposition is not *All men are paragons*; nor can *Some men* be sufficient, since an abstract can only be adequately rendered by a distributed term; but we must say, *All men who approach the ideal*. Universal real propositions, true without qualification, are very scarce; and we often substitute for them *general* propositions, saying perhaps—*generally, though not universally, S is P*. Such general propositions are, in strictness, particular; and the logical rules concerning universals cannot be applied to them without careful scrutiny of the facts.

The marks or predesignations of Quantity commonly used in Logic are: for Universals, *All*, *Any*, *Every*, *Whatever* (in the negative *No* or *No one*, see next Section); for Particulars, *Some*.

Now *Some*, technically used, does not mean *Some only,* but *Some at least* (it may be one, or more, or all). If it meant '*Some only*,' every particular proposition would be an exclusive exponible (chap. ii, section 3); since *Only some men are wise* implies that *Some men are not wise*. Besides, it may often happen in an investigation that all the instances we have observed come under a certain rule, though we do not yet feel justified in regarding the rule as universal; and this situation is exactly met by the expression *Some* (*it may be all*).

The words *Many*, *Most*, *Few* are generally interpreted to mean *Some*; but as *Most* signifies that exceptions are known, and *Few* that the exceptions are the more numerous, propositions thus predesignate are in fact exponibles, mounting to *Some are* and *Some are not*. If to work with both forms be too cumbrous, so that we must choose one, apparently *Few are* should be treated as *Some are not*. The scientific course to adopt with propositions predesignate by *Most* or *Few*, is to collect statistics and determine the percentage; thus, *Few men are wise*—say 2 per cent.

The Quantity of a proposition, then, is usually determined entirely by the quantity of the subject, whether *all* or *some*. Still, the quantity of the predicate is often an important consideration; and though in ordinary usage the predicate is seldom predesignate, Logicians agree that in every Negative Proposition (see Section 2) the predicate is 'distributed,' that is to say, is denied altogether of the subject, and that this is involved in the form of denial. To say *Some men are not brave*, is to declare that the quality for which men may be called brave is not found in any of the *Some men* referred to: and to say *No men are proof against flattery*, cuts off the being 'proof against flattery' entirely from the list of human attributes. On the other hand, every Affirmative Proposition is regarded as having an undistributed predicate; that is to say, its predicate is not affirmed exclusively of the subject. *Some men are wise* does not mean that 'wise' cannot be predicated of any other beings; it is equivalent to *Some men are wise* (*whoever else may be*). And *All elephants are sagacious* does not limit sagacity to

elephants: regarding 'sagacious' as possibly denoting many animals of many species that exhibit the quality, this proposition is equivalent to '*All elephants are* some *sagacious animals*.' The affirmative predication of a quality does not imply exclusive possession of it as denial implies its complete absence; and, therefore, to regard the predicate of an affirmative proposition as distributed would be to go beyond the evidence and to take for granted what had never been alleged.

Some Logicians, seeing that the quantity of predicates, though not distinctly expressed, is recognised, and holding that it is the part of Logic "to make explicit in language whatever is implicit in thought," have proposed to exhibit the quantity of predicates by predesignation, thus: 'Some men are *some* wise (beings)'; 'some men are not *any* brave (beings)'; *etc*. This is called the Quantification of the Predicate, and leads to some modifications of Deductive Logic which will be referred to hereafter. (See Section 5; chap. vii. Section 4, and chap. viii. Section 3.)

Section 2. As to Quality, Propositions are either Affirmative or Negative. An Affirmative Proposition is, formally, one whose copula is affirmative (or, has no negative sign), as *S—is—P, All men—are—partial to themselves.* A Negative Proposition is one whose copula is negative (or, has a negative sign), as *S—is not—P, Some men—are not—proof against flattery*. When, indeed, a Negative Proposition is of Universal Quantity, it is stated thus: *No S is P, No men are proof against flattery*; but, in this case, the detachment of the negative sign from the copula and its association with the subject is merely an accident of our idiom; the proposition is the same as *All men—are not—proof against flattery*. It must be distinguished, therefore, from such an expression as *Not every man is proof against flattery*; for here the negative sign really restricts the subject; so that the meaning is—*Some men at most* (it may be *none) are proof against flattery*; and thus the proposition is Particular, and is rendered—*Some men—are not—proof against flattery*.

When the negative sign is associated with the predicate, so as to make this an Infinite Term (chap. iv. Section 8), the proposition is called an Infinite Proposition, as *S is not-P* (or *p), All men are—incapable of resisting flattery*, or *are—not-proof against*

flattery.

Infinite propositions, when the copula is affirmative, are formally, themselves affirmative, although their force is chiefly negative; for, as the last example shows, the difference between an infinite and a negative proposition may depend upon a hyphen. It has been proposed, indeed, with a view to superficial simplification, to turn all Negatives into Infinites, and thus render all propositions Affirmative in Quality. But although every proposition both affirms and denies something according to the aspect in which you regard it (as *Snow is white* denies that it is any other colour, and *Snow is not blue* affirms that it is some other colour), yet there is a great difference between the definite affirmation of a genuine affirmative and the vague affirmation of a negative or infinite; so that materially an affirmative infinite is the same as a negative.

Generally Mill's remark is true, that affirmation and denial stand for distinctions of fact that cannot be got rid of by manipulation of words. Whether granite sinks in water, or not; whether the rook lives a hundred years, or not; whether a man has a hundred dollars in his pocket, or not; whether human bones have ever been found in Pliocene strata, or not; such alternatives require distinct forms of expression. At the same time, it may be granted that many facts admit of being stated with nearly equal propriety in either Quality, as *No man is proof against flattery*, or *All men are open to flattery*.

But whatever advantage there is in occasionally changing the Quality of a proposition may be gained by the process of Obversion (chap. vii. Section 5); whilst to use only one Quality would impair the elasticity of logical expression. It is a postulate of Logic that the negative sign may be transferred from the copula to the predicate, or from the predicate to the copula, without altering the sense of a proposition; and this is justified by the experience that not to have an attribute and to be without it are the same thing.

Section 3. A. I. E. O.—Combining the two kinds of Quantity, Universal and Particular, with the two kinds of Quality, Affirmative and Negative, we get four simple types of proposition, which it is usual to symbolise by the letters A. I. E. O., thus:

A.	Universal Affirmative	— All S is P.
I.	Particular Affirmative	— Some S is P.
E.	Universal Negative	— No S is P.
O.	Particular Negative	— Some S is not P.

As an aid to the remembering of these symbols we may observe that A. and I. are the first two vowels in *affirmo* and that E. and O. are the vowels in *nego*.

It must be acknowledged that these four kinds of proposition recognised by Formal Logic constitute a very meagre selection from the list of propositions actually used in judgment and reasoning.

Those Logicians who explicitly quantify the predicate obtain, in all, eight forms of proposition according to Quantity and Quality:

U.	Toto-total Affirmative	— All X is all Y.
A.	Toto-partial Affirmative	— All X is some Y.
Y.	Parti-total Affirmative	— Some X is all Y.
I.	Parti-partial Affirmative	— Some X is some Y.
E.	Toto-total Negative	— No X is any Y.
–.	Toto-partial Negative	— No X is some Y.
O.	Parti-total Negative	— Some X is not any Y.
–.	Parti-partial Negative	— Some X is not some Y.

Here A. I. E. O. correspond with those similarly symbolised in the

usual list, merely designating in the predicates the quantity which was formerly treated as implicit.

Section 4. As to Relation, propositions are either Categorical or Conditional. A Categorical Proposition is one in which the predicate is directly affirmed or denied of the subject without any limitation of time, place, or circumstance, extraneous to the subject, as *All men in England are secure of justice*; in which proposition, though there is a limitation of place ('in England'), it is included in the subject. Of this kind are nearly all the examples that have yet been given, according to the form *S is P*.

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A Conditional Proposition is so called because the predication is made under some limitation or condition not included in the subject, as *If a man live in England, he is secure of justice*. Here the limitation 'living in England' is put into a conditional sentence extraneous to the subject, 'he,' representing any man.

Conditional propositions, again, are of two kinds—Hypothetical and Disjunctive. Hypothetical propositions are those that are limited by an explicit conditional sentence, as above, or thus: *If Joe Smith was a prophet, his followers have been unjustly persecuted*. Or in symbols thus:

If A is, B is;
If A is B, A is C;
If A is B, C is D.

Disjunctive propositions are those in which the condition under which predication is made is not explicit but only implied under the disguise of an alternative proposition, as *Joe Smith was either a prophet or an impostor*. Here there is no direct predication concerning Joe Smith, but only a predication of one of the alternatives conditionally on the other being denied, as, *If Joe Smith was not a prophet he was an impostor*; or, *If he was not an impostor, he was a prophet*. Symbolically, Disjunctives may be represented thus:

A is either B or C,
Either A is B or C is D.

Formally, every Conditional may be expressed as a Categorical. For our last example shows how a Disjunctive may be reduced to two Hypotheticals (of which one is redundant, being the contrapositive of the other; see (chap. vii. Section 10). And a Hypothetical is reducible to a Categorical thus: *If the sky is clear, the night is cold* may be read—*The case of the sky being clear is a case of the night being cold*; and this, though a clumsy plan, is sometimes convenient. It would be better to say *The sky being clear is a sign of the night being cold*, or a condition of it. For, as Mill says, the essence of a Hypothetical is to state that one clause of it (the indicative) may be inferred from the other (the conditional). Similarly, we might write: *Proof of Joe Smith's not being a prophet is a proof of his being an impostor*.

This turning of Conditionals into Categoricals is called a Change of Relation; and the process may be reversed: *All the wise are virtuous* may be written, *If any man is wise he is virtuous*; or, again, *Either a man is not-wise or he is virtuous*. But the categorical form is usually the simplest.

If, then, as substitutes for the corresponding conditionals, categoricals are formally adequate, though sometimes inelegant, it may be urged that Logic has nothing to do with elegance; or that, at any rate, the chief elegance of science is economy, and that therefore, for scientific purposes, whatever we may write further about conditionals must be an ugly excrescence. The scientific purpose of Logic is to assign the conditions of proof. Can we, then, in the conditional form prove anything that cannot be proved in the categorical? Or does a conditional require to be itself proved by any method not applicable to the Categorical? If not, why go on with the discussion of Conditionals? For all laws of Nature, however stated, are essentially categorical. 'If a straight line falls on another straight line, the adjacent angles are together equal to two right angles'; 'If a body is unsupported, it falls'; 'If population increases, rents tend to rise': here 'if' means 'whenever' or 'all cases in which'; for to raise a doubt whether a straight line is ever conceived to fall upon another, whether bodies are ever unsupported, or population ever increases, is a superfluity of scepticism; and plainly the hypothetical form has nothing to do with the proof of such propositions, nor with inference from them.

Still, the disjunctive form is necessary in setting out the relation of contradictory terms, and in stating a Division (chap. xxi.), whether formal (*as A is B or not-B*) or material (as *Cats are white, or black, or tortoiseshell, or tabby*). And in some cases the hypothetical form is useful. One of these occurs where it is important to draw attention to the condition, as something doubtful or especially requiring examination. *If there is a resisting medium in space, the earth will fall into the sun; If the Corn Laws are to be re-enacted, we had better sell railways and buy land*: here the hypothetical form draws attention to the questions whether there is a resisting medium in space, whether the Corn Laws are likely to be re-enacted; but as to methods of inference and proof, the hypothetical form has nothing to do with them. The propositions predicate causation: *A resisting medium in space is a condition of the earth's falling into the sun; A Corn Law is a condition of the rise of rents, and of the fall of railway profits*.

A second case in which the hypothetical is a specially appropriate form of statement occurs where a proposition relates to a particular matter and to future time, as *If there be a storm to-morrow, we shall miss our picnic*. Such cases are of very slight logical interest. It is as exercises in formal thinking that hypotheticals are of most value; inasmuch as many people find them more difficult than categoricals to manipulate.

In discussing Conditional Propositions, the conditional sentence of a Hypothetical, or the first alternative of a Disjunctive, is called the Antecedent; the indicative sentence of a Hypothetical, or the second alternative of a Disjunctive, is called the Consequent.

Hypotheticals, like Categoricals, have been classed according to Quantity and Quality. Premising that the quantity of a Hypothetical depends on the quantity of its Antecedent (which determines its limitation), whilst its quality depends on the quality of its consequent (which makes the predication), we may exhibit four forms:

A. *If A is B, C is D;*
I. *Sometimes when A is B, C is D;*
E. *If A is B, C is not D;*
O. *Sometimes when A is B, C is not D*.

But I. and O. are rarely used.

As for Disjunctives, it is easy to distinguish the two quantities thus:

A. *Either A is B, or C is D;*
I. *Sometimes either A is B or C is D.*
But I. is rarely used. The distinction of quality, however, cannot be made: there are no true negative forms; for if we write—

Neither is A B, nor C D,
there is here no alternative predication, but only an Exponible equivalent to *No A is B, and No C is D*. And if we write—

Either A is not B, or C is not D,
this is affirmative as to the alternation, and is for all methods of treatment equivalent to A.

Logicians are divided in opinion as to the interpretation of the conjunction 'either, or'; some holding that it means 'not both,' others that it means 'it may be both.' Grammatical usage, upon which the question is sometimes argued, does not seem to be established in favour of either view. If we say *A man so precise in his walk and conversation is either a saint or a consummate hypocrite*; or, again, *One who is happy in a solitary life is either more or less than man*; we cannot in such cases mean that the subject may be both. On the other hand, if it be said that *the author of 'A Tale of a Tub' is either a misanthrope or a dyspeptic*, the alternatives are not incompatible. Or, again, given that *X. is a lunatic, or a lover, or a poet*, the three predicates have much congruity.

It has been urged that in Logic, language should be made as exact and definite as possible, and that this requires the exclusive interpretation 'not both.' But it seems a better argument, that Logic (1) should be able to express all meanings, and (2), as the science of evidence, must not assume more than is given; to be on the safe side, it must in doubtful cases assume the least, just as it generally assumes a preindesignate term to be of particular quantity; and, therefore 'either, or' means 'one, or the other, or both.'

However, when both the alternative propositions have the same subject, as *Either A is B, or A is C*, if the two predicates are contrary or contradictory terms (as 'saint' and 'hypocrite,' or 'saint' and 'not-saint'), they cannot in their nature be predicable in the same way of the same subject; and, therefore, in such a case 'either, or' means one or the other, but not both in the same relation. Hence it seems necessary to admit that the conjunction 'either, or' may sometimes require one interpretation, sometimes the other; and the rule is that it implies the further possibility 'or both,' except when both alternatives have the same subject whilst the predicates are contrary or contradictory terms.

If, then, the disjunctive *A is either B or C* (*B* and *C* being contraries) implies that both alternatives cannot be true, it can only be adequately rendered in hypotheticals by the two forms—(1) *If A is B, it is not C*, and (2)*If A is not B, it is C*. But if the disjunctive *A is either B or C* (*B* and *C* not being contraries) implies that both may be true, it will be adequately translated into a hypothetical by the single form, *If A is not B, it is C*. We cannot translate it into—*If A is B, it is not C*, for, by our supposition, if '*A is B*' is true, it does not follow that '*A is C*' must be false.

Logicians are also divided in opinion as to the function of the hypothetical form. Some think it expresses doubt; for the consequent depends on the antecedent, and the antecedent, introduced by 'if,' may or may not be realised, as in *If the sky is clear, the night is cold*: whether the sky is, or is not, clear being supposed to be uncertain. And we have seen that some hypothetical propositions seem designed to draw attention to such uncertainty, as—*If there is a resisting medium in space, etc*. But other Logicians lay stress upon the connection of the clauses as the important matter: the statement is, they say, that the consequent may be inferred from the antecedent. Some even declare that it is given as a necessary inference; and on this ground Sigwart rejects particular hypotheticals, such as *Sometimes when A is B, C is D*; for if it happens only sometimes the connexion cannot be necessary. Indeed, it cannot even be probably inferred without further grounds. But this is also true whenever the antecedent and consequent are concerned with different matter. For example, *If the soul is simple, it is indestructible*. How do you know that? Because *Every simple substance is indestructible*. Without this further

ground there can be no inference. The fact is that conditional forms often cover assertions that are not true complex propositions but a sort of euthymemes (chap. xi. Section 2), arguments abbreviated and rhetorically disguised. Thus: *If patience is a virtue there are painful virtues*—an example from Dr. Keynes. Expanding this we have—

Patience is painful;

Patience is a virtue:

therefore Some virtue is painful.

And then we see the equivocation of the inference; for though patience be painful *to learn*, it is not painful *as a virtue* to the patient man.

The hypothetical, '*If Plato was not mistaken poets are dangerous citizens*,' may be considered as an argument against the laureateship, and may be expanded (informally) thus: 'All Plato's opinions deserve respect; one of them was that poets are bad citizens; therefore it behoves us to be chary of encouraging poetry.' Or take this disjunctive, '*Either Bacon wrote the works ascribed to Shakespeare, or there were two men of the highest genius in the same age and country*.' This means that it is not likely there should be two such men, that we are sure of Bacon, and therefore ought to give him all the glory. Now, if it is the part of Logic 'to make explicit in language all that is implicit in thought,' or to put arguments into the form in which they can best be examined, such propositions as the above ought to be analysed in the way suggested, and confirmed or refuted according to their real intention.

We may conclude that no single function can be assigned to all hypothetical propositions: each must be treated according to its own meaning in its own context.

Section 5. As to Modality, propositions are divided into Pure and Modal. A Modal proposition is one in which the predicate is affirmed or denied, not simply but *cum modo*, with a qualification. And some Logicians have considered any adverb occurring in the

predicate, or any sign of past or future tense, enough to constitute a modal: as 'Petroleum is *dangerously* inflammable'; 'English *will be* the universal language.' But far the most important kind of modality, and the only one we need consider, is that which is signified by some qualification of the predicate as to the degree of certainty with which it is affirmed or denied. Thus, 'The bite of the cobra is *probably* mortal,' is called a Contingent or Problematic Modal: 'Water is *certainly* composed of oxygen and hydrogen' is an Assertory or Certain Modal: 'Two straight lines *cannot* enclose a space' is a Necessary or Apodeictic Modal (the opposite being inconceivable). Propositions not thus qualified are called Pure.

Modal propositions have had a long and eventful history, but they have not been found tractable by the resources of ordinary Logic, and are now generally neglected by the authors of text-books. No doubt such propositions are the commonest in ordinary discourse, and in some rough way we combine them and draw inferences from them. It is understood that a combination of assertory or of apodeictic premises may warrant an assertory or an apodeictic conclusion; but that if we combine either of these with a problematic premise our conclusion becomes problematic; whilst the combination of two problematic premises gives a conclusion less certain than either. But if we ask 'How much less certain?' there is no answer. That the modality of a conclusion follows the less certain of the premises combined, is inadequate for scientific guidance; so that, as Deductive Logic can get no farther than this, it has abandoned the discussion of Modals. To endeavour to determine the degree of certainty attaching to a problematic judgment is not, however, beyond the reach of Induction, by analysing circumstantial evidence, or by collecting statistics with regard to it. Thus, instead of 'The cobra's bite is *probably* fatal,' we might find that it is fatal 80 times in 100. Then, if we know that of those who go to India 3 in 1000 are bitten, we can calculate what the chances are that any one going to India will die of a cobra's bite (chap. xx.).

Section 6. Verbal and Real Propositions.—Another important division of propositions turns upon the relation of the predicate to the subject in respect of their connotations. We saw, when discussing Relative Terms, that the connotation of one term often implies that of another; sometimes reciprocally, like 'master' and

'slave'; or by inclusion, like species and genus; or by exclusion, like contraries and contradictories. When terms so related appear as subject and predicate of the same proposition, the result is often tautology—*e.g., The master has authority over his slave; A horse is an animal; Red is not blue; British is not foreign.* Whoever knows the meaning of 'master,' 'horse,' 'red,' 'British,' learns nothing from these propositions. Hence they are called Verbal propositions, as only expounding the sense of words, or as if they were propositions only by satisfying the forms of language, not by fulfilling the function of propositions in conveying a knowledge of facts. They are also called 'Analytic' and 'Explicative,' when they separate and disengage the elements of the connotation of the subject. Doubtless, such propositions may be useful to one who does not know the language; and Definitions, which are verbal propositions whose predicates analyse the whole connotations of their subjects, are indispensable instruments of science (see chap. xxii.).

Of course, hypothetical propositions may also be verbal, as *If the soul be material it is extended*; for 'extension' is connoted by 'matter'; and, therefore, the corresponding disjunctive is verbal—*Either the soul is not material, or it is extended*. But a true divisional disjunctive can never be verbal (chap. xxi. Section 4, rule 1).

On the other hand, when there is no such direct relation between subject and predicate that their connotations imply one another, but the predicate connotes something that cannot be learnt from the connotation of the subject, there is no longer tautology, but an enlargement of meaning—*e.g., Masters are degraded by their slaves; The horse is the noblest animal; Red is the favourite colour of the British army; If the soul is simple, it is indestructible*. Such propositions are called Real, Synthetic, or Ampliative, because they are propositions for which a mere understanding of their subjects would be no substitute, since the predicate adds a meaning of its own concerning matter of fact.

To any one who understands the language, a verbal proposition can never be an inference or conclusion from evidence; nor can a verbal proposition ever furnish grounds for an inference, except as to the meaning of words. The subject of real and verbal

propositions will inevitably recur in the chapters on Definition; but tautologies are such common blemishes in composition, and such frequent pitfalls in argument, that attention cannot be drawn to them too early or too often.