CHAPTER VIII

ORDER OF TERMS, EULER'S DIAGRAMS, LOGICAL EQUATIONS, EXISTENTIAL IMPORT OF PROPOSITIONS

Section 1. Of the terms of a proposition which is the Subject and which the Predicate? In most of the exemplary propositions cited by Logicians it will be found that the subject is a substantive and the predicate an adjective, as in *Men are mortal.* This is the relation of Substance and Attribute which we saw (chap. i. Section 5) to be the central type of relations of coinherence; and on this model other predications may be formed in which the subject is not a substance, but is treated as if it were, and could therefore be the ground of attributes; as *Fame is treacherous, The weather is changeable*. But, in literature, sentences in which the adjective comes first are not uncommon, as *Loud was the applause, Dark is the fate of man, Blessed are the peacemakers*, and so on. Here, then, 'loud,' 'dark' and 'blessed' occupy the place of the logical subject. Are they really the subject, or must we alter the order of such sentences into *The applause was loud, etc*.? If we do, and then proceed to convert, we get *Loud was the applause*, or (more scrupulously) *Some loud noise was the applause*. The last form, it is true, gives the subject a substantive word, but 'applause' has become the predicate; and if the substantive 'noise' was not implied in the first form, *Loud is the applause*, by what right is it now inserted? The recognition of Conversion, in fact, requires us to admit that, formally, in a logical proposition, the term preceding the copula is subject and the one following is predicate. And, of course, materially considered, the mere order of terms in a proposition can make no difference in the method of proving it, nor in the inferences that can be drawn from it.

Still, if the question is, how we may best cast a literary sentence into logical form, good grounds for a definite answer may perhaps be found. We must not try to stand upon the naturalness of expression, for *Dark is the fate of man* is quite as natural as *Man is mortal*. When the purpose is not merely to state a fact, but also to express our feelings about it, to place the grammatical predicate first may be perfectly natural and most effective. But the grounds of a logical order of statement must be found in its adaptation to the purposes of proof and inference. Now general propositions are those from which most inferences can be drawn, which, therefore, it is most important to establish, if true; and they are also the easiest to disprove, if false; since a single negative instance suffices to establish the contradictory. It follows that, in re-casting a literary or colloquial sentence for logical purposes, we should try to obtain a form in which the subject is distributed—is either a singular term or a general term predesignate as 'All' or 'No.' Seeing, then, that most adjectives connote a single attribute, whilst most substantives connote more than one attribute; and that therefore the denotation of adjectives is usually wider than that of substantives; in any proposition, one term of which is an adjective and the other a substantive, if either can be distributed in relation to the other, it is nearly sure to be the substantive; so that to take the substantive term for subject is our best chance of obtaining an universal proposition. These considerations seem to justify the practice of Logicians in selecting their examples.

For similar reasons, if both terms of a proposition are substantive, the one with the lesser denotation is (at least in affirmative propositions) the more suitable subject, as *Cats are carnivores*. And if one term is abstract, that is the more suitable subject; for, as we have seen, an abstract term may be interpreted by a corresponding concrete one distributed, as *Kindness is infectious*; that is, *All kind actions suggest imitation*.

If, however, a controvertist has no other object in view than to refute some general proposition laid down by an opponent, a particular proposition is all that he need disentangle from any statement that serves his purpose.

Section 2. Toward understanding clearly the relations of the terms of a proposition, it is often found useful to employ diagrams; and the diagrams most in use are the circles of Euler.

These circles represent the denotation of the terms. Suppose the proposition to be *All hollow-horned animals ruminate*: then, if we could collect all ruminants upon a prairie, and enclose them with a circular palisade; and segregate from amongst them all the hollow-horned beasts, and enclose them with another ring-fence inside the other; one way of interpreting the proposition (namely, in denotation) would be figured to us thus:

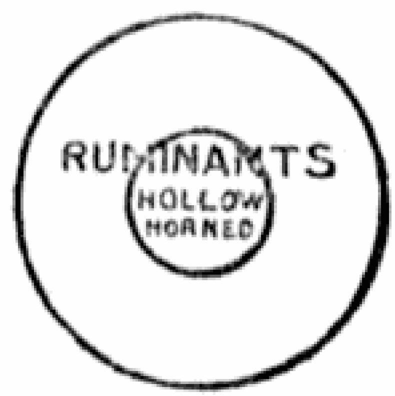

Fig. 1.

An Universal Affirmative may also state a relation between two terms whose denotation is co-extensive. A definition always does this, as *Man is a rational animal*; and this, of course, we cannot represent by two distinct circles, but at best by one with a thick circumference, to suggest that two coincide, thus:

Fig. 2.

The Particular Affirmative Proposition may be represented in several ways. In the first place, bearing in mind that 'Some' means 'some at least, it may be all,' an I. proposition may be represented by Figs. 1 and 2; for it is true that *Some horned animals ruminate*, and that *Some men are rational*. Secondly, there is the case in which the 'Some things' of which a predication is made are, in fact, not all; whilst the predicate, though not given as distributed, yet might be so given if we wished to state the whole truth; as if we say *Some men are Chinese*. This case is also represented by Fig. 1, the outside circle representing 'Men,' and the inside one 'Chinese.' Thirdly, the predicate may appertain to some only of the subject, but to a great many other things, as in *Some horned beasts are domestic*; for it is true that some are not, and that certain other kinds of animals are, domestic. This case, therefore, must be illustrated by overlapping circles, thus:

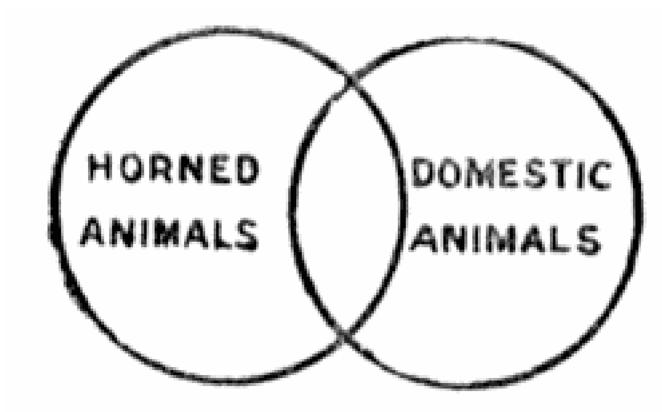

Fig. 3.

The Universal Negative is sufficiently represented by a single Fig. (4): two circles mutually exclusive, thus:

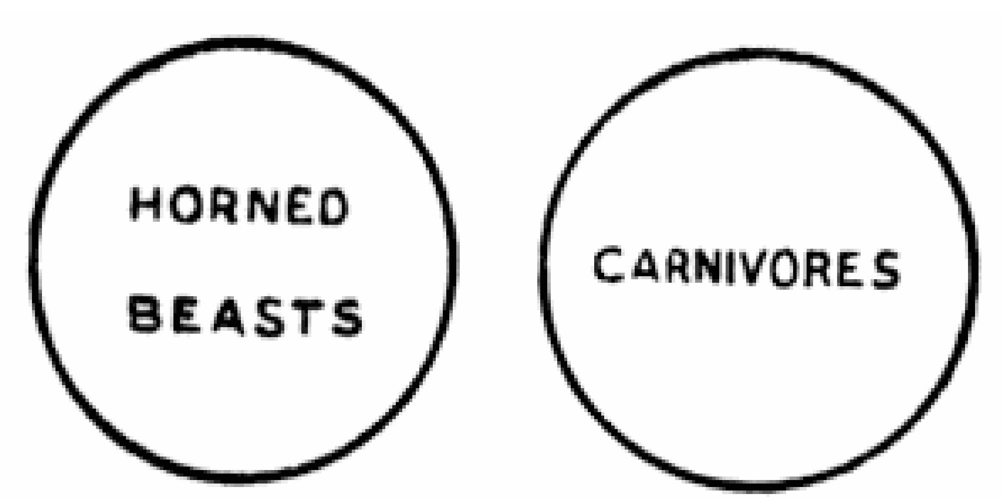

Fig. 4.

That is, *No horned beasts are carnivorous*.

Lastly, the Particular Negative may be represented by any of the Figs. 1, 3, and 4; for it is true that *Some ruminants are not hollow-horned*, that *Some horned animals are not domestic*, and that *Some horned beasts are not carnivorous*.

Besides their use in illustrating the denotative force of propositions, these circles may be employed to verify the results of Obversion, Conversion, and the secondary modes of Immediate Inference. Thus the Obverse of A. is clear enough on glancing at Figs. 1 and 2; for if we agree that whatever term's denotation is represented by a given circle, the denotation of the contradictory term shall be represented by the space outside that circle; then if it is true that *All hollow horned animals are ruminants*, it is at the same time true that *No hollow-horned animals are not-ruminants*; since none of the hollow-horned are found outside the palisade that encloses the ruminants. The Obverse of I., E. or O. may be verified in a similar manner.

As to the Converse, a Definition is of course susceptible of Simple Conversion, and this is shown by Fig. 2: 'Men are rational animals' and 'Rational animals are men.' But any other A. proposition is presumably convertible only by limitation, and this is shown by Fig. 1; where *All hollow-horned animals are ruminants*, but we can only say that *Some ruminants are hollow-horned*.

That I. may be simply converted may be seen in Fig. 3, which represents the least that an I. proposition can mean; and that E. may be simply converted is manifest in Fig. 4.

As for O., we know that it cannot be converted, and this is made plain enough by glancing at Fig. 1; for that represents the O., *Some ruminants are not hollow-horned*, but also shows this to be compatible with *All hollow-horned animals are ruminants* (A.). Now in conversion there is (by definition) no change of quality. The Converse, then, of *Some ruminants are not hollow-horned* must be a negative proposition, having 'hollow-horned' for its subject, either in E. or O.; but these would be respectively the contrary and contradictory of *All hollow-horned animals are* *ruminants*; and, therefore, if this be true, they must both be false.

But (referring still to Fig. 1) the legitimacy of contrapositing O. is equally clear; for if *Some ruminants are not hollow-horned*, *Some animals that are not hollow-horned are ruminants*, namely, all the animals between the two ring-fences. Similar inferences may be illustrated from Figs. 3 and 4. And the Contraposition of A. may be verified by Figs. 1 and 2, and the Contraposition of E. by Fig. 4.

Lastly, the Inverse of A. is plain from Fig. 1—*Some things that are not hollow-horned are not ruminants*, namely, things that lie outside the outer circle and are neither 'ruminants' nor 'hollow-horned.' And the Inverse of E may be studied in Fig. 4—*Some things that are not-horned beasts are carnivorous*.

Notwithstanding the facility and clearness of the demonstrations thus obtained, it may be said that a diagrammatic method, representing denotations, is not properly logical. Fundamentally, the relation asserted (or denied) to exist between the terms of a proposition, is a relation between the terms as determined by their attributes or connotation; whether we take Mill's view, that a proposition asserts that the connotation of the subject is a mark of the connotation of the predicate; or Dr. Venn's view, that things denoted by the subject (as having its connotation) have (or have not) the attribute connoted by the predicate; or, the Conceptualist view, that a judgment is a relation of concepts (that is, of connotations). With a few exceptions artificially framed (such as 'kings now reigning in Europe'), the denotation of a term is never directly and exhaustively known, but consists merely in 'all things that have the connotation.' If the value of logical training depends very much upon our habituating ourselves to construe propositions, and to realise the force of inferences from them, according to the connotation of their terms, we shall do well not to turn too hastily to the circles, but rather to regard them as means of verifying in denotation the conclusions that we have already learnt to recognise as necessary in connotation.

Section 3. The equational treatment of propositions is closely connected with the diagrammatic. Hamilton thought it a great merit of his plan of quantifying the predicate, that thereby every proposition is reduced to its true form—an equation. According to this doctrine, the proposition *All X is all Y* (U.) equates X and Y; the proposition *All X is some Y* (A.) equates X with some part of Y; and similarly with the other affirmatives (Y. and I.). And so far it is easy to follow his meaning: the Xs are identical with some or all the Ys. But, coming to the negatives, the equational interpretation is certainly less obvious. The proposition *No X is Y* (E.) cannot be said in any sense to equate X and Y; though, if we obvert it into *All X is some not-Y*, we have (in the same sense, of course, as in the above affirmative forms) X equated with part at least of 'not-Y.'

But what is that sense? Clearly not the same as that in which mathematical terms are equated, namely, in respect of some mode of quantity. For if we may say *Some X is some Y*, these Xs that are also Ys are not merely the same in number, or mass, or figure; they are the same in every respect, both quantitative and qualitative, have the same positions in time and place, are in fact identical. The proposition 2+2=4 means that any two things added to any other two are, *in respect of number*, equal to any three things added to one other thing; and this is true of all things that can be counted, however much they may differ in other ways. But *All X is all Y* means that Xs and Ys are the same things, although they have different names when viewed in different aspects or relations. Thus all equilateral triangles are equiangular triangles; but in one case they are named from the equality of their angles, and in the other from the equality of their sides. Similarly, 'British subjects' and 'subjects of King George V' are the same people, named in one case from the person of the Crown, and in the other from the Imperial Government. These logical equations, then, are in truth identities of denotation; and they are fully illustrated by the relations of circles described in the previous section.

When we are told that logical propositions are to be considered as equations, we naturally expect to be shown some interesting developments of method in analogy with the equations of Mathematics; but from Hamilton's innovations no such thing results. This cannot be said, however, of the equations of Symbolic Logic; which are the starting-point of very remarkable processes of ratiocination. As the subject of Symbolic Logic, as a whole, lies beyond the compass of this work, it will be enough to give Dr. Venn's equations corresponding with the four propositional forms of common Logic.

According to this system, universal propositions are to be regarded as not necessarily implying the existence of their terms; and therefore, instead of giving them a positive form, they are translated into symbols that express what they deny. For example, the proposition *All devils are ugly* need not imply that any such things as 'devils' really exist; but it certainly does imply that *Devils that are not ugly do not exist*. Similarly, the proposition *No angels are ugly* implies that *Angels that are ugly do not exist*. Therefore, writing *x* for 'devils,' *y* for 'ugly,' and *y*_ for 'not-ugly,' we may express A., the universal affirmative, thus:

A. $xy^{-} = 0$.
That is, *x that is not y is nothing*; or, *Devils that are not-ugly do not exist*. And, similarly, writing *x* for 'angels' and *y* for 'ugly,' we may express E., the universal negative, thus:

E. $xy = 0$.
That is, *x that is y is nothing*; or, *Angels that are ugly do not exist*.

On the other hand, particular propositions are regarded as implying the existence of their terms, and the corresponding equations are so framed as to express existence. With this end in view, the symbol v is adopted to represent 'something,' or indeterminate reality, or more than nothing. Then, taking any particular affirmative, such as *Some metaphysicians are obscure*, and writing *x* for 'metaphysicians,' and *y* for 'obscure,' we may express it thus:

I. $xy = \text{v}$.
That is, *x that is y is something*; or, *Metaphysicians that are obscure do occur in experience* (however few they may be, or whether they all be obscure). And, similarly, taking any particular negative, such as *Some giants are not cruel*, and writing *x* for 'giants' and *y* for 'not-cruel,' we may express it thus:

O. $xy^{-} = \text{v}$.
That is, *x that is not y is something*; or, *giants that are not-cruel do occur*—in romances, if nowhere else.

Clearly, these equations are, like Hamilton's, concerned with denotation. A. and E. affirm that the compound terms xy_ and xy have no denotation; and I. and O. declare that xy_ and xy have denotation, or stand for something. Here, however, the resemblance to Hamilton's system ceases; for the Symbolic Logic, by operating upon more than two terms simultaneously, by adopting the algebraic signs of operations, +,-, x, ÷ (with a special signification), and manipulating the symbols by quasi-algebraic processes, obtains results which the common Logic reaches (if at all) with much greater difficulty. If, indeed, the value of logical systems were to be judged of by the results obtainable, formal deductive Logic would probably be superseded. And, as a mental discipline, there is much to be said in favour of the symbolic method. But, as an introduction to philosophy, the common Logic must hold its ground. (Venn: *Symbolic Logic*, c. 7.)

Section 4. Does Formal Logic involve any general assumption as to the real existence of the terms of propositions?

In the first place, Logic treats primarily of the *relations* implied in propositions. This follows from its being the science of proof for all sorts of (qualitative) propositions; since all sorts of propositions have nothing in common except the relations they express.

But, secondly, relations without terms of some sort are not to be thought of; and, hence, even the most formal illustrations of logical doctrines comprise such terms as S and P, X and Y, or x and y, in a symbolic or representative character. Terms, therefore, of some sort are assumed to exist (together with their negatives or contradictories) *for the purposes of logical manipulation*.

Thirdly, however, that Formal Logic cannot as such directly involve the existence of any particular concrete terms, such as 'man' or 'mountain,' used by way of illustration, is implied in the word 'formal,' that is, 'confined to what is common or abstract'; since the only thing common to all terms is to be related in some way to other terms. The actual existence of any concrete thing can only be known by experience, as with 'man' or 'mountain'; or by methodically justifiable inference from experience, as with 'atom' or 'ether.' If 'man' or 'mountain,' or 'Cuzco' be used to illustrate logical forms, they bring with them an existential import derived from experience; but this is the import of language, not of the logical forms. 'Centaur' and 'El Dorado' signify to us the non-existent; but they serve as well as 'man' and 'London' to illustrate Formal Logic.

Nevertheless, fourthly, the existence or non-existence of particular terms may come to be implied: namely, wherever the very fact of existence, or of some condition of existence, is an hypothesis or datum. Thus, given the proposition *All S is P*, to be P is made a condition of the existence of S: whence it follows that an S that is not P does not exist ($xy^{-} = 0$). On the further hypothesis that S exists, it follows that P exists. On the hypothesis that S does not exist, the existence of P is problematic; but, then, if P does exist we cannot convert the proposition; since *Some P is S* (P existing) would involve the existence of S; which is contrary to the hypothesis.

Assuming that Universals *do not*, whilst Particulars *do*, imply the existence of their subjects, we cannot infer the subalternate (I. or O.) from the subalternans (A. or E.), for that is to ground the actual on the problematic; and for the same reason we cannot convert A. *per accidens*.

Assuming, again, a certain *suppositio* or universe, to which in a given discussion every argument shall refer, then, any propositions whose terms lie outside that *suppositio* are irrelevant, and for the purposes of that discussion are sometimes called "false"; though it seems better to call them irrelevant or meaningless, seeing that to call them false implies that they might in the same case be true. Thus propositions which, according to the doctrine of Opposition, appear to be Contradictories, may then cease to be so; for of Contradictories one is true and the other false; but, in the case supposed, both are meaningless. If the subject of discussion be Zoology, all propositions about centaurs or unicorns are absurd; and such specious Contradictories as *No centaurs play the lyre—Some centaurs do play the lyre*; or *All unicorns fight with lions—Some unicorns do not fight with lions*, are both meaningless, because in Zoology there are no centaurs nor unicorns; and, therefore, in this reference, the propositions are not really contradictory. But if the subject of discussion or *suppositio* be Mythology or Heraldry, such propositions as the above are to the purpose, and form legitimate pairs of Contradictories.

In Formal Logic, in short, we may make at discretion any assumption whatever as to the existence, or as to any condition of the existence of any particular term or terms; and then certain implications and conclusions follow in consistency with that hypothesis or datum. Still, our conclusions will themselves be only hypothetical, depending on the truth of the datum; and, of course, until this is empirically ascertained, we are as far as ever from empirical reality. (Venn: *Symbolic Logic*, c. 6; Keynes: *Formal Logic*, Part II. c. 7: *cf*. Wolf: *Studies in Logic*.)