CHAPTER XVI

The Canons of Direct Induction

Section 1. Let me begin by borrowing an example from Bain (Logic: B. III. c. 6). The North-East wind is generally detested in this country: as long as it blows few people feel at their best. Occasional well-known causes of a wind being injurious are violence, excessive heat or cold, excessive dryness or moisture, electrical condition, the being laden with dust or exhalations. Let the hypothesis be that the last is the cause of the North-East wind's unwholesome quality; since we know it is a ground current setting from the pole toward the equator and bent westward by the rotation of the earth; so that, reaching us over thousands of miles of land, it may well be fraught with dust, effluvia, and microbes. Now, examining many cases of North-East wind, we find that this is the only circumstance in which all the instances agree: for it is sometimes cold, sometimes hot; generally dry, but sometimes wet; sometimes light, sometimes violent, and of all electrical conditions. Each of the other circumstances, then, can be omitted without the N.E. wind ceasing to be noxious; but one circumstance is never absent, namely, that it is a ground current. That circumstance, therefore, is probably the cause of its injuriousness. This case illustrates:– (I) The Canon of Agreement.

If two or more instances of a phenomenon under investigation have only one other circumstance (antecedent or consequent) in common, that circumstance is probably the cause (or an indispensable condition) or the effect of the phenomenon, or is connected with it by causation.

This rule of proof (so far as it is used to establish direct causation) depends, first, upon observation of an invariable connection between the given phenomenon and one other circumstance; and, secondly, upon I. (a) and II. (b) among the propositions obtained from the unconditionality of causation at the close of the last chapter.

To prove that A is causally related to p, suppose two instances of the occurrence of A, an antecedent, and p, a consequent, with concomitant facts or events–and let us represent them thus:

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Antecedents:  A    B    C         A    D    E
Consequents:  p    q    r         p    s    t;
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and suppose further that, in this case, the immediate succession of events can be observed. Then A is probably the cause, or an indispensable condition, of p. For, as far as our instances go, A is the invariable antecedent of p; and p is the invariable consequent of A. But the two instances of A or p agree in no other circumstance. Therefore A is (or completes) the unconditional antecedent of p. For B and C are not indispensable conditions of p, being absent in the second instance (Rule II. (b)); nor are D and E, being absent in the first instance. Moreover, q and r are not effects of A, being absent in the second instance (Rule II. (d)); nor are s and t, being absent in the first instance.

It should be observed that the cogency of the proof depends entirely upon its tending to show the unconditionality of the sequence A-p, or the indispensability of A as a condition of p. That p follows A, even immediately, is nothing by itself: if a man sits down to study and, on the instant, a hand-organ begins under his window, he must not infer malice in the musician: thousands of things follow one another every moment without traceable connection; and this we call ‘accidental.’ Even invariable sequence is not enough to prove direct causation; for, in our experience does not night invariable follow day? The proof requires that the instances be such as to show not merely what events are in invariable sequence, but also what are not. From among the occasional antecedents of p (or consequents of A) we have to eliminate the accidental ones. And this is done by finding or making ‘negative instances’ in respect of each of them. Thus the instance

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A    D    E
p    s    t
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is a negative instance of B and C considered as supposable causes of p (and of q and r as supposable effects of A); for it shows that they are absent when p (or A) is present.

To insist upon the cogency of 'negative instances' was Bacon's great contribution to Inductive Logic. If we neglect them, and merely collect examples of the sequence A-p, this is 'simple enumeration'; and although simple enumeration, when the instances of agreement are numerous enough, may give rise to a strong belief in the connection of phenomena, yet it can never be a methodical or logical proof of causation, since it does not indicate the unconditionalness of the sequence. For simple enumeration of the sequence A-p leaves open the possibility that, besides A, there is always some other antecedent of p, say X; and then X may be the cause of p. To disprove it, we must find, or make, a negative instance of X–where p occurs, but X is absent.

So far as we recognise the possibility of a plurality of causes, this method of Agreement cannot be quite satisfactory. For then, in such instances as the above, although D is absent in the first, and B in the second, it does not follow that they are not the causes of p; for they may be alternative causes: B may have produced p in the first instance, and D in the second; A being in both cases an accidental circumstance in relation to p. To remedy this shortcoming by the method of Agreement itself, the only course is to find more instances of p. We may never find a negative instance of A; and, if not, the probability that A is the cause of p increases with the number of instances. But if there be no antecedent that we cannot sometimes exclude, yet the collection of instances will probably give at last all the causes of p; and by finding the proportion of instances in which A, B, or X precedes p, we may estimate the probability of any one of them being the cause of p in any given case of its occurrence.

But this is not enough. Since there cannot really be vicarious causes, we must define the effect (p) more strictly, and examine the cases to find whether there may not be varieties of p, with each of which one of the apparent causes is correlated: A with p1 B with p11, X with p111. Or, again, it may be that none of the recognised antecedents is effective: as we here depend solely on observation, the true conditions may be so recondite and disguised by other phenomena as to have escaped our scrutiny. This may happen even when we suppose that the chief condition has been isolated: the drinking of foul water was long believed to cause dysentery, because it was a frequent antecedent; whilst observation had overlooked the bacillus, which was the indispensable condition.

Again, though we have assumed that, in the instances supposed above, immediate sequence is observable, yet in many cases it may not be so, if we rely only on the canon of Agreement; if instances cannot be obtained by experiment, and we have to depend on observation. The phenomena may then be so mixed together that A and p seem to be merely concomitant; so that, though connection of some sort may be rendered highly probable, we may not be able to say which is cause and which is effect. We must then try (as Bain says) to trace the expenditure of energy: if p gains when A loses, the course of events if from A to p.

Moreover, where succession cannot be traced, the method of Agreement may point to a connection between two or more facts (perhaps as co-effects of a remote cause) where direct causation seems to be out of the question: e.g., that Negroes, though of different tribes, different localities, customs, etc., are prognathous, woolly-haired and dolichocephalic.

The Method of Agreement, then, cannot by itself prove causation. Its chief use (as Mill says) is to suggest hypotheses as to the cause; which must then be used (if possible) experimentally to try if it produces the given effect. A bacillus, for example, being always found with a certain disease, is probably the chief condition of it: give it to a guinea-pig, and observe whether the disease appears in that animal.

Men often use arguments which, if they knew it, might be shown to conform more or less to this canon; for they collect many instances to show that two events are connected; but usually neglect to bring out the negative side of the proof; so that their arguments only amount to simple enumeration. Thus Ascham in his Toxophilus, insisting on the national importance of archery, argues that victory has always depended on superiority in shooting; and, to prove it, he shows how the Parthians checked the Romans, Sesostris conquered a great part of the known world, Tiberius overcame Arminius, the Turks established their empire, and the English defeated the French (with many like examples)–all by superior archery. But having cited these cases to his purpose, he is content; whereas he might have greatly strengthened his proof by showing how one or the other instance excludes other possible causes of success. Thus: the cause was not discipline, for the Romans were better disciplined than the Parthians; nor yet the boasted superiority of a northern habitat, for Sesostris issued from the south; nor better manhood, for here the Germans probably had the advantage of the Romans; nor superior civilisation, for the Turks were less civilised than most of those they conquered; nor numbers, nor even a good cause, for the French were more numerous than the English, and were shamefully attacked by Henry V. on their own soil. Many an argument from simple enumeration may thus be turned into an induction of greater plausibility according to the Canon of Agreement.

Still, in the above case, the effect (victory) is so vaguely conceived, that a plurality of causes must be allowed for: although, e.g., discipline did not enable the Romans to conquer the Parthians, it may have been their chief advantage over the Germans; and it was certainly important to the English under Henry V. in their war with the French.

Here is another argument, somewhat similar to the above, put forward by H. Spencer with a full consciousness of its logical character. States that make war their chief object, he says, assume a certain type of organisation, involving the growth of the warrior class and the treatment of labourers as existing solely to sustain the warriors; the complete subordination of individuals to the will of the despotic soldier-king, their property, liberty and life being at the service of the State; the regimentation of society not only for military but also for civil purposes; the suppression of all private associations, etc. This is the case in Dahomey and in Russia, and it was so at Sparta, in Egypt, and in the empire of the Yncas. But the similarity of organisation in these States cannot have been due to race, for they are all of different races; nor to size, for some are small, some large; nor to climate or other circumstances of habitat, for here again they differ widely: the one thing they have in common is the military purpose; and this, therefore, must be the cause of their similar organisation. (Political Institutions.)

By this method, then, to prove that one thing is causally connected with another, say A with p, we show, first, that in all instances of p, A is present; and, secondly, that any other supposable cause of p may be absent without disturbing p. We next come to a method the use of which greatly strengthens the foregoing, by showing that where p is absent A is also absent, and (if possible) that A is the only supposable cause that is always absent along with p. Section 2. The Canon of the Joint Method of Agreement in Presence and in Absence.

If (1) two or more instances in which a phenomenon occurs have only one other circumstance (antecedent or consequent) in common, while (2) two or more instances in which it does not occur (though in important points they resemble the former set of instances) have nothing else in common save the absence of that circumstance–the circumstance in which alone the two sets of instances differ throughout (being present in the first set and absent in the second) is probably the effect, or the cause, or an indispensable condition of the phenomenon.

The first clause of this Canon is the same as that of the method of Agreement, and its significance depends upon the same propositions concerning causation. The second clause, relating to instances in which the phenomenon is absent, depends for its probative force upon Prop. II. (a), and I. (b): its function is to exclude certain circumstances (whose nature or manner of occurrence gives them some claim to consideration) from the list of possible causes (or effects) of the phenomenon investigated. It might have been better to state this second clause separately as the Canon of the Method of Exclusions.

To prove that A is causally related to p, let the two sets of instances be represented as follows:

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Instances of Presence.     Instances of Absence.
   A  B   C                 C    H    F
   p  q   r                 r    x    v
   A  D   E                 B    D    K
   p  s   t                 q    y    s
   A  F   G                 E    G    M
   p  u   v                 t    f    u
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Then A is probably the cause or a condition of p, or p is dependent upon A: first, by the Canon of Agreement in Presence, as represented by the first set of instances; and, secondly, by Agreement in Absence in the second set of instances. For there we see that C, H, F, B, D, K, E, G, M occur without the phenomenon p, and therefore (by Prop. II. (a)) are not its cause, or not the whole cause, unless they have been counteracted (which is a point for further investigation). We also see that r, v, q, s, t, u occur without A, and therefore are not the effects of A. And, further, if the negative instances represent all possible cases, we see that (according to Prop. I. (b)) A is the cause of p, because it cannot be omitted without the cessation of p. The inference that A and p are cause and effect, suggested by their being present throughout the first set of instances, is therefore strengthened by their being both absent throughout the second set.

So far as this Double Method, like the Single Method of Agreement, relies on observation, sequence may not be perceptible in the instances observed, and then, direct causation cannot be proved by it, but only the probability of causal connection; and, again, the real cause, though present, may be so obscure as to evade observation. It has, however, one peculiar advantage, namely, that if the second list of instances (in which the phenomenon and its supposed antecedent are both absent) can be made exhaustive, it precludes any hypothesis of a plurality of causes; since all possible antecedents will have been included in this list without producing the phenomenon. Thus, in the above symbolic example, taking the first set of instances, the supposition is left open that B, C, D, E, F, G may, at one time or another, have been a condition of p; but, in the second list, these antecedents all occur, here or there, without producing p, and therefore (unless counteracted somehow) cannot be a condition of p. A, then, stands out as the one thing that is present whenever p is present, and absent whenever p is absent.

Stated in this abstract way, the Double Method may seem very elaborate and difficult; yet, in fact, its use may be very simple. Tyndall, to prove that dispersed light in the air is due to motes, showed by a number of cases (1) that any gas containing motes is luminous; (2) that air in which the motes had been destroyed by heat, and any gas so prepared as to exclude motes, are not luminous. All the instances are of gases, and the result is: motes–luminosity; no motes–no luminosity. Darwin, to show that cross-fertilisation is favourable to flowers, placed a net about 100 flower-heads, and left 100 others of the same varieties exposed to the bees: the former bore no seed, the latter nearly 3,000. We must assume that, in Darwin's judgment, the net did not screen the flowers from light and heat sufficiently to affect the result.

There are instructive applications of this Double Method in Wallace's Darwinism. In chap. viii., on Colour in Animals, he observes, that the usefulness of their coloration to animals is shown by the fact that, "as a rule, colour and marking are constant in each species of wild animal, while, in almost every domesticated animal, there arises great variability. We see this in our horses and cattle, our dogs and cats, our pigeons and poultry. Now the essential difference between the conditions of life of domesticated and wild animals is, that the former are protected by man, while the latter have to protect themselves." Wild animals protect themselves by acquiring qualities adapted to their mode of life; and coloration is a very important one, its chief, though not its only use, being concealment. Hence a useful coloration having been established in any species, individuals that occasionally may vary from it, will generally, perish; whilst, among domestic animals, variation of colour or marking is subject to no check except the taste of owners. We have, then, two lists of instances; first, innumerable species of wild animals in which the coloration is constant and which depend upon their own qualities for existence; secondly, several species of domestic animals in which the coloration is not constant, and which do not depend upon their own qualities for existence. In the former list two circumstances are present together (under all sorts of conditions); in the latter they are absent together. The argument may be further strengthened by adding a third list, parallel to the first, comprising domestic animals in which coloration is approximately constant, but where (as we know) it is made a condition of existence by owners, who only breed from those specimens that come up to a certain standard of coloration.

Wallace goes on to discuss the colouring of arctic animals. In the arctic regions, he says, some animals are wholly white all the year round, such as the polar bear, the American polar hare, the snowy owl and the Greenland falcon: these live amidst almost perpetual snow. Others, that live where the snow melts in summer, only turn white in winter, such as the arctic hare, the arctic fox, the ermine and the ptarmigan. In all these cases the white colouring is useful, concealing the herbivores from their enemies, and also the carnivores in approaching their prey; this usefulness, therefore, is a condition of the white colouring. Two other explanations have, however, been suggested: first, that the prevalent white of the arctic regions directly colours the animals, either by some photographic or chemical action on the skin, or by a reflex action through vision (as in the chameleon); secondly, that a white skin checks radiation and keeps the animals warm. But there are some exceptions to the rule of white colouring in arctic animals which refute these hypotheses, and confirm the author's. The sable remains brown throughout the winter; but it frequents trees, with whose bark its colour assimilates. The musk-sheep is brown and conspicuous; but it is gregarious, and its safety depends upon its ability to recognise its kind and keep with the herd. The raven is always black; but it fears no enemy and feeds on carrion, and therefore does not need concealment for either defence or attack. The colour of the sable, then, though not white, serves for concealment; the colour of the musk-sheep serves a purpose more important than concealment; the raven needs no concealment. There are thus two sets of instances:–in one set the animals are white (a) all the year, (b) in winter; and white conceals them (a) all the year, (b) in winter; in the other set, the animals are not white, and to them either whiteness would not give concealment, or concealment would not be advantageous. And this second list refutes the rival hypotheses: for the sable, the musk-sheep and the raven are as much exposed to the glare of the snow, and to the cold, as the other animals are. Section 3. The Canon of Difference.

If an instance in which a phenomenon occurs, and an instance in which it does not occur, have every other circumstance in common save one, that one (whether consequent or antecedent) occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable condition of the phenomenon.

This follows from Props. I (a) and (b), in chapter xv. Section 7. To prove that A is a condition of p, let two instances, such as the Canon requires, be represented thus:

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A    B    C    |    B    C
p    q    r    |    q    r
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Then A is the cause or a condition of p. For, in the first instance, A being introduced (without further change), p arises (Prop. I. (a)); and, in the second instance, A having been removed (without other change), p disappears (Prop. I. (b)). Similarly we may prove, by the same instances, that p is the effect of A.

The order of the phenomena and the immediacy of their connection is a matter for observation, aided by whatever instruments and methods of inspection and measurement may be available.

As to the invariability of the connection, it may of course be tested by collecting more instances or making more experiments; but it has been maintained, that a single perfect experiment according to this method is sufficient to prove causation, and therefore implies invariability (since causation is uniform), though no other instances should ever be obtainable; because it establishes once for all the unconditionality of the connection

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A    B    C
p    q    r.
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Now, formally this is true; but in any actual investigation how shall we decide what is a satisfactory or perfect experiment? Such an experiment requires that in the negative instance

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B    C
q    r,
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BC shall be the least assemblage of conditions necessary to co-operate with A in producing p; and that it is so cannot be ascertained without either general prior knowledge of the nature of the case or special experiments for the purpose. So that invariability will not really be inferred from a single experiment; besides that every prudent inquirer repeats his experiments, if only to guard against his own liability to error.

The supposed plurality of causes does not affect the method of Difference. In the above symbolic case, A is clearly one cause (or condition) of p, whatever other causes may be possible; whereas with the Single Method of Agreement, it remained doubtful (admitting a plurality of causes) whether A, in spite of being always present with p, was ever a cause or condition of it.

This method of Difference without our being distinctly aware of it, is oftener than any other the basis of ordinary judgments. That the sun gives light and heat, that food nourishes and fire burns, that a stone breaks a window or kills a bird, that the turning of a tap permits or checks the flow of water or of gas, and thousands of other propositions are known to be true by rough but often emphatic applications of this method in common experience.

The method of Difference may be applied either (1) by observation, on finding two instances (distinct assemblages of conditions) differing only in one phenomenon together with its antecedent or consequent; or (2) by experiment, and then, either (a) by preparing two instances that may be compared side by side, or (b) by taking certain conditions, and then introducing (or subtracting) some agent, supposed to be the cause, to see what happens: in the latter case the "two instances" are the same assemblage of conditions considered before and, again, after, the introduction of the agent. As an example of (a) there is an experiment to show that radium gives off heat: take two glass tubes, in one put some chloride of radium, in both thermometers, and close them with cotton-wool. Soon the thermometer in the tube along with radium reads 54° F. higher than the other one. The tube without the radium, whose temperature remains unaltered, is called the "control" experiment. Most experiments are of the type (b); and since the Canon, which describes two co-existing instances, does not readily apply to this type, an alternative version may be offered: Any agent whose introduction into known circumstances (without further change) is immediately followed by a definite phenomenon is a condition of the occurrence of that phenomenon.

The words into known circumstances are necessary to emphasise what is required by this Method, namely, that the two instances differ in only one thing; for this cannot be ascertained unless all the other conditions are known; and this further implies that they have been prepared. It is, therefore, not true (as Sigwart asserts) that this method determines only one condition of a phenomenon, and that it is then necessary to inquire into the other conditions. If they were not known they must be investigated; but then the experiment would not have been made upon this method. Practically, experiments have to be made in all degrees of imperfection, and the less perfect they are, that is, the less the circumstances are known beforehand, the more remains to be done. A common imperfection is delay, or the occurrence of a latent period between the introduction of an agent and the manifestation of its effects; it cannot then be the unconditional cause; though it may be an indispensable remote condition of whatever change occurs. If, feeling out of sorts, you take a drug and some time afterwards feel better, it is not clear on this ground alone that the drug was the cause of recovery, for other curative processes may have been active meanwhile–food, or sleep, or exercise.

Any book of Physics or of Chemistry will furnish scores of examples of the method of Difference: such as Galileo's experiment to show that air has weight, by first weighing a vessel filled with ordinary air, and then filling it with condensed air and weighing it again; when the increased weight can only be due to the greater quantity of air contained. The melting-point of solids is determined by heating them until they do melt (as silver at 1000° C., gold at 1250°, platinum at 2000°); for the only difference between bodies at the time of melting and just before is the addition of so much heat. Similarly with the boiling point of liquids. That the transmission of sound depends upon the continuity of an elastic ponderable medium, is proved by letting a clock strike in a vacuum (under a glass from which the air has been withdrawn by an air pump), and standing upon a non-elastic pedestal: when the clock be seen to strike, but makes only such a faint sound [Pg 220]as may be due to the imperfections of the vacuum and the pedestal.

The experiments by which the chemical analysis or synthesis of various forms of matter is demonstrated are simple or compound applications of this method of Difference, together with the quantitative mark of causation (that cause and effect are equal); since the bodies resulting from an analysis are equal in weight to the body analysed, and the body resulting from a synthesis is equal in weight to the bodies synthesised. That an electric current resolves water into oxygen and hydrogen may be proved by inserting the poles of a galvanic battery in a vessel of water; when this one change is followed by another, the rise of bubbles from each pole and the very gradual decrease of the water. If the bubbles are caught in receivers placed over them, it can be shown that the joint weight of the two bodies of gas thus formed is equal to the weight of the water that has disappeared; and that the gases are respectively oxygen and hydrogen may then be shown by proving that they have the properties of those gases according to further experiments by the method of Difference; as (e.g.) that one of them is oxygen because it supports combustion, etc.

When water was first decomposed by the electric current, there appeared not only oxygen and hydrogen, but also an acid and an alkali. These products were afterwards traced to impurities of the water and of the operator's hands. Mill observes that in any experiment the effect, or part of it, may be due, not to the supposed agent, but to the means employed in introducing it. We should know not only the other conditions of an experiment, but that the agent or change introduced is nothing else than what it is supposed to be.

In the more complex sciences the method of Difference is less easily applicable, because of the greater difficulty of being sure that only one circumstance at a time has altered; still, it is frequently used. Thus, if by dividing a certain nerve certain muscles are paralysed, it is shown that normally that nerve controls those muscles. That the sense of smell in flies and cockroaches is connected with the antennae has been shown by cutting them off: whereupon the insects can no longer find carrion. In his work on Earthworms, Darwin shows that, though sensitive to mechanical tremors, they are deaf (or, at least, not sensitive to sonorous vibrations transmitted through the air), by the following experiment. He placed a pot containing a worm that had come to the surface, as usual at night, upon a table, whilst close by a piano was violently played; but the worm took no notice of the noise. He then placed the pot upon the piano, whilst it was being played, when the worm, probably feeling mechanical vibrations, hastily slid back into its burrow.

When, instead of altering one circumstance in an instance (which we have done our best not otherwise to disturb) and then watching what follows, we try to find two ready-made instances of a phenomenon, which only differ in one other circumstance, it is, of course, still more difficult to be sure that there is only one other circumstance in which they differ. It may be worth while, however, to look for such instances. Thus, that the temperature of ocean currents influences the climate of the shores they wash, seems to be shown by the fact that the average temperature of Newfoundland is lower than that of the Norwegian coast some 15° farther north. Both regions have great continents at their back; and as the mountains of Norway are higher and capped with perennial snow, we might expect a colder climate there: but the shore of Norway is visited by the Gulf Stream, whilst the shore of Newfoundland is traversed by a cold current from Greenland. Again, when in 1841 the railway from Rouen to Paris was being built, gangs of English and gangs of French workmen were employed upon it, and the English got through about one-third more work per man than the French. It was suspected that this difference was due to one other difference, namely, that the English fed better, preferring beef to thin soup. Now, logically, it might have been objected that the evidence was unsatisfactory, seeing that the men differed in other things besides diet–in 'race' (say), which explains so much and so easily. But the Frenchmen, having been induced to try the same diet as the English, were, in a few days, able to do as much work: so that the "two instances" were better than they looked. It often happens that evidence, though logically questionable, is good when used by experts, whose familiarity with the subject makes it good. Section 4. The Canon Of Concomitant Variations.

Whatever phenomenon varies in any manner whenever another phenomenon (consequent or antecedent) varies in some particular manner [no other change having concurred] is either the cause or effect of that phenomenon [or is connected with it through some fact of causation].

This is not an entirely fresh method, but may be regarded as a special case either of Agreement or of Difference, to prove the cause or effect, not of a phenomenon as a whole, but of some increment of it (positive or negative). There are certain forces, such as gravitation, heat, friction, that can never be eliminated altogether, and therefore can only be studied in their degrees. To such phenomena the method of Difference cannot be applied, because there are no negative instances. But we may obtain negative instances of a given quantity of such a phenomenon (say, heat), and may apply the method of Difference to that quantity. Thus, if the heat of a body increases 10 degrees, from 60 to 70, the former temperature of 60 was a negative instance in respect of those 10 degrees; and if only one other circumstance (say, friction) has altered at the same time, that circumstance (if an antecedent) is the cause. Accordingly, if in the above Canon we insert, after 'particular manner,' "[no other change having concurred,]" it is a statement of the method of Difference as applicable to the increment of a phenomenon, instead of to the phenomenon as a whole; and we may then omit the last clause–"[or is connected, etc.]." For these words are inserted to provide for the case of co-effects of a common cause (such as the flash and report of a gun); but if no other change (such as the discharge of a gun) has concurred with the variations of two phenomena, there cannot have been a common cause, and they are therefore cause and effect.

If, on the other hand, we omit the clause "[no other change having concurred,]" the Canon is a statement of the method of Agreement as applicable to the increment of a phenomenon instead of to the phenomenon as a whole; and it is then subject to the imperfections of that method: that is to say, it leaves open the possibilities, that an inquirer may overlook a plurality of causes; or may mistake a connection of two phenomena, which (like the flash and report of a gun) are co-effects of a common cause, for a direct relation of cause and effect.

It may occur to the reader that we ought also to distinguish Qualitative and Quantitative Variations as two orders of phenomena to which the present method is applicable. But, in fact, Qualitative Variations may be adequately dealt with by the foregoing methods of Agreement, Double Agreement, and Difference; because a change of quality or property entirely gets rid of the former phase of that quality, or substitutes one for another; as when the ptarmigan changes from brown to white in winter, or as when a stag grows and sheds its antlers with the course of the seasons. The peculiar use of the method of Variations, however, is to formulate the conditions of proof in respect of those causes or effects which cannot be entirely got rid of, but can be obtained only in greater or less amount; and such phenomena are or course, quantitative.

Even when there are two parallel series of phenomena the one quantitative and the other qualitative–like the rate of air-vibration and the pitch of sound, or the rate of ether-vibration and the colour-series of the spectrum–the method of Variations is not applicable. For (1) two such series cannot be said to vary together, since the qualitative variations are heterogeneous: 512: 576 is a definite ratio; but the corresponding notes, C, D, in the treble clef, present only a difference. Hence (2) the correspondence of each note with each number is a distinct fact. Each octave even is a distinct fact; there is a difference between C 64 and C 128 that could never have been anticipated without the appropriate experience. There is, therefore, no such law of these parallel series as there is for temperature and change of volume (say) in mercury. Similar remarks apply to the physical and sensitive light-series.

We may illustrate the two cases of the method thus (putting a dash against any letter, A’ or p’, to signify an increase or decrease of the phenomenon the letter stands for): Agreement in Variations (other changes being admissible)–

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A   B   C       A'  D   E       A'' F   G
p   q   r       p'  s   t       p'' u   v
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Here the accompanying phenomena (B C q r, D E s t, F G u v) change from time to time, and the one thing in which the instances agree throughout is that any increase of A (A’ or A’‘) is followed or accompanied by an increase of p (p’ or p’‘): whence it is argued that A is the cause of p, according to Prop. III. (a) (ch. xv. Section

7). Still, it is supposable that, in the second instance, D or E may be the cause of the increment of p; and that, in the third instance, F or G may be its cause: though the probability of such vicarious causation decreases rapidly with the increase of instances in which A and p vary together. And, since an actual investigation of this type must rely on observation, it is further possible that some undiscovered cause, X, is the real determinant of both A and p and of their concomitant variations.

Professor Ferri, in his Criminal Sociology, observes: "I have shown that in France there is a manifest correspondence of increase and decrease between the number of homicides, assaults and malicious wounding, and the more or less abundant vintage, especially in the years of extraordinary variations, whether of failure of the vintage (1853-5, 1859, 1867, 1873, 1878-80), attended by a remarkable diminution of crime (assaults and wounding), or of abundant vintages (1850, 1856-8, 1862-3, 1865, 1868, 1874-5), attended by an increase of crime" (p. 117, Eng. trans.). And earlier he had remarked that such crimes also "in their oscillations from month to month display a characteristic increase during the vintage periods, from June to December, notwithstanding the constant diminution of other offences" (p. 77). This is necessarily an appeal to the canon of Concomitant Variations, because France is never without her annual vintage, nor yet without her annual statistics of crime. Still, it is an argument whose cogency is only that of Agreement, showing that probably the abuse of the vintage is a cause of crimes of violence, but leaving open the supposition, that some other circumstance or circumstances, arising or varying from year to year, may determine the increase or decrease of crime; or that there is some unconsidered agent which affects both the vintage and crimes of violence. French sunshine, it might be urged, whilst it matures the generous grape, also excites a morbid fermentation in the human mind.

Difference in Variations may be symbolically represented thus (no other change having concurred):

```
    A    B          A'   B          A''  B
    p    q,         p'   q,         p''  q.
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Here the accompanying phenomena are always the same B/q; and the only point in which the successive instances differ is in the increments of A (A', A'') followed by corresponding increments of p (p', p''): hence the increment of A is the cause of the increment of p.

For examples of the application of this method, the reader should refer to some work of exact science. He will find in Deschanel's Natural Philosophy, c. 32, an account of some experiments by which the connection between heat and mechanical work has been established. It is there shown that "whenever work is performed by the agency of heat" [as in driving an engine], "an amount of heat disappears equivalent to the work performed; and whenever mechanical work is spent in generating heat" [as in rubbing two sticks together], "the heat generated is equivalent to the work thus spent." And an experiment of Joule's is described, which consisted in fixing a rod with paddles in a vessel of water, and making it revolve and agitate the water by means of a string wound round the rod, passed over a pulley and attached to a weight that was allowed to fall. The descent of the weight was measured by a graduated rule, and the rise of the water's temperature by a thermometer. "It was found that the heat communicated to the water by the agitation amounted to one pound-degree Fahrenheit for every 772 foot-pounds of work" expended by the falling weight. As no other material change seems to take place during such an experiment, it shows that the progressive expenditure of mechanical energy is the cause of the progressive heating of the water.

The thermometer itself illustrates this method. It has been found that the application of heat to mercury expands it according to a law; and hence the volume of the mercury, measured by a graduated index, is used to indicate the temperature of the air, water, animal body, etc., in which the thermometer is immersed, or with which it is brought into contact. In such cases, if no other change has taken place, the heat of the air, water, or body is the cause of the rise of the mercury in its tube. If some other substance (say spirit) be substituted for mercury in constructing a thermometer, it serves the same purpose, provided the index be graduated according to the law of the expansion of that substance by heat, as experimentally determined.

Instances of phenomena that do not vary together indicate the exclusion of a supposed cause (by Prop. III (c)). The stature of the human race has been supposed to depend on temperature; but there is no correspondence. The “not varying together,” however, must not be confused with “varying inversely,” which when regular indicates a true concomitance. It is often a matter of convenience whether we regard concomitant phenomena as varying directly or inversely. It is usual to say–’the greater the friction the less the speed’; but it is really more intelligible to say–’the greater the friction the more rapidly molar is converted into molecular motion.’

The Graphic Method exhibits Concomitant Variations to the eye, and is extensively used in physical and statistical inquiries. Along a horizontal line (the abscissa) is measured one of the conditions (or agents) with which the inquiry is concerned, called the Variable; and along perpendiculars (ordinates) is measured some phenomenon to be compared with it, called the Variant.

Thus, the expansion of a liquid by heat may be represented by measuring degrees of temperature along the horizontal, and the expansion of a column of the liquids in units of length along the perpendicular.

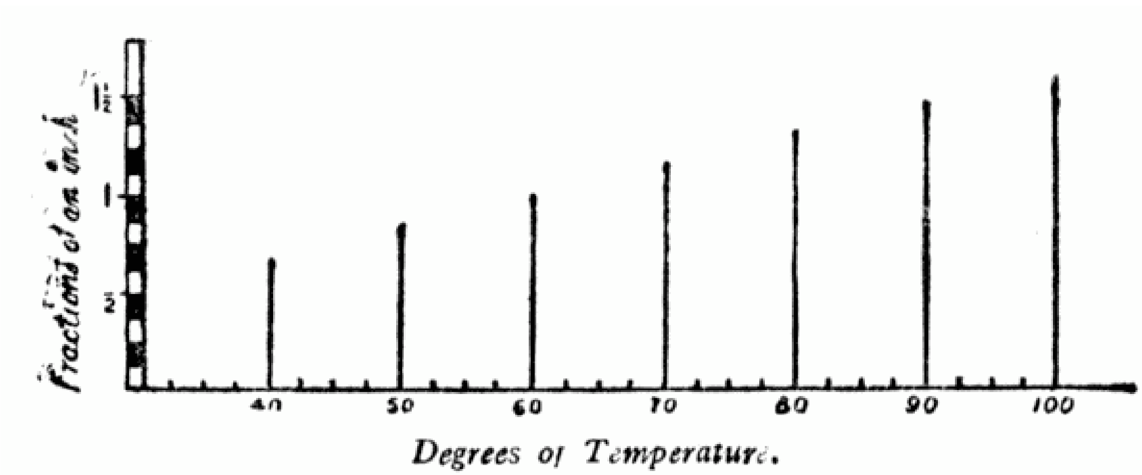

Fig. 9.

In the next diagram (Fig. 10), reduced from one given by Mr. C.H. Denyer in an article on the Price of Tea (Economic Journal, No. 9), the condition measured horizontally is Time; and, vertically, three variants are measured simultaneously, so that their relations to one another from time to time may be seen at a glance. From this it is evident that, as the duty on tea falls, the price of tea falls, whilst the consumption of tea rises; and, in spite of some irregularity of correspondence in the courses of the three phenomena, their general causal connection can hardly be mistaken. However, the causal connection may also be inferred by general reasoning; the statistical Induction can be confirmed by a Deduction; thus illustrating the combined method of proof to be discussed in the next chapter. Without such confirmation the proof by Concomitant Variations would not be complete; because, from the complexity of the circumstances, social statistics can only yield evidence according to the method of Agreement in Variations. For, besides the agents that are measured, there may always be some other important influence at work. During the last fifty years, for example, crime has decreased whilst education has increased: true, but at the same time wages have risen and many other things have happened.

Diagram showing (1)—the average Price of Tea (in bond), but with duty added per lb.; (2)— the rate of Duty; (3)— the consumption per head, from 1809 to 1889.

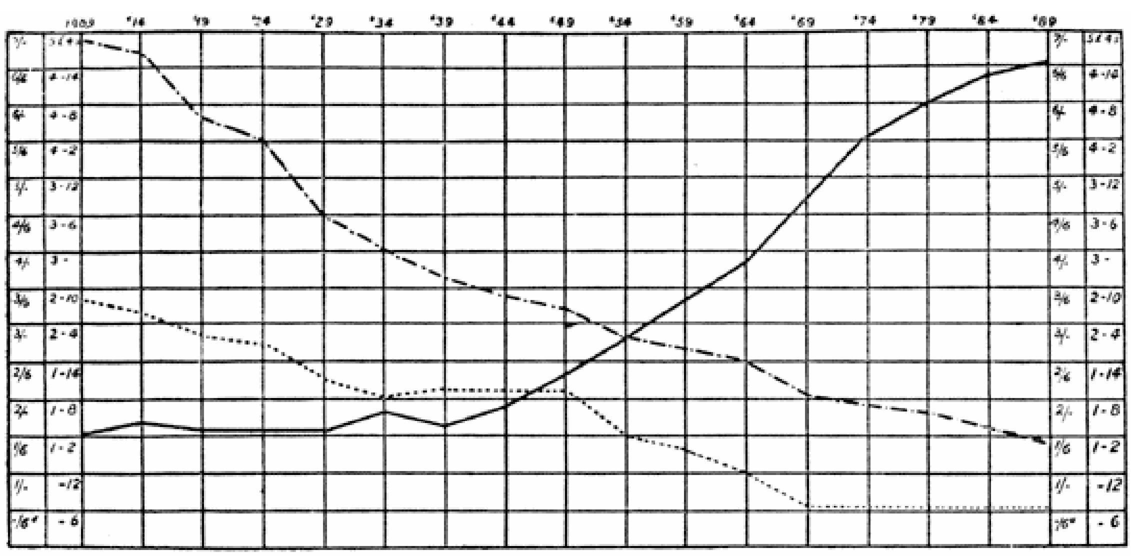

Fig. 10.

One horizontal space = 5 years. One vertical space = 6 pence, or 6 ounces.

It will be noticed that in the diagram the three lines, especially those of Price and Consumption (which may be considered natural resultants, in contrast with the arbitrary fixation of a Tax), do not depart widely from regular curves; and accordingly, assuming the causes at work to vary continuously during the intervals between points of measurement, curves may be substituted. In fact, a curve often represents the course of a phenomenon more truthfully than can be done by a line that zigzags along the exact measurements; because it is less influenced by temporary and extraordinary causes that may obscure the operation of those that are being investigated. On the other hand, the abrupt deviations of a punctilious zigzag may have their own logical value, as will appear in the next section.

In working with the Method of Variations one must allow for the occurrence in a series of 'critical points,' at which sudden and sometimes heterogeneous changes may take place. Every substance exists at different temperatures in three states, gaseous, liquid, solid; and when the change takes place, from one state to another, the series of variations is broken. Water, e.g., follows the general law that cooling is accompanied by decrease of volume between 212° and 39° F.: but above 212°, undergoes a sudden expansion in becoming a gas; and below 39° begins to expand, until at 32° the expansion is considerable on its becoming solid. This illustrates a common experience that concomitant variations are most regular in the 'median range,' and are apt to become irregular at the extremities of the series, where new conditions begin to operate.

The Canon of Variations, again, deals not with sudden irruptions of a cause, force or agent, but with some increase or decrease of an agent already present, and a corresponding increase or decrease of some other phenomenon–say an increase of tax and a rise of price. But there are cases in which the energy of a cause is not immediately discharged and dissipated. Whilst a tax of 6d. per lb. on tea raises the price per lb. by about 6d., however long it lasts, the continuous application of friction to a body may gradually raise its temperature to the point of combustion; because heat is received faster than it is radiated, and therefore accumulates. Such cases are treated by Mill under the title of ‘progressive effects’ (Logic: B. III., c. 15): he gives as an example of it the acceleration of falling bodies. The storage of effects is a fact of the utmost importance in all departments of nature, and is especially interesting in Biology and Sociology, where it is met with as heredity, experience, tradition. Evolution of species of plants and animals would (so far as we know) be impossible, if the changes (however caused) that adapt some individuals better than others to the conditions of life were not inherited by, and accumulated in, their posterity. The eyes in the peacock’s tail are supposed to have reached their present perfection gradually, through various stages that may be illustrated by the ocelli in the wings of the Argus pheasant and other genera of PhasianidÃ_. Similarly the progress of societies would be impossible without tradition, whereby the improvements made in any generation may be passed on to the next, and the experience of mankind may be gradually accumulated in various forms of culture. The earliest remains of culture are flint implements and weapons; in which we can trace the effect of tradition in the lives of our remote forefathers, as they slowly through thousands of years learnt to improve the chipping of flints, until the first rudely shaped lumps gave place to works of unmistakable design, and these to the beautiful weapons contemporary with the Bronze Age.

The Method of Gradations, the arranging of any phenomena to be studied in series, according to the degree in which some character is exhibited, is, perhaps, the most definite device in the Art of Discovery. (Bain: Induction, c. 6, and App. II.) If the causes are unknown it is likely to suggest hypotheses: and if the causes are partly known, variation in the character of the series is likely to indicate a corresponding variation of the conditions. Section 5. The Canon Of Residues.

Subduct from any phenomenon such part as previous inductions have shown to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

The phenomenon is here assumed to be an effect: a similar Canon may be framed for residuary causes.

This also is not a fresh method, but a special case of the method of Difference. For if we suppose the phenomenon to be p q r, and the antecedent to be A B C, and that we already know B and C to have (either severally or together) the consequents q r, in which their efficacy is exhausted; we may regard

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B    C
q    r
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as an instance of the absence of p obtained deductively from the whole phenomenon

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A    B    C
p    q    r
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by our knowledge of the laws of B and C; so that

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A    B    C
p    q    r
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is an instance of the presence of p, differing otherwise from

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B    C
q    r
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in nothing except that A is also present. By the Canon of Difference, therefore A is the cause of p. Or, again, when phenomena thus treated are strictly quantitative, the method may be based on Prop. III. (b), ch. xv. Section 7.

Of course, if A can be obtained apart from B C and directly experimented with so as to produce p, so much the better; and this may often be done; but the special value of the method of Residues appears, when some complex phenomenon has been for the most part accounted for by known causes, whilst there remains some excess, or shortcoming, or deviation from the result which those causes alone would lead us to expect, and this residuary fact has to be explained in relation to the whole. Here the negative instance is constituted by deduction, showing what would happen but for the interference of some unknown cause which is to be investigated; and this prominence of the deductive process has led some writers to class the method as deductive. But we have seen that all the Canons involve deduction; and, considering how much in every experiment is assumed as already known (what circumstances are 'material,' and when conditions may be called 'the same'), the wonder is that no one has insisted upon regarding every method as concerned with residues. In fact, as scientific explanation progresses, the phenomena that may be considered as residuary become more numerous and the importance of this method increases.

Examples: The recorded dates of ancient eclipses having been found to differ from those assigned by calculation, it appears that the average length of a day has in the meanwhile increased. This is a residuary phenomenon not accounted for by the causes formerly recognised as determining the rotation of the earth on its axis; and it may be explained by the consideration that the friction of the tides reduces the rate of the earth's rotation, and thereby lengthens the day. Astronomy abounds in examples of the method of Residues, of which the discovery of Neptune is the most famous.

Capillarity seems to be a striking exception to the principle that water (or any liquid) 'finds its level,' that being the condition of equilibrium; yet capillarity proves to be only a refined case of equilibrium when account is taken of the forces of adhesion exerted by different kinds of bodies in contact.

"Many of the new elements of Chemistry," says Herschel, "have been detected in the investigation of residual phenomena." Thus, Lord Rayleigh and Sir W. Ramsay found that nitrogen from the atmosphere was slightly heavier than nitrogen got from chemical sources; and, seeking the cause of this difference, discovered argon.

The Economist shows that when a country imports goods the chief means of paying for them is to export other goods. If this were all, imports and exports would be of equal value: yet the United Kingdom imports about £400,000,000 annually, and exports about £300,000,000. Here, then, is a residuary phenomenon of £100,000,000 to be accounted for. But foreign countries owe us about £50,000,000 for the use of shipping, and £70,000,000 as interest on the capital we have lent them, and £15,000,000 in commissions upon business transacted for them. These sums added together amount to £135,000,000; and that is £35,000,000 too much. Thus another residuary phenomenon emerges; for whilst foreigners seem to owe us £435,000,000 they only send us £400,000,000 of imports. These £35,000,000 are accounted for by the annual investment of our capital abroad, in return for which no immediate payment is due; and, these being omitted, exports and imports balance. Since this was written the figures of our foreign trade have greatly risen; but the character of the explanation remains the same.

When, in pursuing the method of Variations, the phenomena compared do not always correspond in their fluctuations, the irregular movements of that phenomenon which we regard as the effect may often be explained by treating them as residuary phenomena, and then seeking for exceptional causes, whose temporary interference has obscured the influence of the general cause. Thus, returning to the diagram of the Price of Tea in Section 4, it is clear that generally the price falls as the duty falls; but in Mr. Denyer's more minutely wrought diagram, from which this is reduced, it may be seen that in 1840 the price of tea rose from 3s. 9d. to 4s. 9d. without any increase of duty. This, however, is readily explained by the Chinese War of that year, which checked the supply. Again, from 1869 to 1889 the duty was constant, whilst the price of tea fell as much as 8d. per lb.; but this residuary phenomenon is explained by the prodigiously increased production of tea during that period in India and Ceylon.

The above examples of the method of Residues are all quantitative; but the method is often employed where exact estimates are unobtainable. Thus Darwin, having found certain modifications of animals in form, coloration and habits, that were not clearly derivable from their struggle for existence in relation to other species or to external conditions, suggested that they were due to Sexual Selection.

The 'vestiges' and 'survivals' so common in Biology and Sociology are residuary phenomena. It is a general inference from the doctrine of Natural Selection that every organ of a plant, animal, or society is in some way useful to it. There occur, however, organs that have at present no assignable utility, are at least wasteful, and sometimes even injurious. And the explanation is that formerly they were useful; but that, their uses having lapsed, they are now retained by the force of heredity or tradition. Either they are not injurious enough to be eliminated by natural selection; or they are correlated with other organs, whose utility outweighs their disutility.