

A Short Account of the History of Mathematics



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Rene Descartes

We may consider Descartes as the first of the modern school of mathematics. René Descartes was born near Tours on March 31, 1596, and died at Stockholm on February 11, 1650; thus he was a contemporary of Galileo and Desargues. His father, who, as the name implies, was of good family, was accustomed to spend half the year at Rennes when the local parliament, in which he held a commission as councillor, was in session, and the rest of the time on his family estate of Les Cartes at La Haye. René, the second of a family of two sons and one daughter, was sent at the age of eight years to the Jesuit School at La Flèche, and of the admirable discipline and education there given he speaks most highly. On account of his delicate health he was permitted to lie in bed till late in the mornings; this was a custom which he always followed, and when he visited Pascal in 1647 he told him that the only way to do good work in mathematics and to preserve his health was never to allow anyone to make him get up in the morning before he felt inclined to do so; an opinion which I chronicle for the benefit of any schoolboy into whose hands this work may fall.

On leaving school in 1612 Descartes went to Paris to be introduced to the world of fashion. Here, through the medium of the Jesuits, he made the acquaintance of Mydorge, and renewed his schoolboy friendship with Mersenne, and together with them he devoted the two years of 1615 and 1616 to the study of mathematics. At that time a man of position usually entered either the army or the church; Descartes chose the former profession, and in 1617 joined the army of Prince Maurice of Orange, then at Breda. Walking through the streets there he saw a placard in Dutch which excited his curiosity, and stopping the first passer, asked him to translate it into either French or Latin. The stranger, who happened to be Isaac Beeckman, the head of the Dutch College at Dort, offered to do so if Descartes would answer it; the placard being, in fact, a challenge to all the world to solve a certain geometrical problem. Descartes worked it out within a few hours, and a warm friendship between him and Beeckman was the result. This unexpected test of his mathematical attainments made the uncongenial life of the army distasteful to him, but under family influence and tradition he remained a soldier, and was persuaded at the commencement of the Thirty Years' War to volunteer under Count de Bucquoy in the army of Bavaria. He continued

all this time to occupy his leisure with mathematical studies, and was accustomed to date the first ideas of his new philosophy and of his analytical geometry from three dreams which he experienced on the night of November 10, 1619, at Neuberg, when campaigning on the Danube. He regarded this as the critical day of his life, and one which determined his whole future.

He resigned his commission in the spring of 1621, and spent the next five years in travel, during most of which time he continued to study pure mathematics. In 1626 we find him settled at Paris, “a little well-built figure, modestly clad in green taffety, and only wearing sword and feather in token of his quality as a gentleman.” During the first two years there he interested himself in general society, and spent his leisure in the construction of optical instruments; but these pursuits were merely the relaxations of one who failed to find in philosophy that theory of the universe which he was convinced finally awaited him.

In 1628 Cardinal de Berulle, the founder of the Oratorians, met Descartes, and was so much impressed by his conversation that he urged on him the duty of devoting his life to the examination of truth. Descartes agreed, and the better to secure himself from interruption moved to Holland, then at the height of his power. There for twenty years he lived, giving up all his time to philosophy and mathematics. Science, he says, may be compared to a tree; metaphysics is the root, physics is the trunk, and the three chief branches are mechanics, medicine, and morals, these forming the three applications of our knowledge, namely, to the external world, to the human body, and to the conduct of life.

He spend the first four years, 1629 to 1633, of his stay in Holland in writing *Le Monde*, which embodies an attempt to give a physical theory of the universe; but finding that its publication was likely to bring on him the hostility of the church, and having no desire to pose as a martyr, he abandoned it: the incomplete manuscript was published in 1664. He then devoted himself to composing a treatise on universal science; this was published at Leyden in 1637 under the title *Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences*, and was accompanied with three appendices (which possibly were not issued till 1638) entitled *La Dioptrique*, *Les Météores*, and *La Géométrie*; it is from the last of these that the invention of analytical geometry dates. In 1641 he published a work called *Meditationes*, in which he explained at some length his views on philosophy as sketched out in the *Discours*. In 1644 he issued the *Principia Philosophiae*, the greater part of which was devoted to physical science, especially the laws of motion and the theory of vortices. In 1647 he received a pension from the French court in honour of his

discoveries. He went to Sweden on the invitation of the Queen in 1649, and died a few months later of inflammation of the lungs.

In appearance, Descartes was a small man with large head, projecting brow, prominent nose, and black hair coming down to his eyebrows. His voice was feeble. In disposition he was cold and selfish. Considering the range of his studies he was by no means widely read, and he despised both learning and art unless something tangible could be extracted therefrom. He never married, and left no descendants, though he had one illegitimate daughter, who died young.

As to his philosophical theories, it will be sufficient to say that he discussed the same problems which have been debated for the last two thousand years, and probably will be debated with equal zeal two thousand years hence. It is hardly necessary to say that the problems themselves are of importance and interest, but from the nature of the case no solution ever offered is capable either of rigid proof or of disproof; all that can be effected is to make one explanation more probable than another, and whenever a philosopher like Descartes believes that he has at last finally settled a question it has been possible for his successors to point out the fallacy in his assumptions. I have read somewhere that philosophy has always been chiefly engaged with the inter-relations of God, Nature, and Man. The earliest philosophers were Greeks who occupied themselves mainly with the relations between God and Nature, and dealt with Man separately. The Christian Church was so absorbed in the relation of God to Man as entirely to neglect Nature. Finally, modern philosophers concern themselves chiefly with the relations between Man and Nature. Whether this is a correct historical generalization of the views which have been successively prevalent I do not care to discuss here, but the statement as to the scope of modern philosophy marks the limitations of Descartes's writings.

Descartes's chief contributions to mathematics were his analytical geometry and his theory of vortices, and it is on his researches in connection with the former of these subjects that his mathematical reputation rests.

Analytical geometry does not consist merely (as is sometimes loosely said) in the application of algebra to geometry; that had been done by Archimedes and many others, and had become the usual method of procedure in the works of the mathematicians of the sixteenth century. The great advance made by Descartes was that he saw that a point in a plane could be completely determined if its distances, say x and y , from two fixed lines drawn at right angles in the plane were given, with the convention familiar to us as to the interpretation of positive and negative values; and that though an equation $f(x,y) = 0$ was indeterminate and could be satisfied by an infinite number of values of x and y , yet these values of x and y determined the co-

ordinates of a number of points which form a curve, of which the equation $f(x,y) = 0$ expresses some geometrical property, that is, a property true of the curve at every point on it. Descartes asserted that a point in space could be similarly determined by three co-ordinates, but he confined his attention to plane curves.

It was at once seen that in order to investigate the properties of a curve it was sufficient to select, as a definition, any characteristic geometrical property, and to express it by means of an equation between the (current) co-ordinates of any point on the curve, that is, to translate the definition into the language of analytical geometry. The equation so obtained contains implicitly every property of the curve, and any particular property can be deduced from it by ordinary algebra without troubling about the geometry of the figure. This may have been dimly recognized or foreshadowed by earlier writers, but Descartes went further and pointed out the very important facts that two or more curves can be referred to one and the same system of co-ordinates, and that the points in which two curves intersect can be determined by finding the roots common to their two equations. I need not go further into details, for nearly everyone to whom the above is intelligible will have read analytical geometry, and is able to appreciate the value of its invention.

Descartes's *Géométrie* is divided into three books: the first two of these treat of analytical geometry, and the third includes an analysis of the algebra then current. It is somewhat difficult to follow the reasoning, but the obscurity was intentional. "Je n'ai rien omis," says he, "qu'à dessein ... j'avois prévu que certaines gens qui se vantent de sçavoir tout n'auroient par manqué de dire que je n'avois rien écrit qu'ils n'eussent sçu auparavant, si je me fusse rendu assez intelligible pour eux."

The first book commences with an explanation of the principles of analytical geometry, and contains a discussion of a certain problem which had been propounded by Pappus in the seventh book of his $\square\square\square\square\square\square\square\square$ and of which some particular cases had been considered by Euclid and Apollonius. The general theorem had baffled previous geometers, and it was in the attempt to solve it that Descartes was led to the invention of analytical geometry. The full enunciation of the problem is rather involved, but the most important case is to find the locus of a point such that the product of the perpendiculars on m given straight lines shall be in a constant ratio to the product of the perpendiculars on n other given straight lines. The ancients had solved this geometrically for the case $m = 1, n = 1$, and the case $m = 1, n = 2$. Pappus had further stated that, if $m = n = 2$, the locus is a conic, but he gave no proof; Descartes also failed to prove this by pure geometry, but he shewed that the curve is represented by an equation of the second degree, that is, a conic; subsequently Newton gave an elegant solution of the problem by pure geometry.



In the second book Descartes divides curves into two classes, namely, geometrical and mechanical curves. He defines geometrical curves as those which can be generated by the intersection of two lines each moving parallel to one co-ordinate axis with “commensurable” velocities; by which terms he means that dy/dx is an algebraical function, as, for example, is the case in the ellipse and the cissoid. He calls a curve mechanical when the ratio of the velocities of these lines is “incommensurable”; by which term he means that dy/dx is a transcendental function, as, for example, is the case in the cycloid and the quadratrix. Descartes confined his discussion to geometrical curves, and did not treat of the theory of mechanical curves. The classification into algebraical and transcendental curves now usual is due to Newton.

Descartes also paid particular attention to the theory of the tangents to curves - as perhaps might be inferred from his system of classification just alluded to. The then current definition of a tangent at a point was a straight line through the point such that between it and the curve no other straight line could be drawn, that is, the straight line of closest contact. Descartes proposed to substitute for this a statement equivalent to the assertion that the tangent is the limiting position of the secant; Fermat, and at a later date Maclaurin and Lagrange, adopted this definition. Barrow, followed by Newton and Leibnitz, considered a curve as the limit of an inscribed polygon when the sides become indefinitely small, and stated that the side of the polygon when produced became in the limit a tangent to the curve. Roberval, on the other hand, defined a tangent at a point as the direction of motion at that instant of a point which was describing the curve. The results are the same whichever definition is selected, but the controversy as to which definition was the correct one was none the less lively. In his letters Descartes illustrated his theory by giving the general rule for drawing tangents and normals to a roulette.

The method used by Descartes to find the tangent or normal at any point of a given curve was substantially as follows. He determined the centre and radius of a circle which should cut the curve in two consecutive points there. The tangent to the circle at that point will be the required tangent to the curve. In modern text-books it is usual to express the condition that two of the points in which a straight line (such as $y = mx + c$) cuts the curve shall coincide with the given point: this enables us to determine m and c , and thus the equation of the tangent there is determined. Descartes, however, did not venture to do this, but selecting a circle as the simplest curve and one to which he knew how to draw a tangent, he so fixed his circle as to make it touch the given curve at the point in question, and thus reduced the problem to drawing a tangent to a circle. I should note in passing that he only applied this method to curves which are symmetrical about an axis, and he took the centre of the circle on the axis.



The obscure style deliberately adopted by Descartes diminished the circulation and immediate appreciation of these books; but a Latin translation of them, with explanatory notes, was prepared by F. de Beaune, and an edition of this, with a commentary by F. van Schooten, issued in 1659, was widely read.

The third book of the *Géométrie* contains an analysis of the algebra then current, and it has affected the language of the subject by fixing the custom of employing the letters at the beginning of the alphabet to denote known quantities, and those at the end of the alphabet to denote unknown quantities. [On the origin of the custom of using x to represent an unknown example, see a note by G. Eneström in the *Bibliotheca Mathematica*, 1885, p. 43.] Descartes further introduced the system of indices now in use; very likely it was original on his part, but I would here remind the reader that the suggestion had been made by previous writers, though it had not been generally adopted. It is doubtful whether or not Descartes recognized that his letters might represent any quantities, positive or negative, and that it was sufficient to prove a proposition for one general case. He was the earliest writer to realise the advantage to be obtained by taking all the terms of an equation to one side of it, though Stifel and Harriot had sometimes employed that form by choice. He realised the meaning of negative quantities and used them freely. In this book he made use of the rule for finding the limit to the number of positive and of negative roots of an algebraical equation, which is still known by his name; and introduced the method of indeterminate coefficients for the solution of equations. He believed that he had given a method by which algebraical equations of any order could be solved, but in this he was mistaken. It may also be mentioned that he enunciated the theorem, commonly attributed to Euler, on the relation between the numbers of faces, edges and angles of a polyhedron: this is in one of the papers published by Careil.

Of the two other appendices to the *Discours* one was devoted to optics. The chief interest of this consists in the statement given of the law of refraction. This appears to have been taken from Snell's work, though, unfortunately, it is enunciated in a way which might lead a reader to suppose that it is due to the researches of Descartes. Descartes would seem to have repeated Snell's experiments when in Paris in 1626 or 1627, and it is possible that he subsequently forgot how much he owed to the earlier investigations of Snell. A large part of the optics is devoted to determining the best shape for the lenses of a telescope, but the mechanical difficulties in grinding a surface of glass to a required form are so great as to render these investigations of little practical use. Descartes seems to have been doubtful whether to regard the rays of light as proceeding from the eye and so to speak touching the object, as the Greeks had done,

or as proceeding from the object, and so affecting the eye; but, since he considered the velocity of light to be infinite, he did not deem the point particularly important.

The other appendix, on meteors, contains an explanation of numerous atmospheric phenomena, including the rainbow; the explanation of the latter is necessarily incomplete, since Descartes was unacquainted with the fact that the refractive index of a substance is different for lights of different colours.

Descartes's physical theory of the universe, embodying most of the results contained in his earlier and unpublished *Le Monde*, is given in his *Principia*, 1644, and rests on a metaphysical basis. He commences with a discussion on motion; and then lays down ten laws of nature, of which the first two are almost identical with the first two laws of motion as given by Newton; the remaining eight laws are inaccurate. He next proceeds to discuss the nature of matter which he regards as uniform in kind though there are three forms of it. He assumes that the matter of the universe must be in motion, and that the motion must result in a number of vortices. He states that the sun is the centre of an immense whirlpool of this matter, in which the planets float and are swept round like straws in a whirlpool of water. Each planet is supposed to be the centre of a secondary whirlpool by which its satellites are carried: these secondary whirlpools are supposed to produce variations of density in the surrounding medium which constitute the primary whirlpool, and so cause the planets to move in ellipses and not in circles. All these assumptions are arbitrary and unsupported by any investigation. It is not difficult to prove that on his hypothesis the sun would be in the centre of these ellipses, and not at a focus (as Kepler had shewn was the case), and that the weight of a body at every place on the surface of the earth except the equator would act in a direction which was not vertical; but it will be sufficient here to say that Newton in the second book of his *Principia*, 1687, considered the theory in detail, and showed that its consequences are not only inconsistent with each of Kepler's laws and with the fundamental laws of mechanics, but are also at variance with the laws of nature assumed by Descartes. Still, in spite of its crudeness and its inherent defects, the theory of vortices marks a fresh era in astronomy, for it was an attempt to explain the phenomena of the whole universe by the same mechanical laws which experiment shows to be true on the earth.